# ANALYSIS OF CHANGE MODE I STRESS INTENSITY FACTOR IN ELEMENTS WITH ANGULAR NOTCHES

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#### ABSTRACT

The values of Mode I of stress intensity factors (SIF) are analyzed in the case of normal crack opening depending on angle parameter in the elements having angle notches. The data were obtained both from numerical and polarization-optical experiments with the use of the simplified procedure of the determination of Mode I SIF for angle notches considering the symmetry of investigated models.

## **1 INTRODUCTION**

Stone, ferroconcrete and concrete buildings are able to operate when defects or partial fracture are presented in them, for example, notches with different angles. It is necessary to take into account the influence of angle notches on SIF value while calculating and designing such structures. Different approaches to this problem were described in work of Srinivasa [1]. Numerical-experimental method of the determination of the generalized Mode I SIF is used in this article with the consideration of the singularity modified degree, Williams [2].

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### **2 BASIC THEORETICAL DEPENDENCES**

Some analytical dependences of solid body mechanics were used in this work. In Figure 1 plates with the angle notches under plane stress state are shown, the direct-line co-ordinate system with the center in the top of the notch is given. The axis of the notch is a symmetry axis of a model; there are extreme normal stresses on it. The generalized Mode I SIF  $K_I$  in that case may be determined from a dependence obtained on the basis of Williams' asymptotic solution of Williams [2].

$$K_{I} = \frac{\sigma_{x}(0, y)\sqrt{2\pi}}{v^{\lambda_{1}-1}},$$
 (1)

where  $\sigma_x(0,y)$  - stress on a symmetry axis, y - coordinate of a considered point,  $(\lambda_l-1)$  - the degree of a singularity. Procedure of its determination is given detail in work Albaut [3]. Stress  $\sigma_x(0,y)$  is possible to be determined by different ways, for example, numerically - a finite element method, or experimentally - a polarization-optical method with the help of interference fringe patterns pictures and dependence (2).

$$\sigma_{\max} = n_{\max} \cdot \sigma_0^d \,, \tag{2}$$

where  $\sigma_0^{d}$  - the price of a fringe pattern of a sample material,  $n_{max}$  – the order of a fringe pattern near the concentration source.

Mode I SIF is determined in the classical fracture mechanics (constant value of a singularity degree is -0.5) as

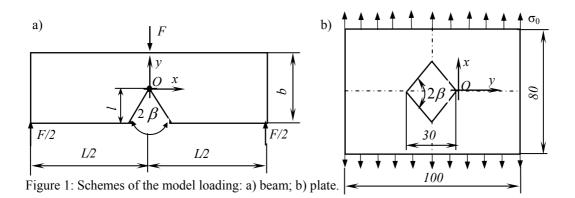
$$K_I = \sigma \sqrt{2\pi} r. \tag{3}$$

Here r - polar coordinate.

#### 3 A BEAM WITH THE ANGLE NOTCH

Values of Mode I SIF are determined in the beams with angle notches by the numericalexperimental method. These values are determined and analyzed by different methods in the process of the beams test on three-dot bending. Angle parameters varied from  $0^0$  up to  $150^0$  in the beams. Thus the results obtained by numerical and polarization-optical experiments were compared.

3.1 Numerical experiment. The determination of the generalized Mode I SIF is executed under three-dot bending of a beam loosened by an angle notch in the tension zone with considering of the modified singularity degree ( $\lambda_I$ -1). The same prismatic models are used in the process of tests conserning material at crack strength. The model geometry and the loading scheme are given in Figure 1a. The following dimensionless values of the model linear parameters and a concentrated force are accepted: t=1 (width), b=10 (height), L=4b (length), l=0,5b (depth of the notch), F=2(force). The field of dimensionless stresses was found by the finite element method. The generalized Mode I SIF K<sub>I</sub> are determined with their use on the basis of dependence (1) for six specimens with values of angle parameters  $2\beta$ : 0<sup>°</sup>, 30<sup>°</sup>, 60<sup>°</sup>, 90<sup>°</sup>, 120<sup>°</sup>, 150<sup>°</sup>. Graph of K<sub>I</sub> depending on the angle value is shown in Figure 3 (the second curve from above).



3.2 Polarization-optical experiment. Two complete sets of beam models were studied by polarization-optical method. The depth of the notch angle in the first batch was made l=0,5b (as in case of numerical experiment), in the second it was l=0,25b. The notch angles were same as in the previous numerical task. Radiuses of a curving at the notch angle tops were identical and are r=0,5mm. The loading scheme of the beams under three-dot bending is shown in Figure 1a.

The beams with notches were investigated under a step-by-step tensioning. Photos of interference fringe patterns are obtained, fragments of some of them are shown in Figure 2. The stress values were obtained as a result of data processing near notches' tops. Their recalculation was carried out with the help of scales of simulation on the beam corresponding to the numerical experiment. The dimensionless values of the generalized Mode I SIF K<sub>1</sub><sup>e</sup> were calculated on the basis of these stresses by dependence (1) using the modified singularity degrees ( $\lambda_1$ -1), Figure 3. In order to analyze and compare the generalized and classical Mode I SIF in both types of beams values K<sub>1</sub> for constant value of a singularity degree is -0.5 were determined on the basis of a dependence of a classical fracture mechanics (3), Figure 3 (dotted lines).

