TEARING ENERGY OF TIRE RUBBER UNDER MODE–I AND MODE-III LOADING

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ABSTRACT

The fundamental principles of the application of fracture mechanics to rubber are briefly discussed. The importance of the problem arises because of the large nonlinear deformation of rubber which introduces difficulties in the solution of the boundary value problem of a cracked body made of rubber. The tear behavior of rubber can be conveniently described by the critical tearing energy which is a characteristic property of the material. The results of an experimental study of determining the crack growth behavior and critical tearing energies of pure tire rubber are presented. Constrained tension and trousers specimens were used for mode-I and mode-III loading, respectively. In the trousers specimens the force necessary to grow the crack varies widely from a maximum value at crack initiation to a minimum value at crack arrest. This result to a stick-slip stable crack propagation, that is, the crack arrest and reinitiates of fairly regular intervals. In the constrained tension tests crack initiation triggers catastrophic growth. Results for the critical tearing energies for mode-I and mode-III are given.

1. INTRODUCTION

The problem of crack growth in elastomers was first studied by the monumental work of Rivlin and Thomas [1]. Many investigators [2-8] have demonstrated that, the critical tearing energy, T_{cr}, is independent of the geometry and dimensions of the test piece and can be considered as a material property. For component design the tearing energy, T, is calculated for a hypothetical crack and is compared with T_{cr} to determine if the crack will propagate. Thomas [8] has shown that the tearing energy depends on the work required to break a unit volume of material in simple tension in the absence of cracks and the diameter of the notch tip, which measures the bluntness of the notch. In the present work results of an experimental study of determining the critical tearing energy of pure tire rubber under mode-I and mode-III loading are presented.

2. TEARING ENERGY

In attempting to find a criterion for growth of a crack in elastomers, application of the critical stress criterion presents considerable difficulties. Indeed, elastomers present large deformation prior to failure, and the solution of the mathematical problem of determining the stress field in the cracked body made of an elastic material is intractable. Furthermore, high stresses are developed to a very small region around the crack tip, so that their measurement cannot be readily carried out.

The Griffith approach can be applied to elastomers since it is not limited to small strains and linear elastic response. However, the reduction of the elastic strain energy in elastomers is not spent only to increase of surface free energy of the cracked body, but is being transformed to other forms of energy, like irreversible deformation of the material. These changes take place in the neighborhood of the crack tip in a relatively small volume of the material compared with the overall dimensions of the body. Thus, it is anticipated that the energy losses in causing an increase of the crack length will be independent of the shape and dimensions of the cracked body and the form of the applied forces. The energy required to grow a crack is characteristic of the material and independent of the test piece geometry. Under such conditions, the Griffith criterion can be applied to elastomers. The region near the tip of the crack will deform very highly with respect to the rest of the body. When the crack of length a in a sheet of thickness t is grown by da an amount of work T_{cr} t dc is done, where T_{cr} is an energy characteristic of the material. When the applied forces do no work during crack growth the crack growth condition is given by [9, 10]

$$-\frac{1}{t}\left(\frac{\partial W}{\partial a}\right)_{\ell} = T_{cr} \tag{1}$$

The suffix l denotes differentiation with constant displacement of the boundaries over which forces are applied. T_{cr} is the critical energy for tearing and is a characteristic property of the material. The tearing energy, T, is defined by the left hand side of Eq. (1). Experimental measurements show that when crack propagation is expressed in terms of the tearing energy, the relation is independent of specimen type and geometry. Fatigue crack growth characteristics are also related to tearing energy. Thomas [8] has shown that

$$T \cong W_b d \tag{2}$$

where W_b is the average energy density at the tip. W_b is the work required to break a unit volume of material in simple extension in the absence of cracks and is an intrinsic material constant. Eq. (2) indicates that T is directly proportional to d.

3. TROUSERS SPECIMEN

The trousers specimen has become a favorite test piece for determination of out-of-plane mode-III critical tearing energy for elastomers. The specimen is a thin rectangular piece cut centrally along its length so that two legs are formed. The legs are pulled in opposite directions out of the plane of the test piece by equal and opposite forces. The expression for tearing energy is

$$T = \frac{2\lambda P}{h} - 2bw \tag{3}$$

where:
P = force on legs of specimen
λ = extension ratio in legs (ratio of length of deformed to underformed leg)
h = specimen thickness
b = width of legs
w = strain energy density in the legs

When the specimen legs can be considered inextensible compared to the tearing $(\lambda \cong 1, w \cong 0)$ Eq. (3) is simplified as

$$T = \frac{2P}{h} \tag{4}$$

The rate of crack propagation, $\dot{a}$ when the specimen legs are inextensible (λ=1) is

$$\dot{a} = \frac{R}{2} \tag{5}$$

where R is the crosshead speed. This means that the rate of tearing is half the crosshead speed of the testing machine.

The force at which the cut of the specimen first grows is measured. Then Eq. (3) or Eq (4) is used to determine the critical tearing energy.

