CRACK DEFLECTION IN CERAMIC LAMINATES

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ABSTRACT

Laminated structures can enhance the resistance of brittle ceramics. The interfaces are expected to deflect cracks, increasing the fracture energy of the laminate compared to a monolithic material and thus raising the apparent toughness. The laminates are made of alternating dense and porous layers. This offers the best chemical compatibility between the laminas and almost no thermal residual stresses. The question is to predict the volume fraction of pores required to cause crack deflection.
The criterion derives from an energy balance and takes into account an additional stress criterion avoiding the drawback met in other approaches: the arbitrary choice of crack increment lengths. The model is based on a two-scale analysis and can be written in terms of Young's modulus and toughness ratios.
Experiments show that a unique function depending on the volume fraction of pores can be used to express the above mentioned ratios. A cubic lattice of spherical voids is assumed. The parameters of the porous ceramic depend linearly on the porosity and vanish at percolation of pores. Then the criterion can be rewritten in terms of a single parameter: the porosity.
Results agree almost well with experiments on SiC and B_4C. The comparison with the He and Hutchinson criterion (HH) shows that it underestimates the correct value.

1 INTRODUCTION

Ceramics are very brittle materials. Laminated structures can enhance their resistance. The interfaces are expected to deflect cracks, increasing the fracture energy of the laminate compared to a monolithic material and thus raising the apparent toughness. The laminates are made of alternating dense and porous layers. This offers the best chemical compatibility between the laminas almost no thermal residual stresses. The question is to predict the volume fraction of pores required to cause crack deflection.
Two ceramics are analysed: Silicon Carbide (SiC) (Blanks et al. 1998 [1], Reynaud 2002 [2]) and Boron Carbide (B_4C) (Tariolle 2004 [3]). Porosity is introduced by adding corn starch or polymer particles in the interlayer. They are burned out during the elaboration process (tape-casting). Dense and porous layers have the same thickness (≈ 100 μm). Laminated specimens made of 20 layers have been tested under 3-point flexure loadings.
The analyses of crack deflections by interfaces are generally based on two models due to He and Hutchinson (1989 [4, 5]). Both are carried out in an unbounded domain made of two elastic materials. In the first one [4], the primary crack lies in one material and impinges on the interface. Two virtual crack extensions are considered, one along the interface and one in the adjacent material. The energy release rates at the tip of these two extensions are compared. The drawback of this approach is the arbitrary choice of the two increment lengths. In the other one [5], the primary crack lies along the interface and the ability of the crack to leave the interface is studied. The two criteria involve the toughness of the materials and of the interface. Curiously, it is often this second paper that is referred to interpret the experimental results of cracks deflection in ceramic laminates, although the main assumption, a long primary interface crack, is not fulfilled.

The criterion we proposed here derives from an energy balance and takes into account an additional stress criterion. This allows avoiding the above mentioned drawback.

2 THE ASYMPTOTICS OF THE PROBLEM

The model is based on a two-scale analysis, the small parameter being the layers thickness. At the macro scale the material is homogenized using a rule of mixture for simplicity. More sophisticated homogenization processes do not bring significant differences in the final results. There is a primary crack which tip undergoes the classical mode I. The antisymmetric mode II is inhibited due to the symmetries.

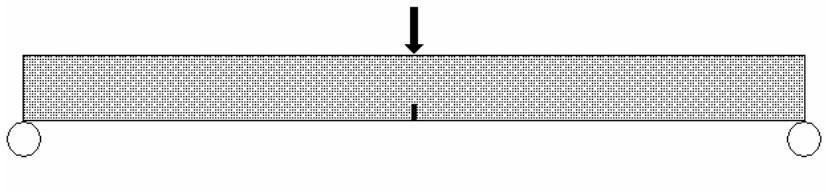

Figure 1: The 3-point flexure loading on the pre-cracked specimen.

Within this framework, the solution (so-called the far field) prior to any crack growth writes

$$\underline{U}^0(x_1,x_2) = \underline{U}^0(0,0) + k_I\sqrt{r}\,\underline{u}_I(\theta) + ... \qquad (1)$$

Here x_1 and x_2 are the Cartesian coordinates and r and θ the polar ones.

Considering now a small crack extension ℓ, the perturbed solution is expressed as a correction brought to the initial term (1)

$$\underline{U}^\ell(x_1,x_2) = \underline{U}^0(x_1,x_2) + \text{small correction decreasing to 0 as } \ell \to 0 \qquad (2)$$

The micro scale is obtained by stretching the domain around the primary crack tip by $1/e$ where e is the layers thickness. Considering the limit $e \to 0$, the problem is now settled in an unbounded domain, so-called inner domain. In order to have tractable computations, this domain is artificially bounded at a large distance (>>1, where 1 is the dimensionless stretched thickness of the layers) of the primary crack tip. Secondly, only few dense (D in figure 1) and porous (P in figure 1) layers (3 or 4) are kept in the vicinity of the primary crack tip, the remaining part being replaced by the homogenized material (H in figure 1).

