FRACTURE MECHANICS MODELLING OF MULTIPLE SITE DAMAGE SCENARIOS

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Abstract

Quantitative risk assessment (QRA) is now required by most aircraft structural integrity programs. One of the key components of an accurate QRA resides in the accuracy of the fracture mechanics modelling of multiple site fatigue damage (MSD) scenarios, which is used to generate the MSD crack growth curves.

This paper presents the results on the development of fracture mechanics models for MSD problems in order to carry out a QRA. To address this complex problem, NRC developed a crack growth analysis (CGA) software, which is capable of calculating the β-solutions for stress intensity factors and the remaining life of MSD in built-up structures (aircraft wing panels). The β-solutions were calculated by superposition of several closed-form and numerical solutions. Comparison between NRC's CGA β-solutions and p-version finite element analysis (FEA) results for various MSD scenarios are presented in this paper. Good agreement was obtained between NRC's CGA results and FEA results.

1. Introduction

Structural risk assessment has now become a critical decision making tool to balance safety, maintenance cost, and availability of aircraft fleets by providing qualitative (i.e. frequent, remote, extremely improbable, etc.) and quantitative (i.e. $>1x10^{-3}$, $< 1x10^{-7}$, etc.) probability of failure.

Structural damages in ageing aircraft are often characterized by the simultaneous presence of fatigue cracks in the same structural component, which is referred to as multiple site fatigue damage (MSD). The capability to accurately and efficiently predict the fatigue life of aircraft structural elements with MSD is therefore essential to perform a quantitative risk assessment (QRA) study. Compared to single crack growth analysis, MSD analysis is more complicated due to the coexistence of multiple cracks within the same structural element and their interaction on each other.

One of the key steps in QRA is to perform probabilistic fracture mechanics analyses by considering the variability of the initial flaw size, crack growth rate, fracture toughness, and maximum stress per flight. One of the popular methods for determining the probably of failure is to perform a Monte Carlo simulation, which involve performing multiple (e.g. 10^6) deterministic fracture mechanics analyses with different initial parameters (initial flaw sizes, material properties, maximum stress, etc.). A simple calculation shows that performing 10^6 simulations at 1 minute per simulation would require almost 2 years of computational time, which is not acceptable. Therefore, the deterministic fracture mechanics model has to be sufficiently fast to keep the computational time to a manageable level. For this reason, numerical approaches involving direct computation using boundary and/or finite element methods were excluded in favour of closed-form and tabular solutions.

In this paper, a methodology is presented to perform fracture mechanics analyses of MSD. The developed methodology was integrated into a crack growth analysis (CGA) software, which is capable of calculating the β-solutions for stress intensity factors and the remaining life of MSD in built-up structures (aircraft wing panels). The results from the CGA software were compared with the p-version finite element analyses through a case study.

2. Methodology

The developed methodology aimed to address the MSD problems in panels with a large number of holes and cracks emanating from the holes, while providing fast and accurate β-solution results for every crack tips. This was achieved by using a combination of closed-form and table look-up β-factors.