4. CONSTRAINED TENSION (SHEAR) SPECIMEN

The constrained tension specimen is a wide strip of rubber material attached along its long edges to rigid grips that constrain its lateral deformation. The tearing energy is

$$T = wl_o \tag{6}$$

The value of w is found from the stress-strain relation of a specimen under conditions of constrained tension like those existing in region B.

5. EXPERIMENTAL

Mode-III Loading

The out-of-plane mode-III critical tearing energy was determined from the trousers test. The trousers specimens had legs 12.6 mm wide and varying thickness ranged from 0.74 to 1.73 mm. Initial results revealed a stick-slip tearing mechanism during crack growth. The applied force necessary to propagate the crack varied widely from a minimum at crack arrest to a maximum at

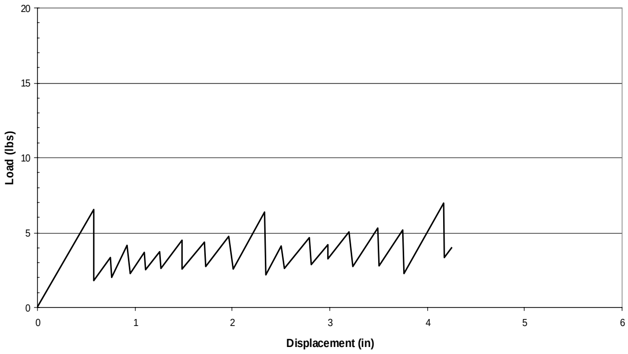

Figure 1: Load versus displacement curves for mode-III

crack extension. To reduce the stick-slip tearing constrained trousers specimens were used. The specimens were reinforced locally with two thin steel shims of different widths bonded on opposite sides of the specimen along the crack. To ensure that the crack propagates along its initial plane direction a shallow groove was cut along the crack ligament on both sides of the specimen.

Fig. 1 shows a typical load-displacement graph during crack growth for a specimen of net thickness along the crack ligament of 1.45 mm after the depth of the two grooves has been accounted for. The loading rate is 25.4 mm/min. The load reaches a maximum value at crack growth and a minimum value at crack arrest. The mode of crack propagation is characterized by an increase of load with no crack growth followed by a sudden decrease of load as the crack propagates unstably and arrests. This pattern is repeated at fairly regular intervals.

Mode-I Loading

A series of constrained tension specimens of dimensions 101.6 x 17.8 x1.9 mm with crack lengths of 38.1, 44.5, 50.4 and 57.1 mm were loaded in an Instron servohydraulic testing machine. Fig. 2 shows the load-displacement curve up to the point of crack initiation for various initial crack lengths. No stable crack growth as in the case of the trousers test was observed. Crack initiation coincided with rapid catastrophic failure. The load-displacement curve presents a nonlinear sigmoid behavior characteristic of rubber. The stiffness of the curve after a small linear part decreases up to a limiting strain after which it increases. Note that the curves approach each other as the crack length increases up to a limiting crack length of 57.1 mm.

6. RESULTS AND DISCUSSION

Mode-III Loading

From the load-displacement records of Fig. 1 we observe that the force gradually increases with displacement until a maximum is reached. At this point, the force drops with increasing displacement until a minimum is reached. The crack at maximum force initiates, while at minimum force arrest. This cycle of crack initiation and arrest repeats itself at fairly regular intervals. This form of crack growth is known as stick-slip tearing. The crack does not propagate in a self-similar manner at constant speed, but stops and reinitiates at fairly regular intervals. The stick-slip mode of crack growth was reduced, hut it was not eliminated with the bonding of the steel shims to the legs of the trousers specimen.

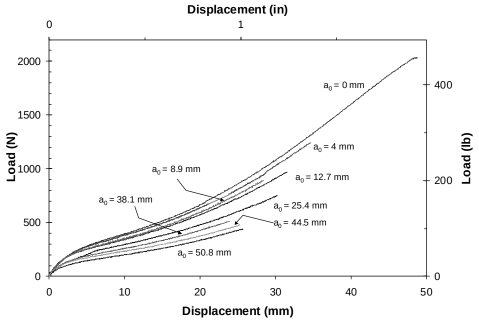

Figure 2: Load versus displacement curves for various initial crack lengths under mode-I

Table 1: Trousers Test Results

(N/mm)			(lb/in)		
Initiation	**Arrest**	**Mean**	**Initiation**	**Arrest**	**Mean**
44	20	32	250	113	181
32	23	27	182	130	156
29	25	27	165	141	153
35	**22**	**29**	**199**	**128**	**163**

An explanation of the observed fluctuation of the force from a maximum value at crack initiation to a minimum value at crack arrest can be provided from Eq. (2). As the applied force increases the diameter, d, of the notch at its tip, which measures the bluntness of the notch, also increases up to a value at which the notch starts to propagate. At this point d reaches a maximum value and since the work W_b is a material constant, Eq. (2) suggests that the tearing energy T, and therefore, the force F (Eq. (4)) for crack growth becomes maximum. As the notch initiates at the maximum load its diameter starts to decrease to a minimum value at arrest, which leads to a minimum value of the applied force.

