INFLUENCE OF CONTINUOUS NUCLEATION OF SECONDARY VOIDS UPON GROWTH AND COALESCENCE OF CAVITIES IN POROUS DUCTILE METALS

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ABSTRACT

We study the influence of the nucleation of secondary small voids upon growth and coalescence of primary large cavities in porous ductile metals. To do so, we consider a simple model, the loading of a hollow sphere subjected to hydrostatic loading whereby the sphere simulates a big, primary void, the presence of secondary voids in the surrounding matrix being accounted for by the Gurson-Tvergaard-Needleman homogenized model for plastic porous materials. Continuous nucleation of small voids in the matrix is described by a simple phenomenological formula proposed by Pineau and Joly for the evolution of porosity due to nucleation. The problem is solved analytically at initial time and numerically at later times, when a non-uniform distribution of secondary voids develops in the matrix. These elements are used to define a simplified model with only a small number internal parameters, describing growth of primary voids coupled with nucleation, growth and coalescence of secondary ones. This study opens the way to the analysis of coalescence of primary voids assisted by secondary ones.

1 INTRODUCTION

The macroscopic mechanical behaviour of a porous plastic metal containing voids of identical size is widely admitted to be adequately described by Gurson's famous model [1], as extended and improved by Tvergaard and Needleman [2, 3]. However, there is strong experimental evidence that two "populations" of cavities with widely different sizes coexist in many materials. Perrin and Leblond [4] used a theoretical approach to show that, prior to coalescence of voids, one can safely ignore this feature and simply use the Gurson-Tvergaard-Needleman (GTN) model with a single internal parameter, the total porosity of both populations of cavities. However the picture is different when coalescence sets in. Indeed it was suggested by Marini *et al.* [5] on experimental grounds, and confirmed by Perrin and Leblond [6] on theoretical grounds, that early coalescence of small cavities near large ones considerably accelerates growth and final coalescence of the latter. It then becomes compulsory to distinguish the contributions of either population to the overall porosity and to use distinct evolution equations for each of them.

Perrin and Leblond's approach to this question [6] was based on the analysis of a simple model problem, a hollow sphere made of plastic porous material and subjected to hydrostatic tension. The central hole represented a single big, primary void; interactions between primary voids were thus disregarded. Presence of a secondary population of smaller cavities in the surrounding matrix was accounted for by the GTN model, which yields the local behaviour of the matrix. Nucleation of small voids was threfore disregarded, although in typical experiments small voids are known to be continuously nucleated during the whole fracture process. The aim of the present paper is to go beyond this approximation, that is to introduce nucleation of secondary voids in the model of Ref. [6]. This is done by using a simple phenomenological formula suggested by Pineau and Joly [7] for the rate of evolution of the porosity arising from nucleation.

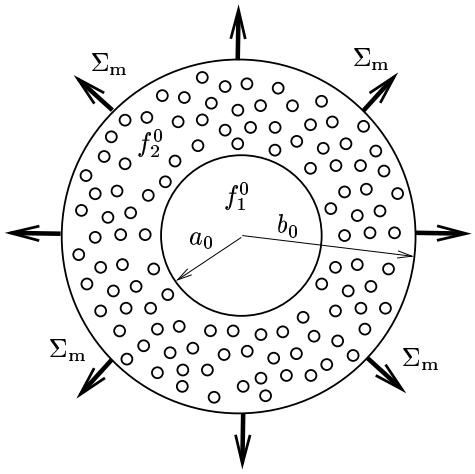

Figure 1: Model problem considered

2 THE MODEL PROBLEM

The model problem is schematized in Fig. 1 at the initial time. The sphere, with internal and external radii a_0 and b_0, is subjected to hydrostatic tensile macroscopic loading $(\Sigma_{\rm m}\,\mathbf{1})$. The porosity of the matrix, which arises from a homogeneous distribution of finely dispersed voids, is denoted f_2^0. The porosity arising from the big central hole is denoted f_1^0, and defined as[1]

$$f_1^0 \equiv (1 - f_2^0)\frac{a_0^3}{b_0^3}. \tag{1}$$

The matrix material is rigid-ideal plastic and obeys the following criterion:

$$\frac{\sigma_{\rm eq}^2}{\sigma_0^2} + q\, f_2^0\ \exp\left(\frac{3}{2}\frac{\sigma_{\rm m}}{\sigma_0}\right) - 1 \leq 0, \tag{2}$$

