TRANSIENT RESPONSE OF CURVED SANDWICH PANELS SUBJECTED TO EXPLOSIONS

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ABSTRACT
In marine applications, sandwich structures can be subjected to underwater explosions or explosions in air. Generally, those structures consist of curved panels with cellular cores. Explosions result in high-pressure pulses that decay rapidly so that the duration of the loading is much shorter than the fundamental natural frequency of the panel. Because of the core is generally a cellular material (foam, honeycomb, balsa) that is relatively soft compared to the facings, the dynamic response consists of two phases. Initially waves propagate through the thickness and the deformation is essentially the deformation of the core. In a second phase, the overall deflection of the panel takes place. This study presents mathematical models for analyzing both phases of the deformation.

1 INTRODUCTION

Weight reduction in ships is very important since it leads to increased payload, speed, and range, as well as a reduction in fuel requirements. There are additional benefits to reducing topside weight of ship structure, such as increasing the sea-keeping ability of the ship via improved stability. Composites have been used on hulls shorter than 200 feet long with great success. In addition to reducing weight, they also offer the capability of integrating absorbing and reflecting materials into topside structures to reduce the electromagnetic signature of a ship. Composite hull forms offer the promise of reduced thermal and acoustic signatures. Composite structural applications are now transitioning into the fleet.

High pressures of short durations applied during explosions can introduce significant permanent deformations and damage to the core of sandwich structures with composite facings. In addition, the core/ facing interfaces, and the facings themselves can be damaged. In addition, because duration of the loading is so short the deformation of the sandwich structure usually consists of two phases. During the first phase stress waves propagate through the thickness of the plate and cause significant deformation of the core. It is then followed by a second phase dominated by the overall bending deflection of the structure. The overall objective of this study is to develop models capable of analyzing the response of the typical marine sandwich structures to explosions.

2 EQUATIONS OF MOTION FOR CIRCULAR CYLINDRICAL SHELLS

Soedel [1] extended the Donnell-Mushtari-Vlasov approach originally proposed for homogeneous and isotropic shells to laminated orthotropic cylindrical shells. Obviously, a number of complicating effects starting with shear deformations, rotary inertia and details of the kinematics of the deformation are not included in that approach. However, its simplicity and applicability to a wide range of problems is attractive in this first attempt to model the transient response of cylindrical panels to explosions. More advanced theories have been developed by many authors starting with Soedel [2] and a comparison of a number of theories was performed by Soldatos [3].

The motion of a cylindrical shell of radius R, in terms of the transverse displacement w and the stress function ϕ is governed by two coupled partial differential equations

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{1}{R^2}\frac{\partial^4 w}{\partial x^2\partial\theta^2}+D_{22}\frac{1}{R^4}\frac{\partial^4 w}{\partial\theta^4}+\frac{1}{R}\frac{\partial^2\phi}{\partial x^2}+\bar{\rho}\ddot{w}=q \quad (1)$$

$$\frac{A_{12}^2-A_{11}A_{22}}{R}\frac{\partial^2 w}{\partial x^2}+A_{11}\frac{\partial^4\phi}{\partial x^4}+\frac{A_{22}}{R^4}\frac{\partial^4\phi}{\partial\theta^4}+\frac{A_{11}A_{22}-A_{12}^2-2A_{12}A_{66}}{A_{66}R^2}\frac{\partial^4\phi}{\partial x^2\partial\theta^2}=0 \quad (2)$$

where the A_{ij} and D_{ij} coefficients are defined in the classical lamination theory for composites and $\bar{\rho}$ is the mass per unit area (ρh for an homogeneous shell). Note that no extension-shear or bending-twisting coupling is considered. That is $A_{16}=A_{26}=D_{16}=D_{26}=0$

3 FREE VIBRATIONS OF SIMPLY SUPPORTED CYLINDRICAL SHELLS

Closed form solutions for the free vibrations are available for closed cylindrical shells and cylindrical panels with simply supported boundary conditions. These solutions are used to examine the effect of curvature on the natural frequencies and on the dynamic response to transient distributed loads.

