A loading-rate dependent cohesive model to simulate concrete fracture

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Abstract

This paper presents a cohesive model endorsed with a viscous term dependent on the crack-opening rate. It can be used to simulate fracture processes that are sensitive to the loading rate. The model is implemented in a finite-element based program using the smeared crack tip method developed by Planas & Elices [which is thoroughly described in [1] G. Ruiz, Int. J. of Fracture 111: 265-282, 2001]. The model is validated against three point bending tests on notched beams of a high-strength concrete performed at five different displacement rates. The results obtained with the model match very well the experimental ones, particularly the maximum load and the work of fracture for the several rates used in the tests. The model has also been used to perform a parametric study that shows that the dependencies of the work of fracture to the size of the specimen and to the length of the ligament are mainly attributable to rate effects.

1 Introduction

The strengthening of quasi-brittle materials like concrete and ceramics at dynamic loading conditions [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] can be attributed to micro-inertia [13, 14] and its moisture content [15, 16]. At relatively low loading rates, i.e., smaller than approximately 10/s, the free water in the micropores exhibits the so-called Stefan effect, which gradually gains a dominant role compared to the inertia effect in causing the strength increase of concrete. In order to explain the loading-rate effect observed in the experiments performed at those relatively low loading rates [17], we propose a viscous cohesive model implemented in a smeared crack tip framework proposed by Planas and Elices [18] and thoroughly explained in [1]. The solution is sought from a triangular system of equations obtained by superposing linear elastic fracture mechanic (LEFM) cases.

The paper is organized as follows. In Section 2, we present the time dependent model with an explanation of the hypothesis established and the method calculation. Afterwards, the model is validated against the experimental results of three-point-bend specimens of HSC. Parametric studies on combined rate and size effect, notch sensitivity on work of fracture are also performed in Section 3. Finally the conclusions are drawn in Section 4.

2 The Time Dependent Cohesive Model

Taking into account the dependence on crack-opening rate, we propose a viscous cohesive-law as follows

$$\sigma = \left[1 + \left(\frac{\dot{w}}{\dot{w_0}} \right)^n \right] f(w) \tag{1}$$

where $\dot{w_0}$ is a reference crack-opening velocity, *n* stands for the rate-dependent index, which describes the degree of viscosity of the material. The function $f(w)$ defines a general static cohesive law. The case of $f(w)$ being linear is shown in Fig. 1. Note that upon initiation, the crack opens at a zero velocity, therefore, obeys the static cohesive law (dotted line in the figure). As the crack-opening speed increases, it takes subsequently the cohesive curve corresponding to its current opening velocity.

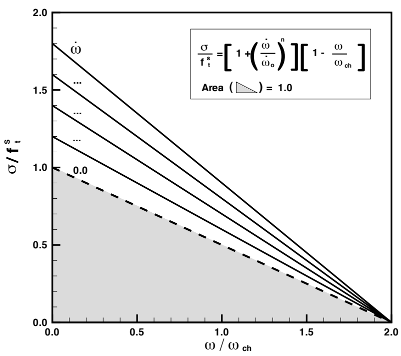

Figure 1: A linear rate-depente cohesive law, where $w_{ch} = G_F / f_t^s$.

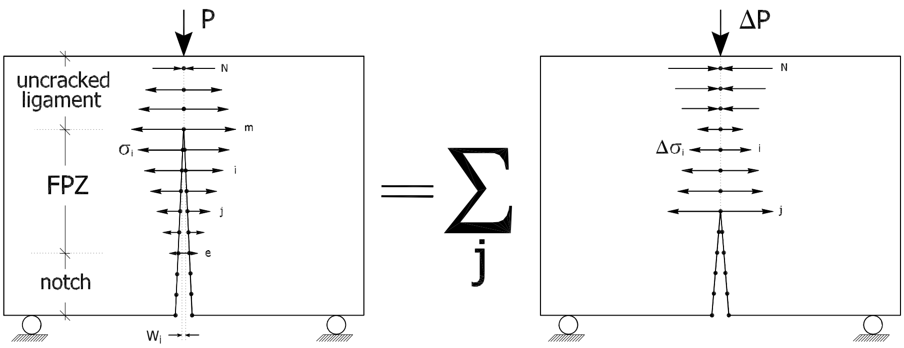

Figure 2: The smeared crack-tip model.