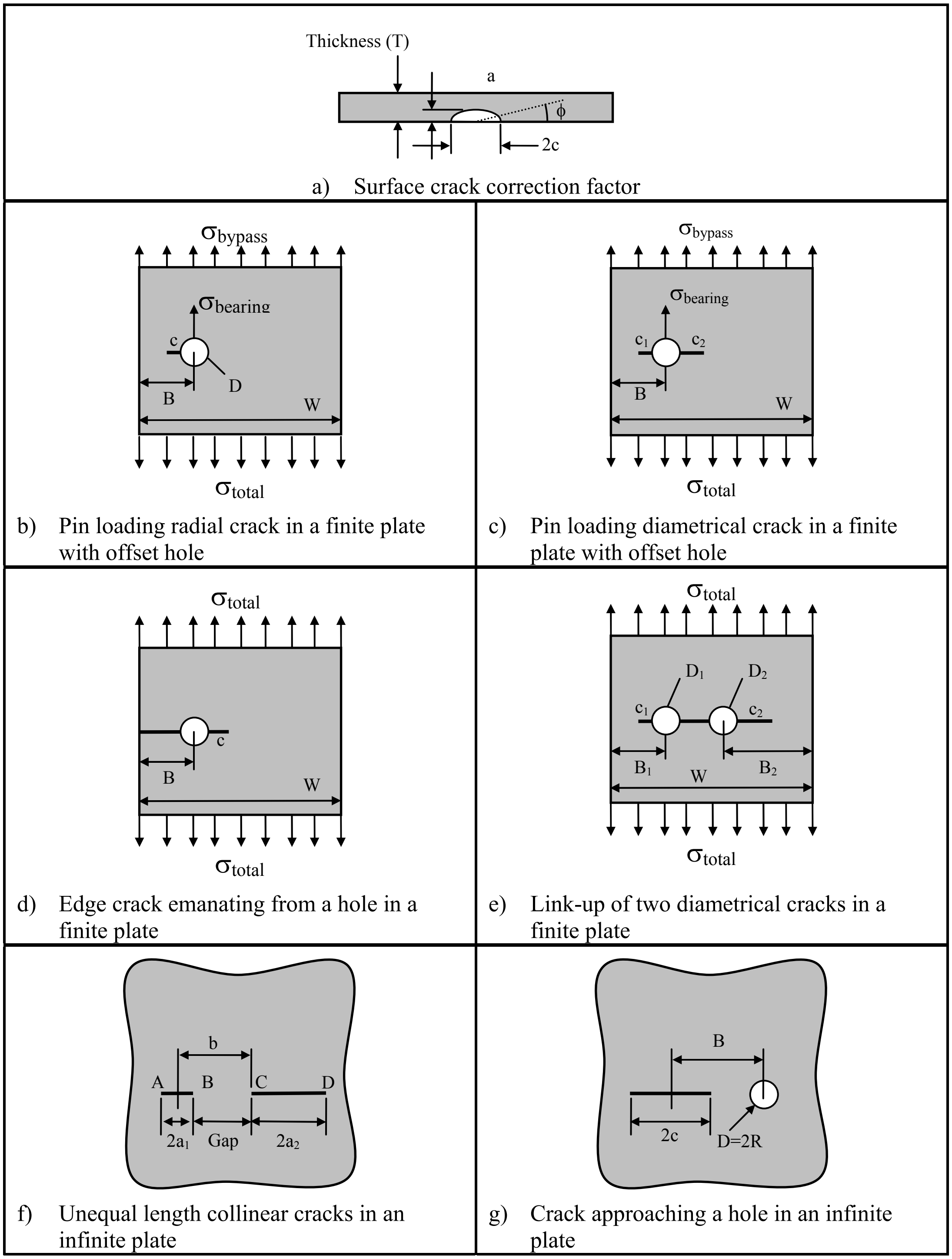
A β-factor library was developed to calculate the β-solutions using the principle of superposition. A minimum set of 7 β-factors, illustrated in Figure 1, was found to be sufficient to cover all possible MSD cases where cracks were nucleated from the holes. Some β-factors illustrated in Figure 1, such as the surface crack [1], the collinear cracks [2], and the crack approaching a hole [3], were directly obtained from the literature. Other β-factors, were obtained by the principle of superposition using various published β-factors such as the edge crack, the unsymmetrical crack, the radial or diametrical cracks emanating from a hole with and without pin load, and others [4-10].

Table look-up method was used for processing some complex closed-form solutions such as the collinear cracks and the crack approaching a hole. The table look-up method uses the pre-computed β-factors stored in a *n*-dimensional table to calculate the β-factor by spline interpolation. This method ensured fast execution of the QRA study during the Monte Carlo simulation.

The β-factor compounding method requires proper identification of the crack information. An algorithm was developed to identify and update the crack information, which includes extracting the location and size of crack, determining if the crack is an edge or a centre crack, determining if the crack is a part-through or through crack, determining the type of interaction with adjacent structural elements (edge, hole, or other cracks), verifying if a crack link-up occurs, and merging the cracks if applicable.

The crack link-up was predicted using the "intuitive link-up criterion" proposed by Swift [11]. This link-up criterion stipulates that ligament failure would occur when the plastic zone of the lead crack touches the plastic zone of adjacent crack.