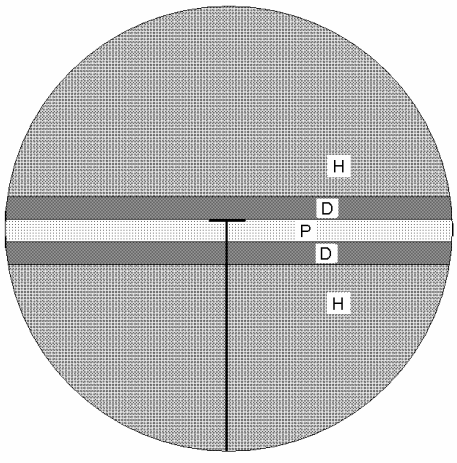

Figure 2: The simplified inner stretched domain.

Using the change of variable $y_i = x_i / e$ ($\rho = r / e$), $\underline{U}^0$ can be expanded as (near field)

$$\underline{U}^0(x_1, x_2) = \underline{U}^0(\ell y_1, \ell y_2) = \underline{U}^0(0,0) + k_I \sqrt{e}\, \underline{W}(y_1, y_2, 0) + ... \tag{3}$$

where the 0 in $\underline{W}$ recalls that there is no crack extension. The function $\underline{W}$ is solution to the following problem

$$\begin{cases} -\nabla.\sigma = 0 \text{ (equilibrium)} \\ \sigma = C : \nabla \underline{W} \text{ (constitutive equation, } C \text{ is the elastic operator)} \\ \sigma.\underline{n} = 0 \text{ along the crack faces (boundary condition)} \\ \underline{W} \text{ behaves like } \sqrt{\rho}\,\underline{u}(\theta) \text{ at infinity} \end{cases} \tag{4}$$

The elastic operator C takes different values in the dense and porous layers and in the homogenized remaining part and ∇ refers to derivatives with respect to y_1 and y_2. The condition at infinity is the matching with the mode I term involved in the far field.

Similarly, a crack extension ℓ (a deflection at the interface porous/dense is illustrated in figure 2) leads to the following expansion

$$\underline{U}^{\ell}(x_1, x_2) = \underline{U}^{\ell}(\ell y_1, \ell y_2) = \underline{U}^0(0,0) + k_I \sqrt{e}\, \underline{W}(y_1, y_2, \mu) + ... \tag{5}$$

where $\mu = \ell / e$ is the dimensionless crack extension length. Here $\underline{W}$ must fulfil the same system of equations (4), the tension free condition (4_3) being extended to the faces of the crack extension.

3 THE DEFLECTION CRITERION

Within this framework, the leading term of the change in potential energy between the two states (prior to and following a crack extension) writes

$$\delta W = k_I^2 [A(E_p / E_d, \mu) - A(E_p / E_d, 0)]\, e \tag{6}$$

where E_d and E_p are respectively the Young's modulus of the dense and porous ceramics (the Poisson's ratio plays a minor role), A is numerically derived from the displacement field $\underline{W}$ using a contour integral (Leguillon 2002 [6]). A necessary condition for the crack growth is a consequence of an energy balance

$$\delta W \geq G^c \ell \Rightarrow k_I^2 \frac{A(E_p / E_d, \mu) - A(E_p / E_d, 0)}{\mu} \geq G^c \tag{7}$$

where G^c is the toughness in the direction of fracture. This expression must be considered twice, once for a deflection (index *def* in the following) and once for a penetration in the next layer (index *pen*). Deflection is promoted if the above inequality holds for deflection while it is wrong for penetration, it leads to

$$\frac{\mu_{pen}}{\mu_{def}} \frac{A_{def}(E_p / E_d, \mu_{def}) - A(E_p / E_d, 0)}{A_{pen}(E_p / E_d, \mu_{pen}) - A(E_p / E_d, 0)} \geq \frac{G_p^c}{G_d^c} \tag{8}$$

where G_d^c and G_p^c are the toughness of the dense and porous ceramics. It is assumed here that the toughness along the interface between the dense and porous ceramics is that of the porous material (note that if the interface was stronger then the crack would grow within the porous medium at a short distance of the interface). Clearly the dimensionless crack increment lengths play a role in the above relation. In this step, we make the following reasonable additional assumption, if the crack penetrates the next layer then it breaks it completely: $\ell_{pen} = e \Rightarrow \mu_{pen} = 1$. The deflected extension length remains to be determined. It could be done using a maximum stress criterion

