Cyclic Elasto-Plastic Fracture Diagram for a Specimen with Crack

L.A. Sosnovskiy 1, A.V. Bogdanovich 2, S.S. Sherbakov 3
1 S&P Group TRIBO-FATIGUE, Gomel, Belarus; 2 Grodno Yanka Kupala State University, Grodno, Belarus; 3 Belarusian State University Minsk, Belarus

1. Introduction

Analytical and experimental method for the estimation of crack growth resistance under cyclic elasto-plastic deformation based on measuring of local plastic strain near the crack tip and is proposed.

2. Diagram Construction Method

Considering that plastic steel was subjected to test an estimation of applicability of basic formulas of linear elastic fracture mechanics was made. Observance of flat deformation conditions was checked by criteria [1-4]:

\[ K_I \leq K^*_I = \sqrt{\frac{t_0 \sigma_{0.2}^2}{2.5}}; \]
\[ \psi = \frac{t_0 - t_{\phi}}{t_0} \cdot 100 \% \leq 1.5 \%. \]

where \( K_I \) is stress intensity factor (SIF); \( t_0 \) is a nominal thickness of the compact specimen; \( t_{\phi} \) is a thickness of the compact specimen with the account of elasto-plastic strains; \( \sigma_{0.2} \) is yield strength (offset = 0.2%) of a material; \( \psi \) is relative contraction of cross-section of the specimen (Fig. 1).

It has appeared that conditions Eq. (1) and Eq. (2) are not satisfied for the investigated steel in the upper part of the fatigue crack growth diagram. Formulas of linear elastic fracture mechanics for the estimation of SIF value of the standard compact tension specimen [1-3]

\[ K_{I_{\text{max}}} = \frac{P_{\text{max}} \sqrt{l}}{t_0 B} \cdot Y\left(\frac{l}{B}\right), \]

where \( P_{\text{max}} \) is the maximum load of a cycle; \( l \) is the measured length of a crack; \( t_0, B \) are the sizes of a dangerous section of the specimen (Fig. 1); \( Y\left(\frac{l}{B}\right) \) is the correction function which considers geometry of the specimen and its scheme of loading:

\[ Y\left(\frac{l}{B}\right) = 29.6 - 185.5\left(\frac{l}{B}\right) + 655.7\left(\frac{l}{B}\right)^2 - 1017\left(\frac{l}{B}\right)^3 + 638.9\left(\frac{l}{B}\right)^4, \]

is also correct for elastic deformation under preservation of flat deformation conditions. In order to apply them to elasto-plastic domain it is necessary to correct them for plasticity.

It can be realized by taking into account in Eq. (4) the actual sizes of dangerous cross-section of the specimen, i.e. those sizes that take place under plastic deformation [5-8].
Let us multiply and divide the relation \( l / B \) by \( t_0 \) value; thus \( l / B = (l / B)(t_\psi / t_0) \). Means, \( Y \left( l / B \right) = Y \left[ (l / B)(t_\psi / t_0) \right] \). At elastic deformation this equality is identical. Taking into account plastic deformation of dangerous cross-section of the specimen in function \( Y \) it is necessary to accept the actual thickness \( t_\psi = t_0 - \varphi \) of the specimen, where \( \varphi \) is a lateral component of plastic strain (contraction) of cross-section, i.e. we can write [5]:

\[
Y \left( \frac{l}{B} \frac{t_\psi}{t_0} \right) = Y \left( \frac{F_I}{F_0} \right) = Y(\omega_F),
\]

(5)

where \( F_0 \) is the nominal (before deformation) area of dangerous cross-section of the specimen; \( F_I \) is the area damaged by a crack with a length \( l \) and defined with the account of the plastic deformation of cross-section. It means that by introduction of Eq. (5) into Eq. (3) and Eq. (4) we obtain a technique of SIF calculation for elasto-plastic domain [5-8]:

\[
K_{I_{\max}}^F = \frac{P_{\max}}{t_0 \sqrt{B}} \frac{\omega_F^{1/2}}{\omega_F^{1/2} Y(\omega_F)};
\]