The rate-dependent cohesive law in Eqn.1 is implemented in the framework of the smeared crack-tip method, which considers a cohesive crack as a superposition of a series of stress-free crack in a linear-elastic body, see Fig. 2. The

Table 1: Hardened HSC mechanical properties.

	$E_c^{(a)}$ (GPa)	$f_t^{s,(b)}$ (MPa)	$G_F^{s,(c)}$ (N/m)	$\ell_{ch}^{(d)}$ (mm)	$\beta_H^{(e)}$	ν
Mean	33.9	5.2	128	160.47	0.6232	0.25
Std	1.2	0.5	-	-	-	-

(a) cylinder axial compression tests;
(b) splitting cylinder (Brazilian) tests;
(c) obtained by curve adjust of experimental data [17];
(d) $\ell_{ch} = E_c G_F^s/(f_t^s)^2$;
(d) $\beta_H = D/\ell_{ch}$.

distributions of nodal stresses, nodal crack opening and the load-displacement curve can be obtained through a coefficient matrix RD, known a priori through any commercial finite element code, for example ANSYS. The nodal crack opening velocity is obtained through the rate relation

$$\dot{w} = \frac{w_j - w_{j-1}}{\delta_j - \delta_{j-1}} \dot{\delta} \tag{2}$$

where $\dot{}$ represents the rate, index j corresponds to variables at loading step j.

3 Numerical Results and Discussion

3.1 Material Characterization

Independent tests were performed to characterize the high strength concrete used in the experimental program. The main mechanical properties are summarized in Table 1. Measured mean values and standard deviation are also shown. Note that the static value of the apparent fracture energy, $G_F = G_F^s$, considered a true material property, was obtained by curve adjust of experimental data from the means values of the specific fracture energy calculated for each set of specimens subject to the same loading applied rate [17].

3.2 Numerical validation

As aforementioned, we validate the current model against the experimental results performed on notched HSC concrete beams in TPB configuration, according Ruiz et al. [17]. All of the calculations have been performed using the beam geometry shown in Fig.3, except in the parametric study presented in Subsection 3.3 where the geometry dimensions are proportionally scaled to account for size effect in the peak load varying the Hillerborg's brittleness number β_H [19]. The coefficients of the matrix RD was computed using a finite element mesh with six-node triangular elements and the middle section was discretized into 100 equal divisions (i.e. N=100 in Fig. 2). A bilinear cohesive law recommended i [20] is adopted for the validation calculations.

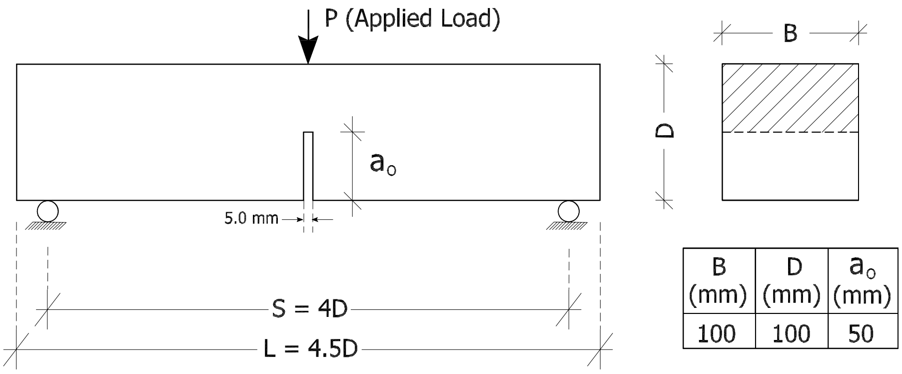

Figure 3: The geometry for a three-point-bend beam.

The rate dependence index, $n = 0.16$, was obtained through curve fitting of the experimental non-dimensional peak load as a function of the loading applied rate. Subsequently the parameter $\dot{w}_0 = 9.65 \times 10^{-2}$ [mm][s]$^{-1}$ was calculated in a semi-inverse manner by fitting it with the priori n. The numerical and experimental comparison for peak load and work of fracture is shown in Fig.4. Almost perfect agreement is shown for peaks loads and work of fracture at relatively higher loading range. The discrepancy for the work of fracture may be attributed to drying of concrete as the test duration covered around 8 hours at the slowest case.

The complete load-displacement curve comparison is shown in Fig.5 for four of the five tested loading rates. We observe that the proposed numerical model faithfully reproduce the experimental results.