Other criteria, such as the critical crack tip opening angle (CTOA) [12] and the T-integral [13] have been proposed to predict the crack link-up but require advanced elasto-plastic finite element analyses.



a) Surface crack correction factor

b) Pin loading radial crack in a finite plate with offset hole

c) Pin loading diametrical crack in a finite plate with offset hole

d) Edge crack emanating from a hole in a finite plate

e) Link-up of two diametrical cracks in a finite plate

f) Unequal length collinear cracks in an infinite plate

g) Crack approaching a hole in an infinite plate

Figure 1. Schematic representation of the β-factors used to solve MSD problem by the principle of superposition.

The β-factors compounding method flow chart is provided in Figure 2. The compounding method includes two main tasks: the calculation of the crack interaction effect (Figure 1f) and the superposition of other β-factors.

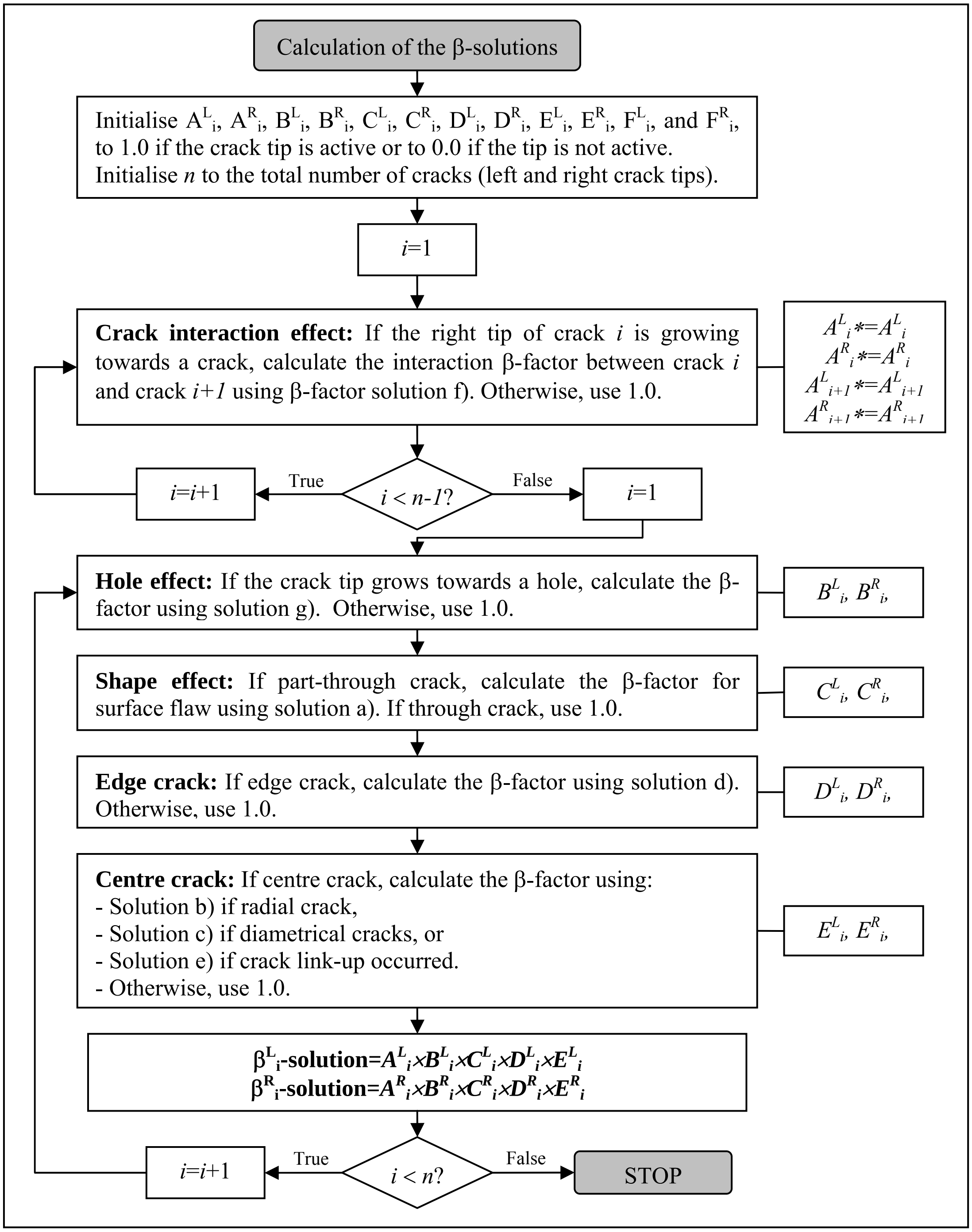


Figure 2. Generic β-solution compounding methodology for MSD problems.

