AN ALTERNATIVE FULLY STOCHASTIC APPROACH TO DETERMINE THE LIFETIME AND INSPECTION SCHEME OF A COMPONENT

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ABSTRACT
Traditionally, most aircraft components are designed according to two different philosophies: the Fail-Safe and Damage Tolerance approach. Both concepts cover a different part of the lifetime and are based on so-called deterministic models, in which the model parameters are constants (single-valued). In order to compensate for neglecting the natural variability of the model parameters (e.g. scatter in material parameters) and other uncertainties scatter and safety factors are applied explicitly and implicitly (e.g. by means of an assumed initial crack length). The results obtained with both approaches can be very conservative, although the reliability of the design remains unknown.

Another better way of dealing with this variability of the model parameters is by means of a stochastic analysis, adding an extra dimension to the deterministic analysis, by introducing a range of values that can occur with their chance on occurrence. However, performing a stochastic Damage Tolerance or Durability analysis does not make much sense, since the most important stochastic parameter, initial crack length distribution, is unknown.

In this paper, an alternative life approach will be presented by which the lifetime and inspection scheme of a component can be determined in a fully stochastic manner, covering the crack initiation period as well as the crack growth period in a realistic way. The approach can serve as an alternative for the currently approaches, especially the Safe-Life and Damage Tolerance approaches, resulting in more realistic predictions of the lifetime and inspection scheme. The method is demonstrated by a realistic example in Grooteman [1].

1 ALTERNATIVE STOCHASTIC LIFE APPROACH (SLAP)
During development of the Damage Tolerance approach it was realised that defining an initial crack length $a_i$ that is based on pre-service inspection would give very conservative results. This realisation led to the concept of Equivalent Initial Flaw Sizes (EIFS) for making crack growth calculations from the start of service to failure. EIFS are substitutes for any real (and unknown) initial damage in the structure. The problem is that EIFS values and distributions are obtained by back-calculation, using macrocrack growth models, from the life distributions (varying) and critical crack length (fixed) of similar components. The macrocrack growth models are unable to describe the real behaviour and size distributions of cracks that grow from any (small) initial damage. The resulting EIFS distribution therefore lacks any relation with reality, and more important, cannot be verified afterwards.

Instead of starting the life analysis at the start of the service life, another approach is to start the analysis at the end of the service life by constructing the failure distribution. This distribution (unlike to the EIFS distribution) can be verified afterwards using inspection data that becomes available during the service life as will be demonstrated in this paper. In the design stage this distribution will be unknown, but with a limited number of tests and experience from the past a conservative lower bound can be generated (Dodson [2]). Based on this distribution a conservative estimate of the inspection scheme can be obtained guaranteeing the required safety level.
In order to subsequently reduce the inspection effort, the obtained conservative failure distribution has to be updated when service life information (failure and non-failure data) becomes available. Even before reaching the initial inspection the current service lives of the various components can be used to obtain an improved estimate of the failure distribution and subsequently the inspection scheme, thereby reducing the conservatism of the approach. In this way an adaptive scheme can be constructed leading to a minimal inspection effort for the required safety level.

These considerations led to the following alternative fully stochastic life approach called **SLAP** (*Stochastic Life Approach*). The approach, presented in Figures 1–5, enables the lifetime and inspection scheme of a structural component to be determined and covers both the crack initiation and growth periods realistically, i.e. without the need for EIFS values and distributions. The approach consists of the following three steps:

1. Construct the failure distribution
2. Backward crack growth analyses
3. Forward crack growth analyses, including inspections.

The basic methodology pertaining to these steps will be discussed next. Grooteman [1] gives more details, discussing the approach for a realistic application.

**Step 1: Construct the failure distribution**

First, the failure distribution has to be obtained, e.g. by means of a Weibull analysis, see figure 1. The initial failure distribution should be a conservative estimate (lower bound) based on a limited set of test data, and should be updated during the lifetime of the component by service data (failures and non-failures) as they become available, to improve safety. For more details the reader is referred to Grooteman [1] and Dodson [2].