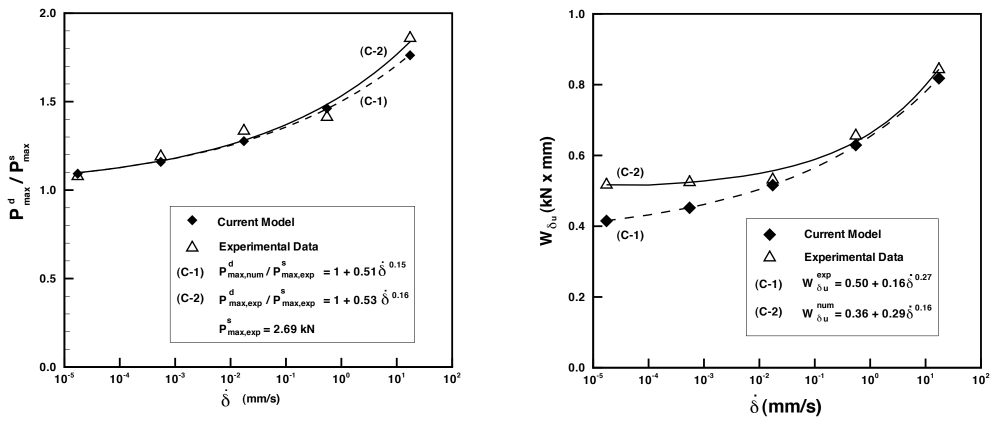

Figure 4: Curves adjusting and comparison between predicted and experimental results for the work of fracture ($W_{\delta u}$) at different load loading rates.

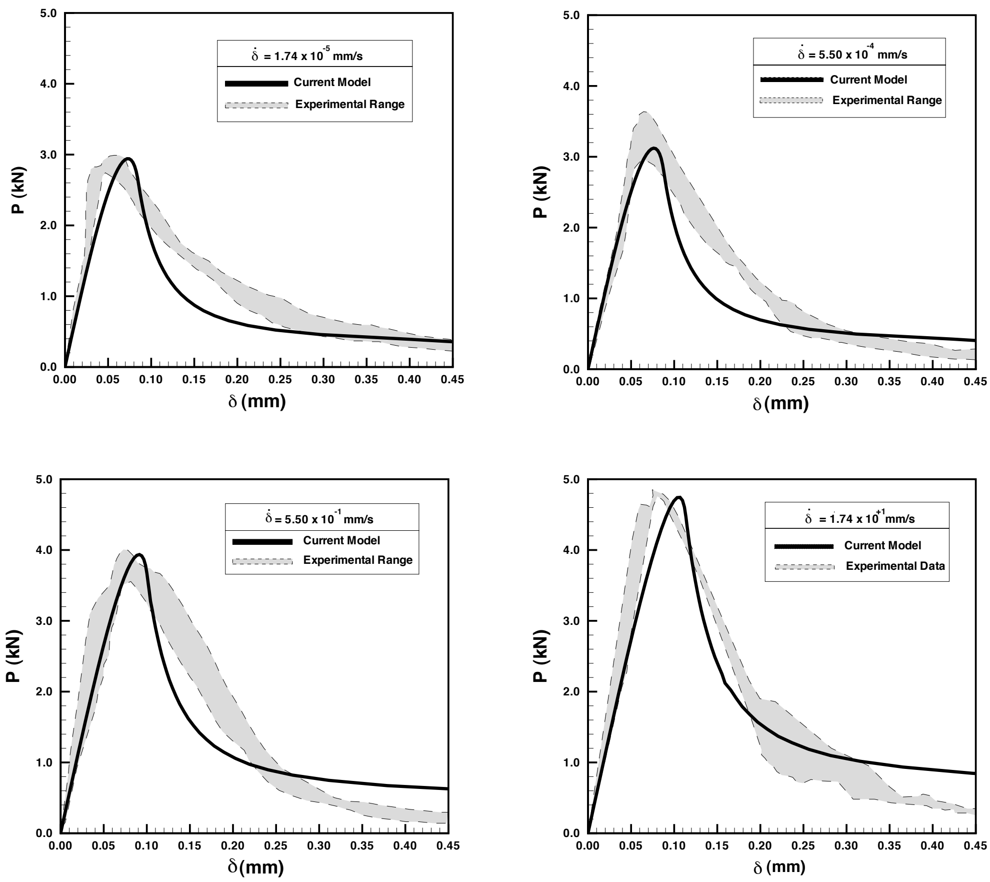

Figure 5: Numerical and experimental load vs. displacement curves at different loading rates.