The integration of the β-factors compounding method into the MSD crack growth analysis flow process is presented in Figure 3. This methodology was implemented into the CGA software developed by the NRC Canada.

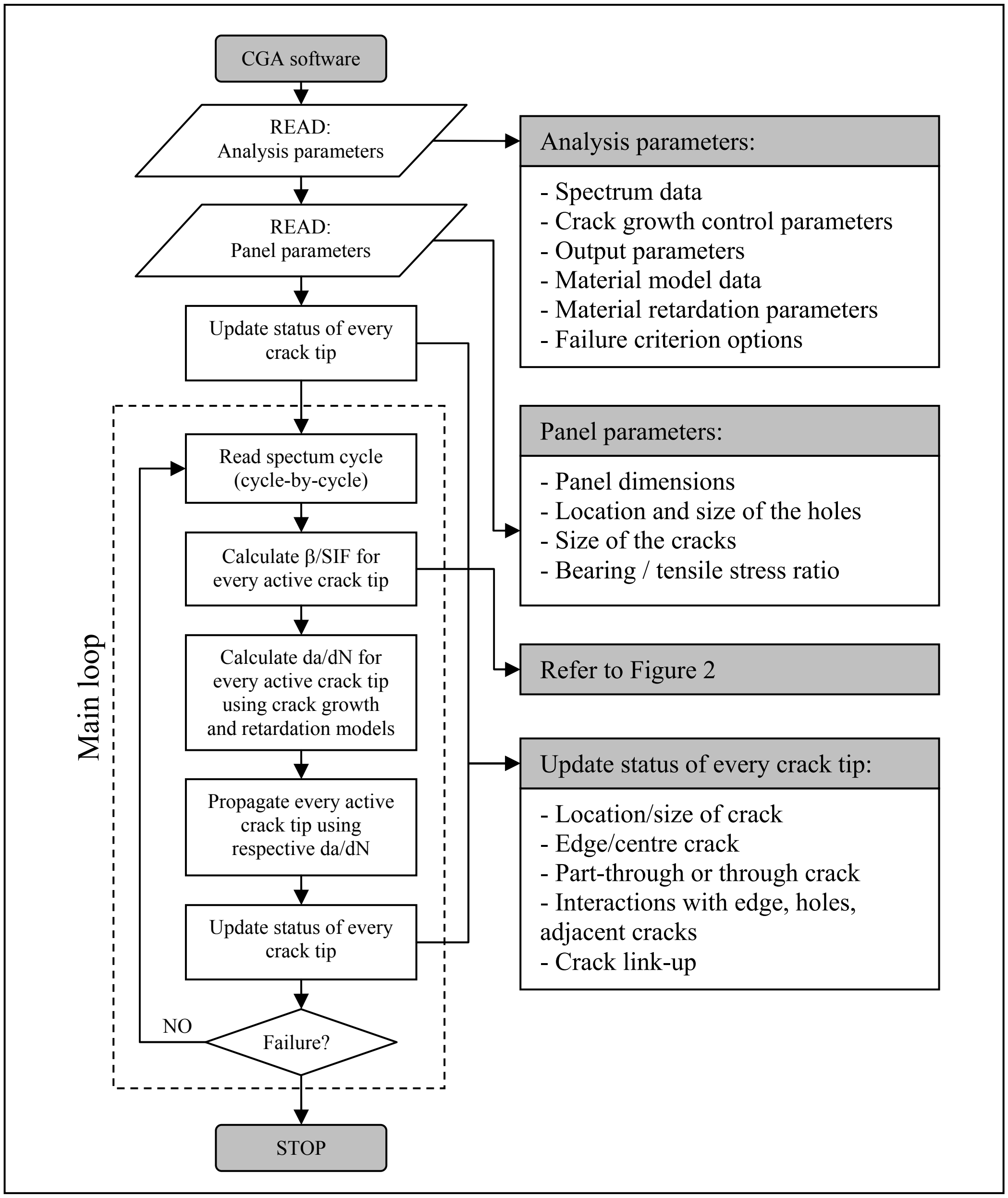


Figure 3. MSD crack growth analysis procedure.