3.1 Closed simply supported shell

The boundary conditions for a simply supported closed circular shell

$$w(0,\theta,t)=w(L,\theta,t)=0, \qquad \frac{\partial^2 w}{\partial x^2}(0,\theta,t)=\frac{\partial^2 w}{\partial x^2}(L,\theta,t)=0 \quad (3)$$

and the governing equations are satisfied by

$$w=W_{mn}\sin\frac{m\pi x}{L}\cos n(\theta-\gamma)\sin\omega_{mn}t \qquad \phi=\Phi_{mn}\sin\frac{m\pi x}{L}\cos n(\theta-\gamma)\sin\omega_{mn}t \quad (4)$$

Substitution into the governing equations (Eq. 1,2) yields

$$\begin{bmatrix}\alpha_{11}-\bar{\rho}\omega_{mn}^2 & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{bmatrix}\begin{Bmatrix}W_{mn}\\ \Phi_{mn}\end{Bmatrix}=0 \quad (5)$$

The natural frequencies are given by

$$\omega_{mn}^2=\left(\alpha_{11}-\frac{\alpha_{12}\alpha_{21}}{\alpha_{22}}\right)/\bar{\rho} \quad (6)$$

Taking W_{mn} = 1, the mode shapes are given by $\Phi_{mn}=-\alpha_{21}/\alpha_{12}$.

3.2 Simply supported cylindrical panel

The boundary conditions for a simply supported closed circular shell

$$w(0,\theta,t)=w(L,\theta,t)=w(x,0,t)=w(x,\beta,t)=0 \quad (7)$$

$$\frac{\partial^2 w}{\partial x^2}(0,\theta,t)=\frac{\partial^2 w}{\partial x^2}(L,\theta,t)=\frac{\partial^2 w}{\partial \theta^2}(x,0,t)=\frac{\partial^2 w}{\partial \theta^2}(0,\beta,t)=0 \tag{8}$$

and the governing equations are satisfied by

$$w = W_{mn}\sin\frac{m\pi x}{L}\sin\frac{n\pi\theta}{\beta}\sin\omega_{mn}t \qquad \text{and} \qquad \phi=\Phi_{mn}\sin\frac{m\pi x}{L}\sin\frac{n\pi\theta}{\beta}\sin\omega_{mn}t \tag{9}$$

The α coefficients in Eq. are now

$$\alpha_{11}=\frac{\pi^4}{L^4}\left[D_{11}m^4+2(D_{12}+2D_{66})m^2n^2r^2+D_{22}n^4r^4\right], \qquad \alpha_{12}=-\frac{m^2\pi^2}{RL^2} \tag{10}$$

$$\alpha_{21}=-\left(A_{12}^2-A_{11}A_{22}\right)\frac{m^2\pi^2}{RL^2}, \quad \alpha_{22}=\frac{\pi^4}{L^4}\left[A_{11}m^4+A_{22}n^4r^4+\frac{A_{11}A_{22}-A_{12}^2-2A_{12}A_{66}}{A_{66}}m^2n^2r^2\right] \tag{11}$$

where $r = L/(R\beta)$ is the aspect ratio of the panel. For a given combination of m and n, we recognize the modal stiffness of a flat plate with the same dimensions. Therefore, in Eq. 6, the first term represents the natural frequency of the flat plate and the second term represents the effect of the curvature of the panel. For homogeneous isotropic panels, the non-dimensional frequencies $\overline{\omega}_{mn}$ defined as

$$\overline{\omega}_{mn}^2=\omega_{mn}^2\Big/\left\{\frac{\pi^4Eh^2}{12\rho(1-\nu^2)(R\beta)^4}\right\} \tag{12}$$

are written as

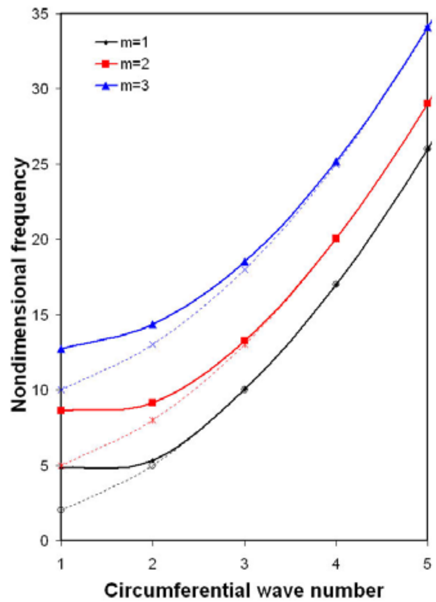
$$\overline{\omega}_{mn}^2=\left[m^2+n^2\right]^2+\frac{12(1-\nu^2)}{\pi^4}\frac{L^2}{h^2}\beta^2\frac{m^4}{\left[m^2+n^2\right]^2} \tag{13}$$

Eq. (13) indicates that: (1) as the shell becomes shallow $(\beta\to 0)$, the effect of curvature vanishes, as expected; (2) for higher modes, the first term on the right hand side of Eq. 13 dominates and the effect of curvature becomes negligible again. The evolution of the natural frequencies as m and n increase is illustrated in Fig. 1 for square panels $(L = R\beta)$ with either $\beta=\pi/2$ or $\beta=\pi/12$. As indicated by Eq. 13, the effect of curvature is significant for modes with low values of n, the circumferential wave number and it is more pronounced for deep shells than for shallow shells. An additional trend observed from Fig. 1 but also more clearly from Table 1 is that, for deeper shells, several of the lowest modes have very close natural frequencies. Therefore, by contrast with square plates for which the transient response is dominated by the first mode, it is expected that, for shells, several modes will participate in the dynamic response.

