SIZE EFFECT ON COMPRESSIVE STRENGTH OF CONCRETE

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ABSTRACT

It is important to consider an effect of a member size when estimating the ultimate strength of a concrete member loaded for all types of loading conditions. As well known as size effect, the strength of a member tends to decrease when the member sizes increase. Therefore, due to the recent increased interest in the topic of size effect of concrete this research focuses on the size effect of two main classes of compressive strength of concrete: pure axial compressive strength and flexural compressive strength.

First, the fracture mechanics type size effect on the axial compressive strength of unnotched and notched cylindrical concrete specimens were studied, with the diameter, and the height/diameter ratio and initial notch length considered as the main parameters. For this purpose, theoretical, experimental, and statistical analyses were conducted. Size effect equations were proposed to predict the compressive strength of cylindrical concrete specimens with various diameters and height/diameter ratios. The proposed equations show good agreement with the existing test results for concrete cylinders.

Second, the size, length, and depth variations of a flexural compressive member have been studied experimentally. A series of C-shaped specimens subjected to axial compressive load and bending moment were tested. The shape of specimens and the test procedures used were similar to those of Hognestad et al.. The test results are curve fitted using Levenberg-Marquardt's Least Square Method (LSM) to obtain parameters for Modified Size Effect Law (MSEL) by Kim et al.. The analysis results show that the effect of specimen size, length, and depth on ultimate strength was apparent.

1 INTRODUCTION

Currently, researchers accept the conclusion that the failure of concrete loaded in tension is caused by strain localization resulting in a finite size fracture process zone (FPZ). In the last few years, many researchers have started to realize that the strain localization also occurs for concrete specimens loaded in compression. Unlike failure caused by pure tension loading which usually takes place in a relatively narrow localized zone, compressive loading failure occurs within a larger damage zone. The compressive failure shows a similar failure mechanism as tensile failure. In compressive loading failure, the failure is caused by the distributed splitting cracks in the direction of member length as the lateral deformation increases during the failure progression. However, the compressive failure mechanism is more complex than tensile failure mechanism. Size effect of compressive failure is not as distinct as in tensile failure, because the formation of microcracks in compressive failure is distributed in a wider region than in tensile failure.

Presently, most design codes for concrete structures do not consider the effect of size. Since quasibrittle materials fail by formation of cracks, size effect has to be implemented. In compressive failure of quasibrittle materials, the size effect is quite apparent. Though the behavior of compressive failure has been studied extensively, the failure mechanism and its size effect have been insufficiently studied when compared to tensile failure mechanism. However, endeavoring studies by few researchers have continuously progressed the field. And, the experimental data currently available for proper analyses of size effect is lacking at best. From the few available experimental data, it is apparent that the compressive strength decreases as specimen sizes increase.

The focus of this study is to further develop and clarify compressive size effect in quasibrittle materials. Compressive strength of concrete can be mainly classified into two main classes: pure axial compressive strength and flexural compressive strength. In the case of pure axial compressive strength of concrete, there have abundant experimental data from past studies to derive its size effect characteristic. However, for the case of flexural compressive strength of concrete, experiments must be performed to obtain sufficient data to study its size effect.

2 THEORETICAL INVESTIGATION ON SIZE EFFECT

Bazant [1] derived size effect law (SEL) from the dimensional analysis and similitude arguments for geometrically similar structures of different sizes with initial crack considering the energy balance at crack propagation in concrete. Thereafter, introducing the size independent strength σ_o ($=\alpha f_t'$), Kim et al. [2] and Kim & Eo [3] proposed a modified size effect law (MSEL), which was also proposed by Bazant [4], Bazant [5], and Bazant & Xiang [6] in different approach, given by

$$\sigma_N = \frac{Bf_t'}{\sqrt{1+d/\lambda_o d_a}} + \alpha f_t' \tag{1}$$

where σ_N is nominal strength, d is characteristic dimension, f_t' is direct tensile strength of concrete cylinder, d_a is maximum aggregate size, and B and λ_o are empirical constants. And, α is an empirical constant less than unity.

In eq. (1), the width of crack band l_o is empirically found to be related to the maximum aggregate size, e.g., $l_o = \lambda_o d_a$ in which λ_o is an approximate constant with values between of 2.0 and 3.0 (Kim et al. [7-9]). In the regression analysis, this constant is selected as 2.0. In this study, tensile strength f_t' in Eq. (1) is substituted with compressive strength f_t' for the regression analysis of flexural compressive size effect.