3. Example calculation

A MSD panel test case study was simulated to verify the developed MSD β-factor compounding methodology. The stress intensity factors (SIF) baseline results were calculated for every crack tip using StressCheck from ESRD. StressCheck uses the p-version finite method, which provides estimation of the error in energy norm as a function of the polynomial degree of the element (p-level), ranging from p=1 to p=8. StressCheck computes the SIF using the contour integral method. To efficiently process the data, a Visual Basic Application (VBA) interface was developed to output the SIF at every crack tip as a function of the p-level. The SIF error estimated by StressCheck is less than 0.6% for the data presented in this paper.

A panel with MSD is presented in Figure 4. Diametrical cracks with initial sizes of 1 mm (0.039") were assumed for the 4 holes located on the left, while no crack was assumed for the fifth hole. A pin load was applied on the first hole from the left with a bearing / bypass stress ratio (BBR) of 2.0.

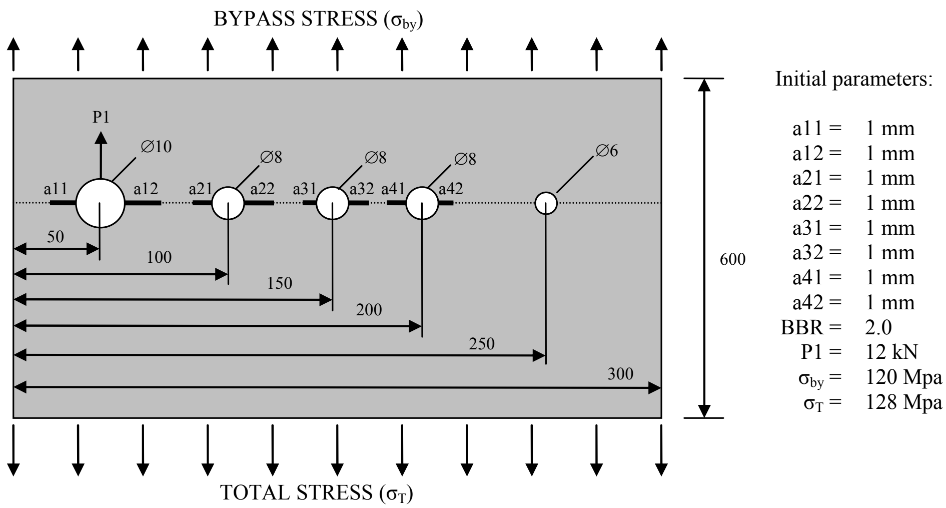


Figure 4. Parametric MSD panel case study. Dimensions are in millimetres (mm).

The β-solutions for all crack tips were calculated from StressCheck SIF results using the following equation:

$$\beta = \frac{K}{\sigma_{by}\sqrt{\pi a_0}}, \qquad (1)$$

where K is the stress intensity factor, σ_{by} is the bypass gross stress, and a_0 is half the crack length for centre cracks and full length for edge cracks.

The case study was first analysed using the NRC's CGA software and 16 sets of crack sizes (a_{11}, a_{12}, a_{21}, etc.) were extracted and used to generate 16 StressCheck finite element models (FEM). For the purpose of this study, through-the-thickness cracks were assumed.

The β-solutions computed from NRC's CGA software and StressCheck are compared in Figure 5 for the lead crack (a_{11}/a_{12}) and in Figure 6 for the second crack from the left (a_{21}/a_{22}) as identified in Figure 4. The StressCheck results were obtained with the parametric FEMs shown in Figure 7.

Overall, good agreement was obtained between the β-solution results of the lead crack (a_{11}/a_{12}) and the second crack (a_{21}/a_{22}) for a small crack size. Some discrepancies were observed for the second crack as the crack size increased. However, this difference is expected to have negligible effect on the predicted life. One possible reason for the discrepancies is the load redistribution that occurs in the presence of large cracks. Before merging the lead crack with the second crack, the lead crack was 70 mm (23% of the panel net section) compared to 20 mm for the second crack. It is likely that the second crack will be affected by a significant reduction of the net section due to the presence of the lead crack. Some of this effect might be included by the use of the crack interaction solution (Figure 1f), but the effect of the load redistribution is not included as this solution was developed for an infinite plate. An additional β-factor is currently under investigation to include the effect of net section loss.