(3a)

\[
Y(\omega_F) = 29,6 - 185,5(\omega_F) + 655,7(\omega_F)^2 - 1017(\omega_F)^3 + 638,9(\omega_F)^4.
\]

(4a)
Thus Eq. (4a) considers not only geometry of the specimen and its scheme of loading but also integrally the size of plastic strain in dangerous cross-section. And in Eq. (3a) the local measure of damage of a specimen with a crack $\omega_F = F_1 / F_0$ that has not only geometrical meaning but also physical content is introduced. This measure unambiguously defines the life of an object with a crack [9]. It should also be stressed out that the measure $\omega_F$ is defined taking into account plastic strain of dangerous cross-section.

According to the developed approach [5-8] whole process of elasto-plastic deformation and destruction are described by means of the cyclic elasto-plastic fracture diagram for a specimen with a crack (CEPF-diagram). This diagram is built in SIF coordinates $K_i^F$ and absolute $\varphi$ or relative $\psi$ contraction. Lateral component of plastic strain of the specimen in the zone of crack growth (contraction) is defined as a difference between nominal $t_0$ and actual $t_\varphi$ values of thickness of the specimen, i.e. $\varphi = t_0 - t_\varphi$ (see Fig. 1,a); its relative value is $\psi = \varphi / t_0$. Thus SIF $K_i^F$ is calculated using formulas of linear elastic fracture mechanics, but adjusted for plasticity of the investigated material. For example Eq. (3a), (4a) are used for calculation of SIF of the compact specimen (see Fig. 1,a).

There are two types of CEPF-diagram [7-8]. If for calculation of $K_i^F$ value we conditionally accept that the maximum load in rupture process remains constant (and it is really possible if the test machine is rigid enough or loading rate is high), then $OBCS$ diagram (see Fig. 1, b) is obtained which resembles letter $D$ taking into account ordinates axis. Therefore it is named $D$-diagram. If for calculation of $K_i^F$ we consider decrease of loading in rupture process of a specimen (when the test machine has rather low rigidity or the rate of loading is low) diagram $OB_1C_1S_1$ (see Fig. 1, b) is obtained. As the form of this diagram reminds letter $Q$ it is named $Q$-diagram.

The CEPF-diagram generally consists of two curves: a curve of cyclic elasto-plastic destruction (sections $OBC$ in $D$-diagram and $OB_1C_1$ in $Q$-diagram) and a curve of quasi-static destruction (rupture) (sections $CS$ in $D$-diagram and $C_1S_1$ in $Q$-diagram). In points $C$ and $C_1$ the crack reaches the critical size $l_c$ to which limiting contraction $\varphi_c$ and limiting SIF value - cyclic fracture toughness (values of $K_{Fc}$ in $D$-diagram and $K_{Fc}^*$ in $Q$-diagram) correspond. There is a division of the specimen into two parts in corresponding points $S$ and $S_1$ , thus takes place a maximum limiting widening $\varphi_s$ it’s dangerous cross-section on which we define other limiting SIF value - the quasi-static fracture toughness (size $K_{Fsc}$ in the $D$-diagram; $K_i^F = 0$ in this point in $Q$-diagram). Crossing of $CS$ curve with an axis of ordinates gives one more parameter of crack growth resistance $K_0^F$ (see Fig. 1, b). The maximum of $Q$-diagram on an SIF axis (point $B_1$) corresponds to the beginning of cyclic rupture and is characterized by parameter $K_a^F$; parameter $K_{cv}^F$
in the $Q$-diagram corresponds to the beginning of quasi-static rupture; it is not a characteristic point of this diagram, but it corresponds to the beginning of sharp lifting of curve $OBC$ (a point $B$ in the $D$-diagram). In a case of "ideally plastic fracture" the curve of cyclic elasto-plastic destruction is transformed to a straight line 1. In a case of "ideally brittle fracture" ($\phi = 0$) this curve coincides with an axis of ordinates. The line 2 divides areas of quasi-brittle and elasto-plastic destructions. Thus the analysis of is viscous-brittle transition, for example, at change of the sizes of a specimen or test temperature is possible by means of CEPF-diagram.