3 SIZE EFFECT ON AXIAL COMPRESSIVE STRENGTH

3.1 Cylindrical specimens without an initial notch

From the statistical analyses of existing experimental data of Gornnerman [10] (172 specimens), Blanks and McNamara [11] (26 specimens), U.S. Department of the Interior [12] (20 specimens), Kesler [13] (337 specimens), and Murdock and Kesler [14] (123 specimens), the empirical constants in eq. (2) were determined. In this case, data numbers of specimens with $h/d = 2$ and $h/d \neq 2$ are 222 and 456, respectively, and the range of the maximum aggregate size is between 12.7 and 76.2 mm.

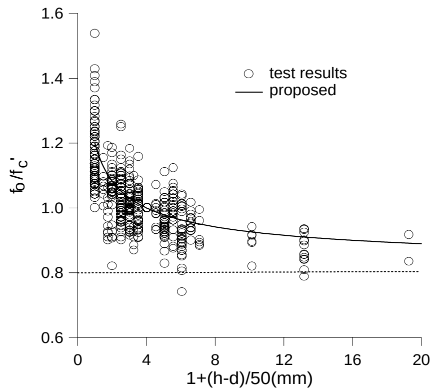

Figure: 1 Relationship between relative concrete strength (f_o/f_c') and $1+(h-d)/50$.

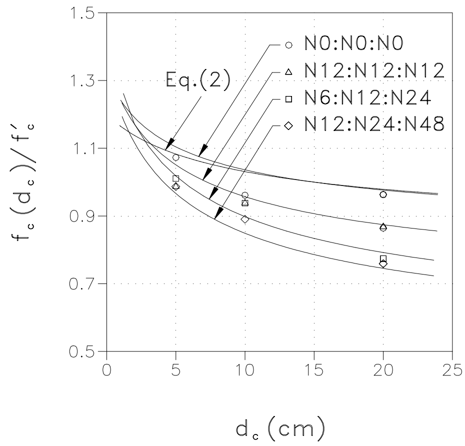

Figure 2: Comparison of model equations obtained from MSEL and experimental results.

For this purpose, the effects of the maximum aggregate size on the FPZ were considered and the concept of characteristic length was newly introduced. From statistical analyses, the following equation was derived using the test results.

$$f_o = \frac{0.4 f_c'}{\sqrt{1 + (h-d)/50}} + 0.8 f_c' \tag{2}$$

where, f_o and f_c' are in MPa, and h and d are in mm. The comparison indicates that the proposed equation give a good prediction. Figure 1 shows the relationship between f_o/f_c' and $1+(h-d)/50$. From the same figure, it can be seen that most of data are concentrated in a certain particular range since the diameters of most cylinders used in tests were 76, 100, and 150mm. When the value of h/d approaches 1.0, it is shown that the scatter of data is increased due to the effects of confinement and energy release zone. Figure 1 also shows the compressive strength of concrete would be 80% of the laboratory test results, since the confinement effects by frictional force would be negligible if the ratio, h/d, becomes very large.

3.2 Cylindrical specimens with an initial notch

A test is carried out to determine the adequate notch length, which enforces Mode I crack opening mechanism leading to failure. When a notch length to maximum aggregate size ratio is 4.0, the normal direction displacement to the initial notch is greater than the parallel direction displacement. Based on the foregoing discussion, it is apparent that we can enforce Mode I failure mechanism using this specimen and experimental procedure. For this reason, notch length to maximum aggregate size ratios of 9.2 and 18.5 (i.e., 12.0 cm and 24.0 cm) in φ100×750 mm cylindrical specimens are used.

To differentiate size effect between the cases with and without an initial notch, tests on specimens without an initial notch are also conducted. To confirm whether or not a size effect is present, MSEL is used. LSM regression analyses (IMSL Library) are carried out for maximum stress values obtained from the tests. The results are given in Table 1. Figure 2 shows $f_c(d_c)/f_c'$ as a function of the diameter d_c. In this figure, the data points represent the mean value of the experimental results of the same case.

Table 1: Results obtained from MSEL and LSM.