From the values of the applied force at crack initiation and arrest we can calculate the corresponding critical values of tearing energy. Since the legs of the trousers specimens have been reinforced with steel shims the deformation of the legs is negligible ($\lambda = 1$, $w = 0$). Thus, Eq. (4) can be used for the calculation of the critical tearing energy, T_{cr}. Results of the initiation, arrest and average tearing energy for a crosshead displacement rate of 51 mm/min and a shim separation distance b = 38 mm are shown in Table 1.

From Fig 1, we observe that the scatter in the force values at crack arrest is smaller than at crack initiation. This result indicates that the tear initiation energy provides a measure of the resistance of the rubber to tearing but cannot be considered as an inherent material property. On the contrary, the arrest critical tearing energy corresponds to a fairly constant crack tip diameter and is an inherent material property. Due to the great difference between the initiation and arrest critical tearing energies, calculation of an average tearing energy is physically inappropriate. A series of specimens with thickness ranging from 0.74 to 1.73 mm were tested to study the effect of specimen thickness on the tearing energy. It is found that the tearing energy is independent of specimen thickness.

The effect of crack growth rate on the tearing energy was studied. The rate of applied load varied between 5.1 and 254 mm/min, which means that the crack propagated at a speed half of these values (Eq. (5)). It was found that as the crack propagation rate increases, the fluctuation of the load decreases at both maximum and minimum values, and therefore, the crack grows in a more stable manner at higher crack propagation rates. Furthermore, for all crack propagation rates it was found that the scatter of force values at crack arrest is smaller than at crack initiation. The variation of the arrest and initiation tearing energies versus the rate of loading is shown in Fig. 3. Note that both tearing energies increase as the loading rate increases. However, the arrest energy increases at a much slower rate than the initiation tearing energy. This result in conjunction with the stability of tearing energies at crack arrest suggests that the critical tearing energy at crack arrest can be considered as an inherent material property.

Mode-I Loading

The load-displacement curves of Fig. 2 approach each other and tend to a limiting curve as the crack length increases up to a value of 50.8 mm. For that crack length Eq. (6) can be used for the determination of the critical tearing energy under mode-I loading. A value of 31.3 N/mm was

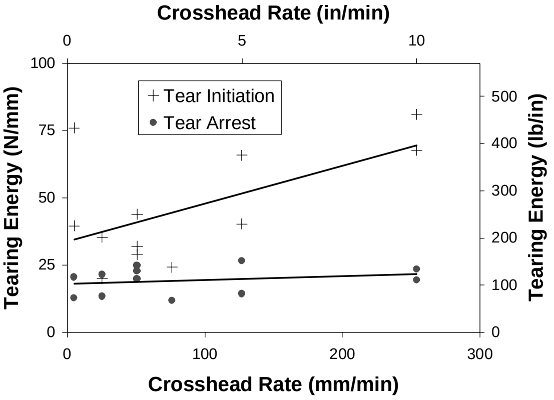

Figure 3: Tearing energy versus crosshead rate

obtained**.** This value is close to the critical tearing energy at initiation for mode-III loading.

REFERENCES

1. Rivlin, R.S. and Thomas, A.G., Rupture of Rubber. I. Characteristic Energy for Tearing, Journal of Polymer Science, Vol. 10, pp. 291-318, 1953.
2. De, D. and Gent, A.N., Tear Strength of Carbon-Black-Filled Compounds, Rubber Chemistry and Technology, Vol. 69, pp. 834-850, 1996.
3. Kadir, A. and Thomas, A.G., Tear Behavior of Rubbers over a Wide Range of Rates, Rubber Chemistry and Technology, Vol. 54, pp. 15-23, 1981.
4. Lake, G.J., Effect of Crack Tip Sharpness on the Strength of Vulcanized Rubbers, Journal of Polymer Science: Part B: Polymers Physics, Vol. 25, 1157-1190, 1987.
5. Lake, G.J. Fatigue and Fracture of Elastomers, Rubber Chemistry and Technology, Vol. 68, pp. 435-460, 1995.
7. Stacer, R.G., Yanyo, L.C. and Kelley, F.N., Observations on the Tearing of Elastomers, Rubber Chemistry and Technology, Vol. 58, pp. 421-435, 1985.
8. Thomas, A.G., The Development of Fracture Mechanics for Elastomers, Rubber Chemistry and Technology, Vol. 67, pp. G50-G60, 1994.
9. Gdoutos, E.E., Fracture Mechanics Criteria and Applications, Kluwer Academic Publishers, Dordrecht, `1990.
10. Gdoutos, E.E., Fracture Mechanics – An Introduction, Kluwer Academic Publishers, Dordrecht, 1993.