(Leguillon 2002 [6]). For simplicity, we assume here that $\mu_{def} = \mu_{pen} = 1$. Nevertheless, complete computations have been done (Cherti Tazi 2004 [7]) and it has been observed that the deflection length increases with the porosity but that the final results are not strongly modified.
Finally the criterion takes the simplified form

$$g(E_p / E_d) \geq G_p^c / G_d^c \tag{9}$$

with

$$g(E_p / E_d) = \frac{A_{def}(E_p / E_d, 1) - A(E_p / E_d, 0)}{A_{pen}(E_p / E_d, 1) - A(E_p / E_d, 0)} \tag{10}$$

It is clear from this expression that the crucial point in this criterion is the knowledge of the ratio of the elastic and fracture parameters in terms of the porosity.

4 CHARACTERISTIC PARAMETERS OF THE POROUS CERAMIC

The two figures 3 and 4 show that a unique function depending on the volume fraction of pores V can be used to express the elastic and fracture parameters of the porous material. It can be either

$$E_p = H(V)E_d \ ; \ G_p^c = H(V)G_d^c \ \text{ with } \ H(V) = 1 - 6V/\pi \tag{11}$$

or

$$E_p = K(V)E_d \ ; \ G_p^c = K(V)G_d^c \ \text{ with } \ K(V) = 1 - 4S/\pi = 1 - (6V/\pi)^{2/3} \tag{12}$$

In both cases a cubic lattice of spherical voids is assumed and the parameters of the porous ceramic vanish at percolation of pores ($V = \pi/6 = 0.52$). In the first case the parameters depend linearly on the volume fraction of pores V, whereas they depend linearly on the largest surface fraction of pores S (the fracture surface) in the second case.
As a consequence, the deflection criterion (9) rewrites

$$h(V) \geq H(V) \ \text{ with } \ h(V) = g(1 - 6V/\pi) \quad \text{or} \quad k(V) \geq K(V) \ \text{ with } \ k(V) = g(1 - (6V/\pi)^{2/3}) \tag{13}$$

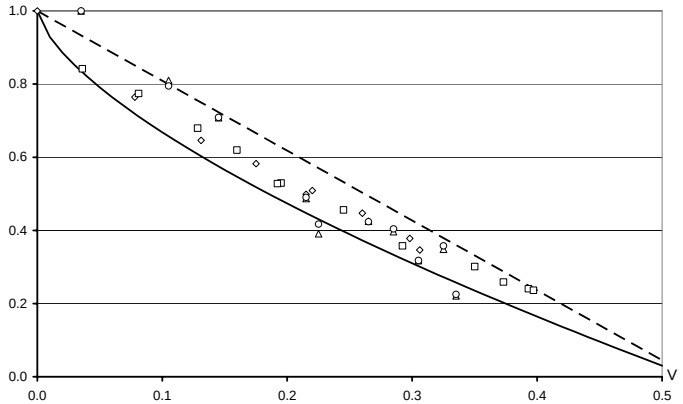

Figure 3: Young's modulus ratio vs. porosity V : SiC with Polyamide particles (diamonds), SiC with corn starch particles (squares), B_4C with corn starch particles (triangles). Shear modulus ratio vs. porosity V : B_4C with corn starch particles (circles). The dashed line is the function $H(V)$, the solid line is the function $K(V)$.

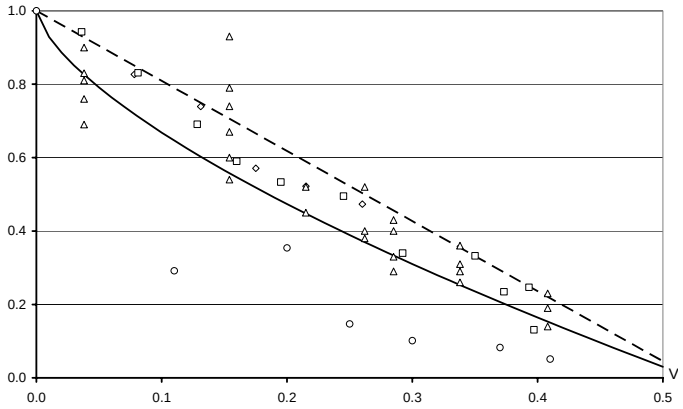

Figure 4: Toughness ratio vs. porosity V : SiC with Polyamide particles (diamonds), SiC with corn starch particles (squares), B_4C with corn starch particles (circles), SiC with PTFE particles (triangles) (data from Blanks et al. 1998 [1]). $H(V)$ dashed line, $K(V)$ solid line.