Three sources of discrepancies related to the finite element analyses were identified:

- The applied bearing load distribution method differed between StressCheck (sinusoidal load distribution) and the CGA software [10]. This affected only the small crack results, which is shown in Figure 5 for crack lengths smaller than 8 mm.
- The FEMs were automatically generated using StressCheck parametric functionality. The parametric mesh does not fully control the mesh density and aspect ratio when approaching features such as an adjacent hole, an edge, or crack tips. This issue is clearly illustrated in Figure 7d when the crack tip a_{12} approaches crack tip a_{21}. The correctness of the results might be affected by the distorted mesh even if a good accuracy was achieved at a certain convergence level (p-level versus SIF).
- The calculated SIF was slightly dependant on the radius of the integration path (contour integral method). A sensitivity study carried-out on the calculated SIF at a_{11} showed that the selection of the radius influenced the calculated SIF by approximately 5% when varying the radius between $0.15 \times a_{11}$ and $0.75 \times a_{11}$.

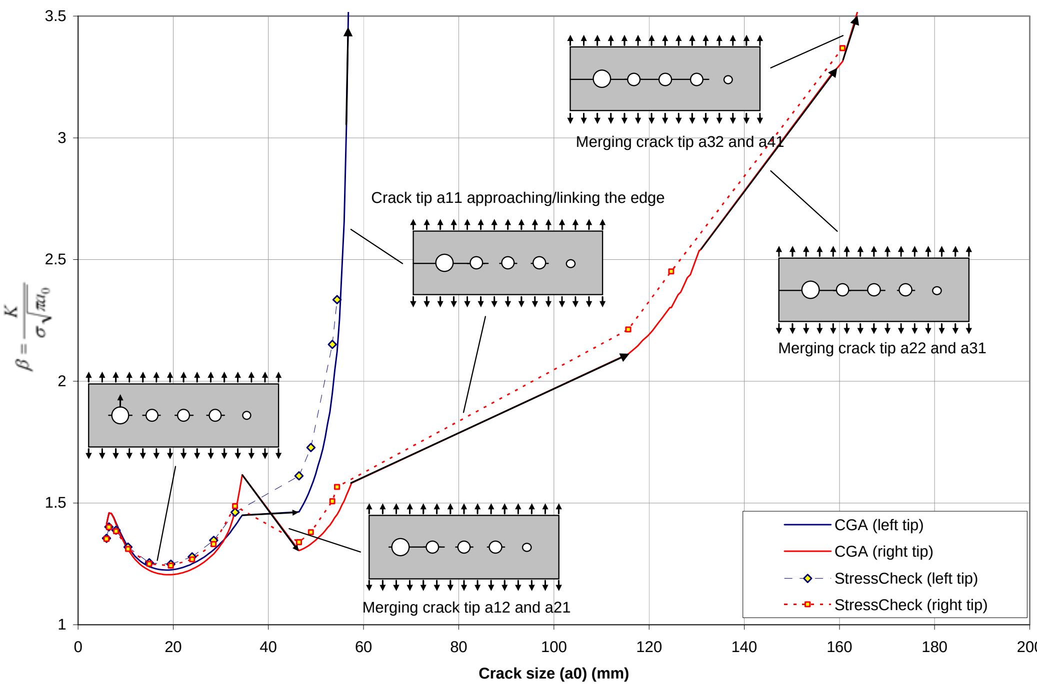


Figure 5. Comparison between the lead crack (a_{11} / a_{12}) β-solutions obtained using NRC's CGA software and StressCheck finite element analyses.