4 TRANSIENT RESPONSE OF SIMPLY SUPPORTED PANELS TO PRESSURE LOADING

Using the modal expansion method, the dynamic response of a simply supported panel can be written as

$$\begin{Bmatrix} w \\ \phi \end{Bmatrix}=\sum_{m=1}^{M}\sum_{n=1}^{N}c_{mn}\begin{Bmatrix} 1 \\ \Phi_{mn}\end{Bmatrix} \tag{14}$$



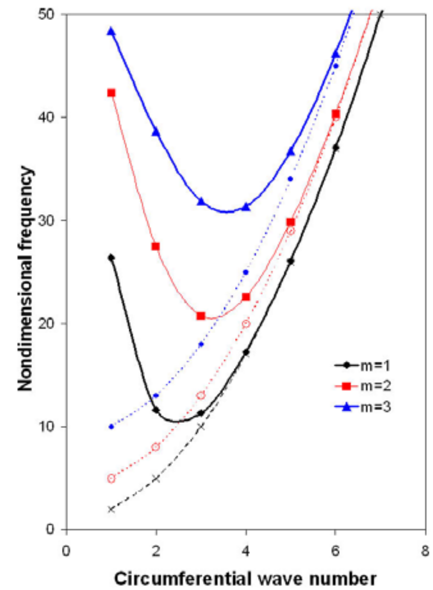


Figure 1: Non-dimensional natural frequencies of square isotropic homogeneous simply supported panels (a) $\beta = \pi/12$; b) $\beta = \pi/2$)

and the modal response is governed by

$$\ddot{c}_{mn} + \omega^2_{mn} c_{mn} = \frac{4}{\rho L \beta} q_{mn} \tag{15}$$

where $q_{mn} = \int_0^L \int_0^\beta q \sin\frac{m\pi x}{L} \sin\frac{n\pi\theta}{\beta} dx d\theta$. Closed form solutions where obtained for many types of transient pressure loadings including: (a) $q = p_o e^{-t/t_o}$ which is often used to represent the pressure

Table 1: First six non-dimensional natural frequencies for square isotropic homogeneous panels with $\beta = 0{,}15{,}90^o$

		Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Flat plate	m	1	1	2	2	1	3
	n	1	2	1	2	3	1
	$\overline{\omega}_{mn}$	2	5	5	8	10	10
Square panel $\beta = 15^o$	m	1	1	2	2	1	3
	n	1	2	1	2	3	1
	$\overline{\omega}_{mn}$	4.81555	5.29854	8.61247	9.12189	10.0383	12.7372
Square panel $\beta = 90^o$	m	1	1	1	2	2	1
	n	3	2	4	3	4	5

	$\overline{\omega}_{mn}$	11.2987	11.6466	17.2792	20.7576	22.5974	26.0786

caused by underwater explosions and (b) $q = p_o(1 - t/t_o)e^{-\alpha t/t_o}$ for explosions in air. The parameter t_o characterizes the duration of the loading. Results indicate that, when $\omega_{mn} t_o < 0.5$, the response depends essentially on the impulse applied $\left(I = \int q(\tau)d\tau\right)$ regardless of the shape of the loading. When the duration of the loading is larger than the period for that particular mode of vibration, a totally different type of response is obtained in which the amplitude of the response depends on the maximum force applied during the loading phase.

5 INITIAL RESPONSE

The initial phase of the deformation takes place before the overall bending deflections of the panel can occur. Here, two cases of practical interest are considered. First, we assume that the applied pressure is low enough for the core to behave elastically during the entire process. The propagation of elastic waves through the thickness of the laminate is studied using the method of characteristics. In the second case, the external pressure is assumed to be sufficiently large to induce crushing of the core from the onset. Other possibilities exist but are not considered here.