5 NUMERICAL RESULTS AND CONCLUSION

The function g (10) and the toughness ratio are plotted vs. the Young's moduli ratio (figure 5) at a porous/dense interface (a similar analysis shows that no deflection can occur at the dense/porous interface). It is simply assumed that the two ratios follow the same rule (whatever this rule).
The criterion can be also plotted vs. the porosity. It is illustrated in the two following figures. In the first one (figure 6), the elastic and fracture parameters depend linearly on the volume fraction of pores, whereas in the second (figure 7) they depend linearly on the surface fraction of pores. Clearly the predicted porosity that causes crack deflection (arrows in figures 6 and 7) is above 40% in both cases. It is in a good agreement with the experiments of Reynaud [2] and Tariolle [3] while the He and Hutchinson approach underestimates it. Blanks et al. [1] found a slightly lower value of the porosity (between 34% and 44%), an explanation could be that their interfaces between porous and dense ceramics are weaker (impurities) than those tested in [2] and [3].

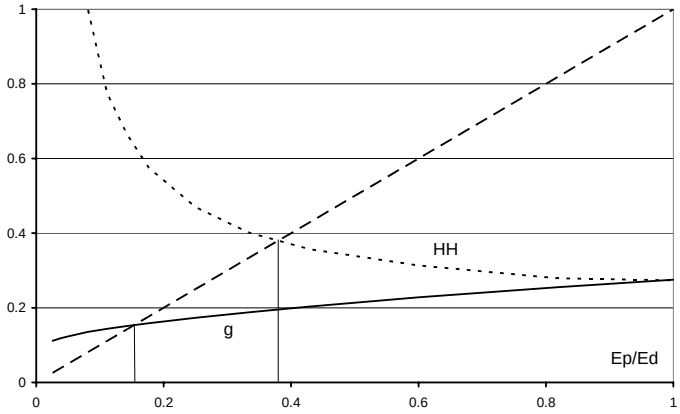

Figure 5: The function g (solid line) vs. the Young's moduli ratio E_p / E_d and the He and Hutchinson approach (HH, dotted line) compared to the toughness ration G_p^c / G_d^c (dashed line).

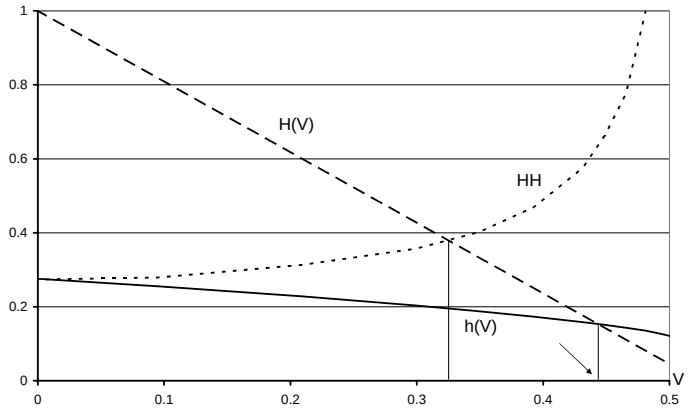

Figure 6: The function h (solid line) vs. the volume fraction of pores and the He and Hutchinson approach (HH, dotted line) compared to the toughness ration $H(V) = G_p^c / G_d^c$ (dashed line).

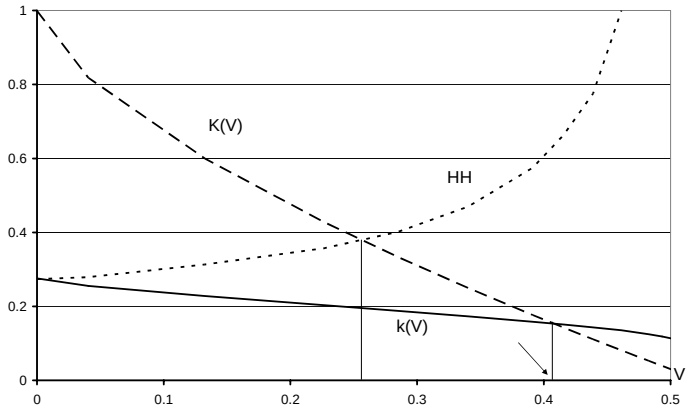

Figure 7: The function k (solid line) vs. the volume fraction of pores and the He and Hutchinson approach (HH, dotted line) compared to the toughness ration $K(V) = G_p^c / G_d^c$ (dashed line).

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