It is offered three expressions for the analytical description of $OBC$ curve at $D$-diagram [5, 7-8]. The first is a power equation

$$K_i^F = K_{th}^F \cdot \varphi^{m_1}$$  \hspace{1cm} (6)

where $m_1$ is a parameter of cyclic hardening ($0 \leq m_1 \leq 1$); $K_{th}^F$ is a plasticity threshold, i.e. SIF value below which the plastic strains in a crack top do not influence its value. Parameters $m_1$ and $K_{th}^F$ are defined on experimental dependence in co-ordinates $\lg K_{lmax}^F - \lg (\varphi / \varphi_{th})$.

The second dependence for the description of a curve of cyclic elasto-plastic destruction $OBC$ looks like:

$$K_i^F = K_w \left( \frac{\psi - \psi_t}{\psi_c - \psi} \right)^{m_2} \quad \text{if} \quad \psi_t < \psi < \psi_c,$$  \hspace{1cm} (7)

where $K_w$ is the parameter which is subject to definition; $m_2$ is parameter of hardening; $\psi_t$ is a relative contraction of a specimen, corresponding to the beginning of yield of a material at an axial tension. If $K_i^F = K_w$, $2 \psi = \psi_c + \psi_t$ or $\psi = (\psi_c + \psi_t) / 2$. Hence parameter $K_w$ is such SIF value which corresponds to relative size of contraction $\psi = (\psi_c + \psi_t) / 2$. And as $\psi_t \to 0$ and for plastic materials $\psi_c >> \psi_t$ so parameter $K_w$ can be defined for them as such value $K_i^F$ which corresponds to half of limiting contraction ($\psi_c / 2$). Practically value $K_w$ is defined also as value corresponding to value $\lg [(\psi - \psi_t) / (\psi_c - \psi)] = 0$ at representation of $OBC$ curve (see Fig. 1, b) in co-ordinates $\lg K_i^F - \lg [(\psi - \psi_t) / (\psi_c - \psi)]$, and value of parameter $m_2$ can be found from the same graph as a tangent of a angle of an inclination of the received straight line to an axis of abscises.

For obtaining the third expression it is accepted that experimental points in an average part of $OBC$ part of the $D$-diagram are approximated by a straight line in co-ordinates $\lg K_i^F - \lg (\psi / \psi_t)$. The equation of this straight line at transition to usual co-ordinates is transformed to power dependence of a kind

$$K_i^F = K_t \left( \frac{\psi}{\psi_t} \right)^{m_3}$$  \hspace{1cm} (8)
where $K_t$, $m_3$ are parameters. Practically value $K_t$ is defined on a point of crossing of the specified straight line with an axis of ordinates in double logarithmic coordinates, and value of parameter $m_3$ is found as a tangent of an angle of an inclination of this straight line to an axis of abscises.

Fig. 2 shows CEPF-diagrams (analytical description of which was given above) for compact specimens of different thickness made of the plastic carbonic steel. Influence of specimen sizes on deformation characteristics of crack growth resistance is visible on Fig. 2. And in Fig. 3 the same diagrams are combined in the form of one dependence SIF-specimen contraction by means of the offered similarity transformation. It is shown how the stated approach can be used for an estimation of pipes survivability.

Fig. 2. CEPF-diagrams for the carbonic steel constructed by the results of tests of compact samples of 10 (1), 20 (2) and 40 (3) mm thickness.
Fig. 3. Generalized CEPF-diagrams for the carbonic steel constructed by the results of tests of compact samples of 10 (1), 20 (2) and 40 (3) mm thickness

References