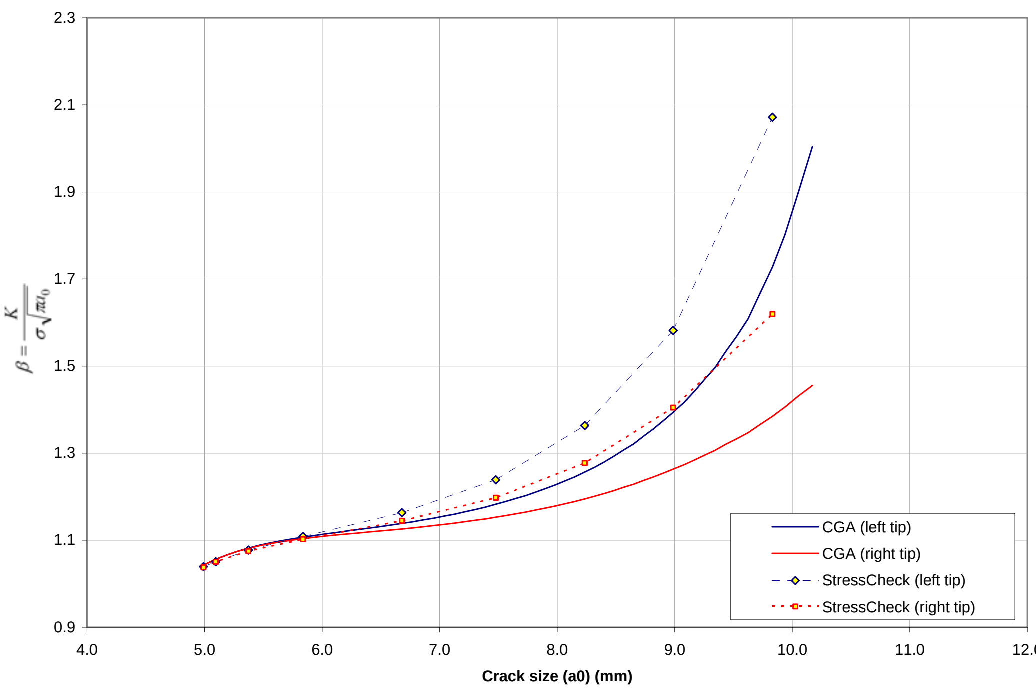
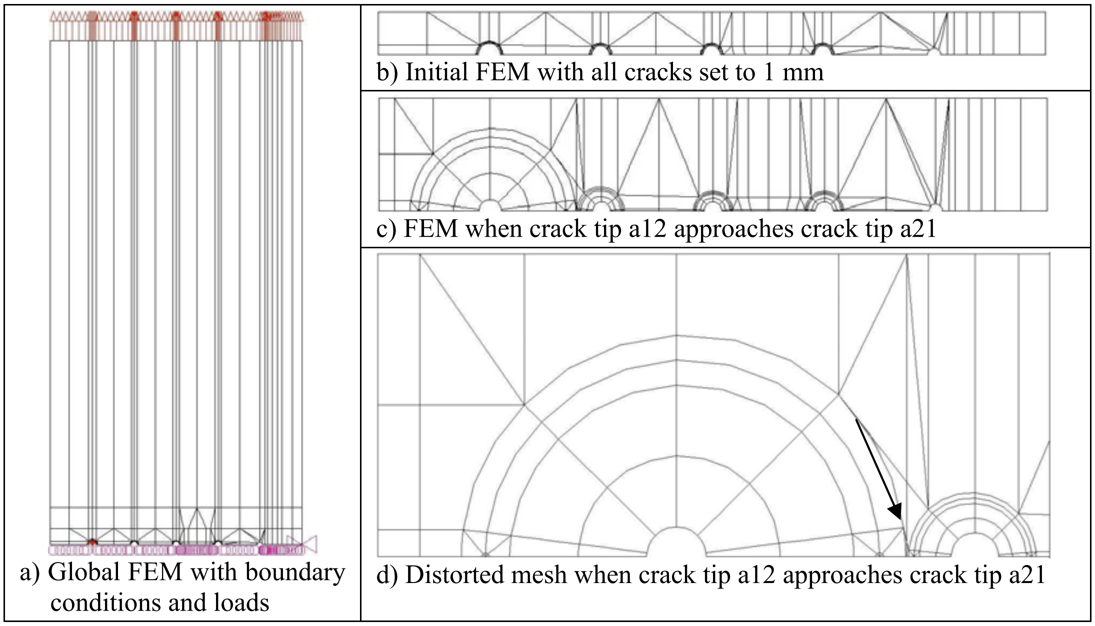


Figure 6. Comparison between the second crack (a_{21} / a_{22}) β-solutions obtained using NRC's CGA software and StressCheck finite element analyses.



a) Global FEM with boundary conditions and loads

b) Initial FEM with all cracks set to 1 mm

c) FEM when crack tip a12 approaches crack tip a21

d) Distorted mesh when crack tip a12 approaches crack tip a21

Figure 7. StressCheck finite element model.