5.1 Elastic wave propagation through the thickness

In this example we consider a sandwich plate in which the facings have the following properties: E_3= 9.7 GPa, ρ_f = 1614 kg/m^3, h_f = 0.762 mm. For the honeycomb core: E_3= 1.005 GPa, ρ_c = 139.22 kg/m^3, and h_c= 12.7 mm. This plate is subjected to a 1 MPa uniform step pressure. The stress at the top-facing/core interface obtained using the method of characteristics can be written as

$$T = p_o T_R \sum_{i=0}^{n} (-R)^i H\langle t - (2i+1)t_f \rangle \qquad (16)$$

where t_f is the travel time trough the facing and H is the Heaviside function. The reflection and transmission coefficients at the interface are $R = (z_c - z_f)/(z_c + z_f)$ and $T_R = 2z_c/(z_c + z_f)$ respectively. At the top-facing/core interface, the transverse normal stress increases progressively even though a step load is applied (Fig. 2). An approximate solution is obtained by modeling the top facing as a rigid body. During the first 9.76 μs, no reflected waves propagating towards the left have reached the top facing/core interface. Therefore, the core provides a resistance that is proportional to the particle velocity. The displacement and the compressive stress at the top facing/core interface predicted by this model are

$$u = \frac{p_o}{z_c} t - \frac{p_o}{z_c^2} \rho_f h_f \left[1 - \exp\left(-\frac{z_c}{\rho_f h_f} t\right)\right] \quad \text{and} \quad T = z_c \dot{u} = p_o \left[1 - \exp\left(-\frac{z_c}{\rho_f h_f} t\right)\right] \qquad (17,18)$$

respectively. Fig. 2 indicates that Eq. 18 provides a very good approximation for the evolution of stress at that point. With the approximate model, the combination of governing parameters that define the ramping up of the stress is defined the time constant $t^* = \rho_f h_f / z_c$ so that $T = p_o\left(1 - e^{-t/t^*}\right)$. When t=3t* the stress T has reached $0.95p_o$. The exact displacement history at the interface can be obtained from Eq.(16) and compared with the prediction by the approximate model (Eq. 17) to show that these two solutions are indistinguishable in this case (Fig.2).

5.2 Propagation of shock waves in core

Under compression, the deformation of cellular materials used as core in sandwich structures typically consists of three phases: (1) the initial linear elastic phase until a peak stress or a crushing stress level is reached; (2) the crushing phase in which deformation proceeds under nearly constant stress; and (3) the consolidation phase in which stress increases rapidly with strain. When the applied pressure is sufficiently large to cause core crushing, shock waves propagate through the thickness of the the core generating high stresses and damage. A model is presented to analyze this phase of the deformatikon.

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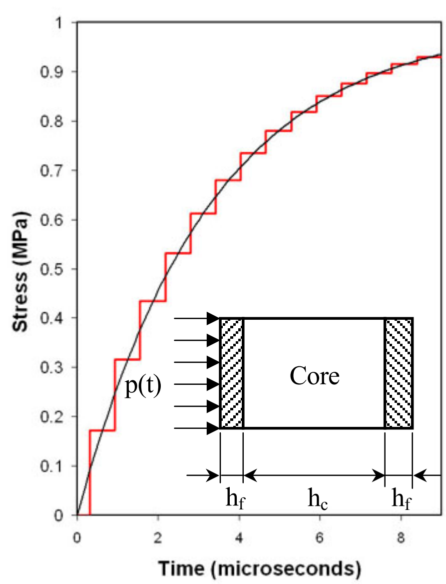

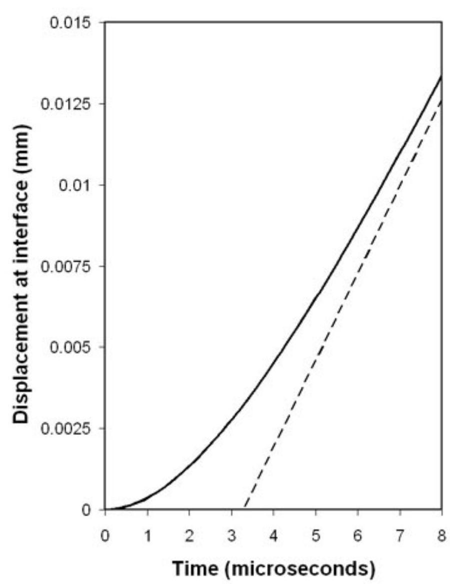

Figure 2: Uniform step pressure on sandwich plate: (a) Stress at the top facing/core interface (red line: exact solution, black line: rigid body approximation); (b) Displacement at interface (solid line: displacement, dashed line: asymptote)