An important concept is that the scatter introduced by material properties, load spectrum, etc., is included, *de facto*, in this one failure distribution, and therefore need not be characterised separately. Moreover, an estimate of the scatter present in the components can be updated easily by using information from service. This is a very important advantage over using the variability of all the analysis parameters, since this information is often hard to acquire, if at all. Furthermore, a limited number of random variables is a very attractive concept, especially for engineering use.

![Figure 1: Step 1 of the SLAP philosophy](image-url)
Step 2: Backward crack growth analyses: Determine initial inspection time and corresponding crack length distribution

a) Back-calculations are done starting from the failure distribution of the component. However, these back-calculations are not extrapolated to time zero, as in the EIFS approach, but only until a detectable crack length has been reached, $a_{det}$ in figure 2: this is comparable to the first stage of back-calculation using the Damage Tolerance philosophy. The resulting distribution ($PDF-a_{det}$ in figure 2) corresponds to an estimate of the distribution which describes the time it takes for cracks of length $a_{det}$ to become present in a certain percentage of the components. N.B.: the individual back-calculations are deterministic, since all the variability is included in the failure distribution. In Grooteman [1] it is demonstrated that an optimal choice of $a_{det}$ can be found by analysing values in the range covered by the POD distribution.

![Figure 2: Step 2a of the SLAP philosophy](image)

b) When a certain threshold percentage ($p_{th}$, e.g. 0.1 %, Fig. 3) of these detectable cracks becomes present, the initial (threshold) inspection becomes timely. An inspection before $p_{th}$ is unfeasible, since any cracks would be difficult to find with a reasonable chance of detection. Determining the initial inspection $t_{initial}$ in this way results in a realistically conservative estimate that automatically covers the crack initiation, micro- and short-crack periods, without the need to model them. This is a great advantage, since there are no well-established engineering models for fatigue crack initiation and microcrack growth, and even the modelling of short crack growth in real structures is also problematical. (N.B.: short crack models are not required because current in-service inspection techniques cannot detect cracks reliably at sizes below 1-2 mm, which is beyond the short crack regime.)
c) The crack length distribution function \((PDF_{t_{\text{initial}}})\) at \(t_{\text{initial}}\) can be obtained at the same time as in b), by extrapolating all the back-calculations down to \(t_{\text{initial}}\). As before, there is no need for crack initiation and micro- and short-crack growth models. For short cracks this might not be completely true when considering the lower tail of the \(PDF_{t_{\text{initial}}}\) distribution. However, this part of the PDF will not contribute to the probability of failure, discussed in the next step, and is therefore irrelevant here.
Step 3: Forward crack growth analyses including inspections: Determine repeat inspections

In the third and last step a stochastic forward crack growth analysis is performed, starting from the time $t_{\text{initial}}$ and the crack distribution ($PDF-I_{\text{initial}}$ in figure 5). During this analysis a repeat inspection scheme (denoted by the crosses in the figure) is simulated by using the Probability Of Detection (POD) function appropriate to the applied inspection method (see Grooteman [1] for details on the POD). As before, the individual crack growth calculations are deterministic.

Once a crack has been "found" by a crack growth analysis, the component is assumed to be replaced (or repaired). There will then be another period $t_{\text{initial}}$ in which a new crack will initiate and grow. Starting from the end of this period, a crack growth analysis can be done using a new crack length value drawn from the crack length distribution function ($PDF-I_{\text{initial}}$). However, it can often be assumed that a replaced or repaired component will survive until the economic life of the overall structure is reached, since this will normally be less than twice the crack initiation period.

Performing many of these crack growth calculations (of which two examples are shown in figure 5) finally results in a Probability Of Failure (POF) value, with the number of calculations depending on the stochastic method applied. This POF value can be compared with the required safety level, and if unsatisfactory the calculations can be redone with different repeat inspection schemes. Similarly, POF values can be obtained for different inspection methods and repeat inspection schemes until the required safety level is attained. Also, one can choose between fixed-interval or variable-interval repeat inspection schemes. If necessary, all these possibilities can be investigated in order to determine an optimum inspection scheme.

Figure 5: Schematic overview of the SLAP philosophy

Each crack growth calculation stops when the component has failed, when cracks have been shown to be non-detectable by all inspections, or when the economic lifetime of the component has been reached, depicted by $t_{\text{economic}}$ in figure 5.
REFERENCES