Specimen series	B	$\lambda_o d_a$	α
N6:N12:N24	0.92	2.6	0.48
N12:N24:N48	0.90	2.6	0.44
N0:N0:N0	0.52	2.6	0.80
N12:N12:N12	0.72	2.6	0.63

The results indicate a strong size effect. In Fig. 2, it is found that size effect between specimens with an initial notch and without an initial notch N0 show a considerable difference. Specimen N12 with a constant initial notch length regardless of specimen size shows that notch length to cylinder height ratio increases with decreasing cylinder diameter and vice versa. Accordingly, the compressive strength at failure decreases as the specimen diameter decreases. In addition, strength reduction level decreases for increasing diameter. Specimens (N12; N24; and N48) with the largest notch length would show more apparent reduction phenomenon than other specimens, which have smaller initial notch length than this specimen. However, Kim et al. [9], Markeset and Hillerborg [15], and Jansen and Shah [16] experimentally showed that the strength reduction is independent of the specimen size when the specimen height/diameter or length/depth is greater than a constant value (i.e., 2.0~2.5 and 3.0 for cylinder and C-shaped specimens, respectively). In this study, we concluded that the compressive strength decreases to a certain level with an increase in a notch length. However, beyond that level, it is nearly consistent with the literature. Namely, the comparison of $f_c(d_c)/f_c'$ for specimens N12; N24; and N48 and N6; N12; and N24 shows a similar difference.

When test results on specimen N0 without an initial notch are compared with eq. (2) (Kim et al. [7]), it can be seen that compressive strengths of specimens with a diameter greater than 10.0 cm show similar values as shown in Fig. 2. When the diameter is approximately 5.0 cm, however, the results show a difference since maximum aggregate size is different. Specifically, eq. (2) is obtained from tests with maximum aggregate size of approximately 25 mm. However, in this study, the aggregate size of 13 mm is used.

4 SIZE EFFECT ON FLEXURAL COMPRESSIVE STRENGTH

4.1 Size effect of flexural compressive strength

In order to obtain an analytical equation, which can predict the flexural compressive strength of C-shaped specimens at failure, MSEL is used, and LSM regression analyses are carried out with 20 test data points. Equation (3) is obtained from the analyses. The results are given in Fig. 3.

$$\sigma_N = \frac{0.70 f_c'}{\sqrt{1 + c/2.60}} + 0.47 f_c' \tag{3}$$

where nominal flexural compressive strength σ_N and uniaxial compressive strength f_c' are in MPa, and depth of C-shaped specimen c is in cm.

From the size effect law of Bazant and nonlinear regression analyses with the same data, eq. (4) is obtained.

$$\sigma_N = \frac{0.96 f_c'}{\sqrt{1 + c/22.27}} \tag{4}$$

In the previous section (Kim et al. [7]), eq. (2) was proposed to obtain the compressive strength of cylindrical concrete specimens with various diameters and *h/d*.

Figure 3 shows the value σ_N / f_c' as a function of the depth c which is measured from the neutral axis to compressive edge of member. In this figure, the hallow circular data points, the thick solid line, the thin solid line, and the dashed line represent experimental data, and the results from eq. (3), eq. (4), and eq. (2), respectively, as illustrated.

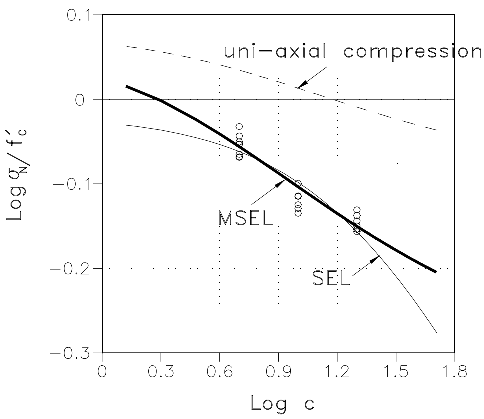

Figure 3: Comparisons of experiments with various equations.

From Fig. 3, the results indicate a strong size effect condition. Especially, the new eq. (3) shows the best agreement with the experimental results. The reduction of flexural compressive strength at failure as the specimen size increases is stronger than that for uniaxial compressive strength. This is due to a FPZ for uniaxial compressive strength of cylinders is larger than that for flexural compressive strength of C-shaped specimens. Comparing eqs. (3) and (4), it can be seen that the size effect by eq. (3) is able to better agree with the experimental result than eq. (4). It is observed that for specimens having no initial crack or notch the use of the MSEL to predict their behavior is appropriate.