4. Concluding remarks

It was demonstrated through the case study that NRC's β-solution compounding methodology provides results which are in good agreement with results obtained using finite element analyses (StressCheck). Also, the computational speed and accuracy currently achieved by NRC's CGA software allow performing QRA in a reasonable time.

Some sources of discrepancy were identified for the β-solution compounding methodology and the finite element modelling. Based on the comparison, an additional correction factor needs to be developed to consider the net section loss in the presence of large cracks in a panel.

The finite element results were used as a comparative baseline to investigate the correctness of the β-solution compounding methodology, although some issues were encountered such as the mesh quality when approaching other features (hole, edge, or adjacent crack tips) and the SIF sensitivity to the radius of the integration path.

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6. References

[1] J. C. Newman, Jr., I. S. Raju, Stress-intensity factor equations for cracks in three-dimensional finite bodies, NASA Langley Research Center, NASA TM 83200, August 1981, 54 p.

[2] D. P. Rooke, D. J. Cartwright, Compendium of Stress Intensity Factors, Her Majesty's Stationery Office, ISBN 0117713368, 1976, 330 p.

[3] M. Isida, On the Determination of Stress Intensity Factors for some Common Structural Problems, Engineering Fracture Mechanics, vol. 2, 1970, pp. 61-79.

[4] J. P. Gallagher, F. J. Giessler, A. P. Berens, R. M. Engle, Jr., USAF Damage tolerant design handbook: guidelines for the analysis and design of damage tolerant aircraft structures, Wright-Patterson AFB, Ohio, Air Force Wright Aeronautical Labs, report AFWAL-TR-82-3073, May 1984, 835 p.

[5] P.C. Miedlar, A. P. Berens, A. Gunderson, J. P. Gallagher, USAF Damage tolerant design handbook: guidelines for the analysis and design of damage tolerant aircraft structures, Wright-Patterson AFB, Ohio, Air Force Wright Aeronautical Labs, report AFRL-VA-WP-TR-2003-3002, November 2002, 716 p.

[6] H. Tada, P. C. Paris, G.R. Irwin, The stress analysis of cracks handbook, Paris Productions Inc. and Del Research Corp., St-Louis, Missouri, 1985.

[7] Y. Murakami (editor) Stress intensity factors handbook, The Society of Materials Science, Japan, Pergamon Press, New York, 1992.

[8] J. A. Harter, An alternate closed-form stress intensity solution for single part-through and through-the-thickness cracks at offset holes, AFRL-VA-WP-TR-1999-3001, December 1998, 29 p.

[9] T. R. Brussat, Technical Note: Mode I Stress Intensity for a Radial Crack at a Hole with Arbitrary Pressure Distribution, Engineering Fracture Mechanics, vol. 14, July 1980, pp. 233-235.

[10] D. J. Cartwright, A. P. Parker, Opening Mode Stress Intensity Factors for Cracks in Pin-Loads Joints, International Journal of Fracture, vol. 18, no. 1, January 1982, pp. 65-78.

[11] T. Swift, Damage tolerance capability, International Journal of Fatigue, Volume 16, Issue 1, January 1994, Pages 75-94.

[12] J. C. Newman, Jr, D. S. Dawicke, M. A. Sutton, C. A Bigelow, A fracture criterion for widespread cracking in thin-sheet aluminum alloys, 17th ICAF, Stockholm, june 1993, p. 443–468.

[13] L. Wang, W. T. Chow, H. Kawai, S. N. Atluri, Prediction of widespread fatigue damage threshold, Proceeding of the FAA/NASA Symposium on Continued Airworthiness of Aircraft Structures, Atlanta, GA, 1996,

[14] J. A. Harter, Damage Tolerance Application of Multiple Through Cracks in Plates With and Without Holes, report AFRL-VA-WP-TR-2004-3112, October 2004, 190 p.