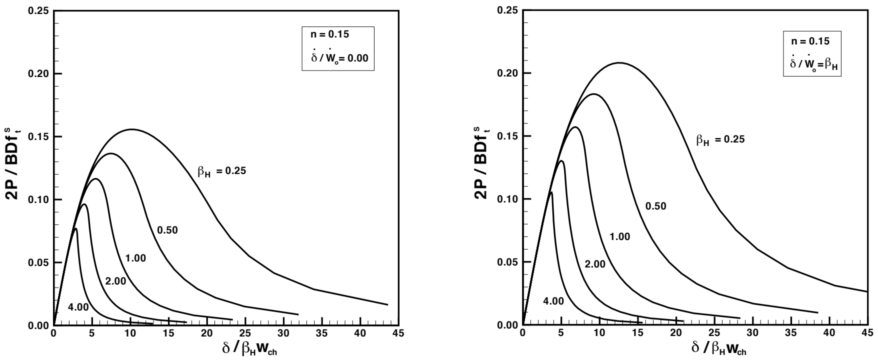

Figure 6: Combined size and rate effect

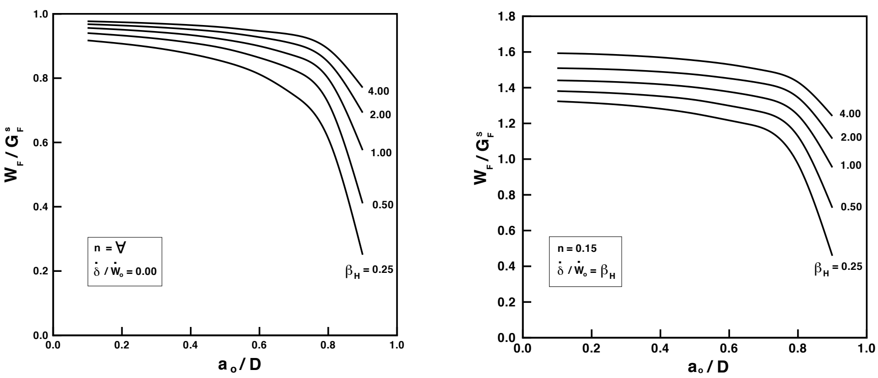

Figure 7: Notch sensitivity on the work of fracture.

3.3 Parametric studies

In Fig.6, we plot the non-dimensional load-displacement curves varying Hillerborg's brittleness number β_H from 0.25 to 4 for static ($\dot{\delta}/\dot{w}_0 = 0$) and dynamic cases (set $\dot{\delta}/\dot{w}_0 = \beta_H$ and fix the rate index n as 0.15). Note that the non-dimensional strength decreases with the size for both cases, even though the largest beam $\beta_H = 4$ was loaded with a rate which is 16 times of that of the smallest beam ($\beta_H = 0.25$). In other words, in the range of loading rate studied, the size effect plays a dominant rule compared to the rate effect.

In Fig.7, we present the sensitivity of the notch size on the work of fracture normalized by the specific fracture energy in both static and dynamic cases. Note that on one hand, in order to facilitate the fracture localization, the notch size should not be too small; on the other hand, it should not be larger than 50% of the beam depth in order to avoid considerable errors in measuring fracture energy. This verifies the recommendations of RILEM [21] that the notch size should be between 15% to 50% of the beam size when using this kind of loading configuration to measure specific fracture energy. For the dynamic cases, due to the rate effect, the work of fracture is significantly larger than the static fracture energy, therefore ceases to be a material parameter.

4 Summary and Conclusions

We have implemented a viscous cohesive model in the framework of a smeared-crack-tip method. The methodology is validated against high strength concrete (HSC) beams in three point bending test (TPBT) configuration. A parametric study had also been realized to demonstrated how the parameters related with the time dependent model reflect the fracture response of the material. Based on the findings of this research, the following conclusions can be drawn:

- The results numerically obtained match very well the experimental ones, particularly the maximum load and the work of fracture for the several loading rates used in the tests.
- The model can explicitly represent the rate dependency of the peak load and of the work of fracture, such that the peak load and the apparent work of fracture increase monotonically with increasing loading rate.
- Based in the parametric study we can conclude that the work of fracture per unit area, W_F, increases significantly with the loading rate.
- Finally the numerical results shown that the model, in spite of its simplicity, provides a general approach to reflect the experimentally documented fact that the crack propagation and so the parameters governing the fracture process are dependent on the loading rate.

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