4.2 Length and depth effect of flexural compressive strength

Markeset [17] and Markeset & Hillborg [15] experimentally showed that the post-peak energy per unit area is independent of the specimen length when the slenderness is greater than approximately

2.50 for concrete cylinders. Jansen and Shah [16] also experimentally showed that pre-peak energy per unit cross-sectional area increases proportionally with specimen length and post-peak energy per unit cross-sectional area does not change with specimen length for lengths greater than 20.0 cm in concrete cylinders. In this study, we conclude that flexural compressive strength does not change for specimens having a length greater than 30.0 cm for C-shaped reinforced concrete specimens as shown in Fig. 4.

In order to obtain an analytical equation, which predicts the flexural compressive strength of C-shaped specimens for length effect at failure, MSEL is used. Then, LSM regression analyses are performed on the results of the 11 test data for length effect. Equation (5) is obtained from the analyses and the results are graphed and shown in Fig. 4.

$$\sigma_N(h) = \frac{0.70 f_c'}{\sqrt{1 + h/2.6\left(1.59/h^{0.37}\right)}} + 0.47 f_c' \quad (h/c \le 3.0) \tag{5.a}$$

$$\sigma_N(h) = 0.75 f_c' \quad (h/c \ge 3.0) \tag{5.b}$$

where length of C-shaped specimen h is in cm. If the ratio of length and depth h/c is greater than or equal to 3.0, then this ratio h/c shall be 3.0.

To develop an equation for depth effect, LSM regression analyses are also performed on the 8 results from the depth effect series. All techniques and notations are same as for length effect. Equation (6) is obtained from the analyses and the results are graphically shown in Fig. 5.

$$\sigma_N(c) = \frac{0.70 f_c'}{\sqrt{1 + c/2.6\left(4.17/c^{0.53}\right)}} + 0.47 f_c' \tag{6}$$

where depth of C-shaped specimen c is in cm. Figure 4 shows the value $\sigma_N(h)/f_c'$ as a function of the h/c. And Fig. 5 shows the value $\sigma_N(c)/f_c'$ as a function of the depth c. The hollow circular data points and the thick solid line in Figs. 4 and 5 represent experimental data and analytical results from eqs. (5) and (6), respectively. Figure 4 indicates a strong length dependent size effect. Equation (5) shows a good agreement with the experimental results. For a h/c greater than 3.0, the failure strength approaches a constant value of 0.75. Figure 5 shows a distinct depth dependent size effect when normalized with the compressive strength f_c'. Equation (6) shows a reasonable agreement with the experimental results.

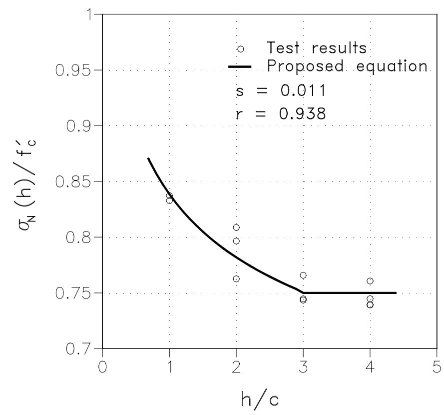

Figure 4: Normalized nominal strength with compressive strength versus ratio of length to depth.

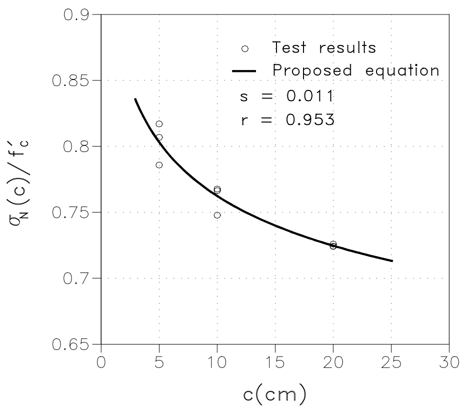

Figure 5: Normalized nominal strength with compressive strength versus depth.

6 CONCLUSIONS

From studies for size effect on compressive strength of concrete, the following conclusions are drawn.

1. Model equations for predicting of the compressive strength of concrete cylinders with and without an initial notch are suggested based on nonlinear fracture mechanics.
2. Size effect on flexural compressive strength is apparent, i.e., the flexural compressive strength at failure decreases as the specimen size increases. New parameter values of MSEL are suggested which better predicts the "reduction phenomena" of the strength. Size effect for flexural compressive strength in C-shaped specimens is more distinct than that for uniaxial compressive strength of cylinders.
3. Length effect is apparent (i.e., the flexural compressive strength at failure decreases as the specimen length increases). Depth effect is also distinct. New parameter values of MSEL are suggested which better predicts the "reduction phenomena" of the strength.

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