3.3 Analysis. It is possible to note that all graphs practically coincide at a notch angle from  $0^0$  up to  $60^0$ , but as the angle further increases experimentally determined  $K_I^e$  become much bigger than numerical  $K_I^n$ . This difference achieves its maximum value at the notch angle  $\beta$ =150<sup>0</sup>.

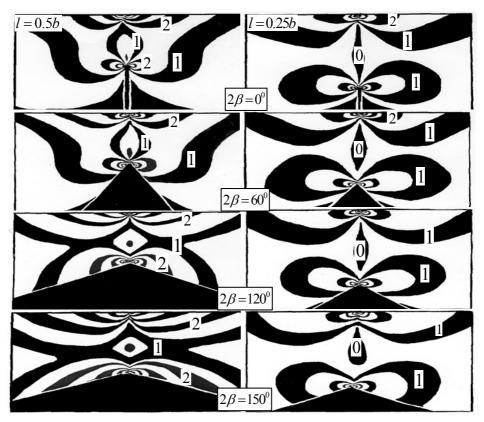
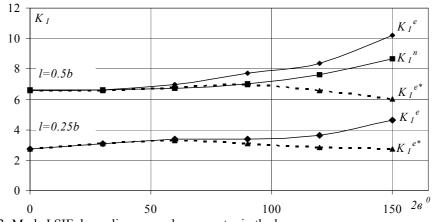
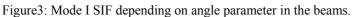


Figure 2: The fragments of some fringe patterns pictures.





## **4 ELEMENTS WITH RHOMBIC CUTS**

Plane elements of constructions were modelled. They may be made of any isotropic building material (metal, concrete, stone). Piesooptical organic glass was used as a model material both in numerical and in polarization-optical experiments. Values of Mode I SIF were determined in the plates with the symmetric rhombic cuts in the center depending on a rhomb angle.

4.1 Numerical experiment. The calculation of six symmetric plane models having identical dimensions was executed by the finite element method. Rhombs with identical horizontal diagonals were cut out on their symmetry axes, and the size of vertical diagonal varied from zero (crack) up to 1.33 from the value of the horizontal one. The loading scheme with nominal tension load  $\sigma_0$  in a vertical direction is shown in Figure 1a. Angle  $2\beta$  varied discretely from  $0^0$  up to  $106^0$ . Fields of stresses were obtained as a result of calculation. Graphs of Mode I SIF values depending on the rhomb angle were drawn on the basis of the data of numerical experiment, Figure 5: with the use of dependence (1)  $K_I^n$  (continued curve) and dependence (3)  $K_I^{n^*}$  (dotted curve).

4.2 Polarization-optical experiment. A complete set of six models, identical to those used in numerical experiment (Figure 1a) was investigated by photoelastic method. Its results in the form of interference fringe patterns are shown in Figure 4. Graphs of generalized  $K_I^e$  and classic  $K_I^{e^*}$  depending on the cut angle were drawn with the use of fringe patterns pictures and dependences (1) and (3), Figure 5.

4.3 Analysis of results. The character of the behavior of curves, Figure 5, constructed as a result of numerical and polarization-optical experiments is different. While analyzing graphs it's possible to note that cut angle of  $0^0$  up to  $42^0$  all graphs practically coincide and under further increasing of angle  $K_1^e$  becomes mach bigger than  $K_1^n$ .

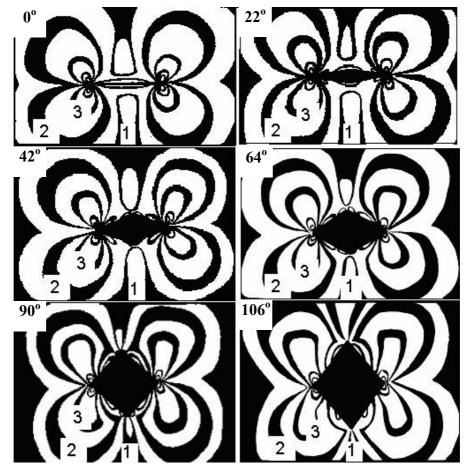


Figure 4: The fragments of some fringe patterns pictures in the models with the rhombic cuts.

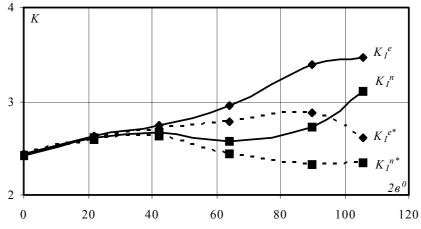


Figure 5: Mode I SIF depending on cut angle of the rhomb.

## **5 RESULTS**

The analysis of change of Mode I SIF obtained by numerical-experimental investigations shows that for the cut angle from  $0^0$  up to  $60^0$  it is possible to determine K<sub>I</sub> using dependence of a classical fracture mechanics with the singularity degree -0.5.

The simplified method of the determination of Mode I SIF was used with the help of asymptotic dependence (1) for elements with cut angle on the basis of analytical, numerical and experimental methods of the analysis based on of their different combination. The fact, that on a symmetry axis there are extremes of normal stresses is taken into account. The value of Mode I SIF can be determined with their help.

This simplified procedure may be applied for the solution of applied building problems and recommended for the use in the development of engineering designing methods of building structures with cracks, cuts or angle notches.

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