where $\sigma_{\rm eq}$ and $\sigma_{\rm m}$ denote the local equivalent and mean stresses, σ_0 the yield stress of the void-free material, and q Tvergaard's parameter [2]; the value $q = 4/e \approx 1.47$ derived by Perrin and Leblond [4, 6] from both differential and self-consistent schemes is used in this work. Equation (2) represents a simplified version of the GTN criterion leading to results quite similar, although not completely identical, to those obtained with the original criterion while allowing for much simpler calculations. The local strain rate $\mathbf{d}$ is given by the flow rule associated to the yield criterion (2) through normality. The porosity rate in the matrix, arising from both growth and nucleation of voids, is given by

$$\dot{f}_2 = (1 - f_2^0)\,{\rm tr}\,\mathbf{d} + A\,\dot{\epsilon}, \qquad (1 - f_2^0)\,\sigma_0\,\dot{\epsilon} = \boldsymbol{\sigma} : \mathbf{d} \tag{3}$$

where A is a material parameter. The simple expression $A\dot{\epsilon}$ of the porosity rate due to nucleation was proposed by Pineau and Joly [7] on experimental basis. A more elaborate expression involving dependence of the parameter A upon ϵ was proposed by Chu and Needleman [8], but that used here seems sufficient in view of the general lack of experimental data concerning nucleation.

[1] The factor $(1 - f_2^0)$ here is introduced in order to make porosities additive, as discussed by Perrin and Leblond [4, 6].

3 ANALYTICAL SOLUTION AT THE INITIAL INSTANT

The problem can be solved analytically at the initial time, when the porosity in the matrix is still uniform; the results are quite simple in the limit of small porosities. First, the stresses are obtained through combination of the radial equilibrium equation and the yield criterion (assuming the entire volume to be plastic); the result reads, in spherical coordinates r, θ, ϕ:

$$\left\{ \begin{array}{l} v \equiv \dfrac{\sigma_{\theta\theta} - \sigma_{rr}}{\sigma_0} = \dfrac{1}{2\mu}\left(-\mu - 1 + \sqrt{\mu^2 + 6\mu + 1}\right) \quad , \quad \mu \equiv f_2^0 \dfrac{r^3}{a_0^3} \\ u \equiv \dfrac{\sigma_{rr}}{\sigma_0} = \dfrac{2}{3}\left(-v + \log \dfrac{1 - v^2}{q f_2^0}\right). \end{array} \right. \tag{4}$$

Also, $\Sigma_{\mathrm{m}} = \sigma_{rr}(r = b_0) = \sigma_0 u_b$ where $u_b \equiv u(r = b_0)$. The (radial) velocity $\mathcal{U}$ is next deduced from the flow rule; the result reads

$$\mathcal{U} = \frac{a_0^2 \dot{a}}{r^2 v}. \tag{5}$$

Finally the porosity rate in the matrix is deduced from eqn (3) and reads:

$$\dot{f}_2 = \frac{\dot{a}}{a_0}\left[\frac{3}{2} f_2^0 \left(1 + \frac{\mu + 3}{\sqrt{\mu^2 + 6\mu + 1}}\right)(1 + Au) + 2A\frac{a_0^3}{r^3}\right]. \tag{6}$$

In particular, at $r = a_0$, this formula reduces to

$$\dot{f}_2(r = a_0) = \frac{\dot{a}}{a_0}\left(6 f_2^0 + 2A\right) = 2\frac{\dot{f}_1}{f_1^0}\left(f_2^0 + \frac{A}{3}\right). \tag{7}$$

4 MECHANISM OF DAMAGE AROUND THE CENTRAL HOLE

In fact it can be shown that an equation similar to (7), with f_1^0 and f_2^0 replaced by f_1 and $f_2(r = a)$, holds at all instants. Time integration then yields

$$f_2(r = a) = f_2^0 \left(\frac{f_1}{f_1^0}\right)^2 + \frac{A}{3}\left[\left(\frac{f_1}{f_1^0}\right)^2 - 1\right]. \tag{8}$$

This equation shows that small voids located at the boundary of the big central void grow much more rapidly than the latter; this effect was first pointed out by Perrin and Leblond [6] in the absence of nucleation, and is of course further enhanced by nucleation. It follows that these small voids may reach coalescence before the big central one. If such is the case, a spherical shell of ruined matter will develop around the big void, thus enhancing its growth rate.

5 NUMERICAL SOLUTION AFTER THE INITIAL INSTANT

In order to study this damage mechanism in more detail, one must obtain the solution after the initial instant. The porosity then becomes non-uniform in the matrix and the problem is no longer amenable to analytic solution. It is therefore solved numerically. The positions of discretized material points and the corresponding porosities are calculated incrementally in time, the stresses serving as auxiliary variables. Following a simplified variant of Tvergaard and Needleman [3]'s idea, coalescence of small cavities is simulated by assuming that the material is suddenly and completely ruined when

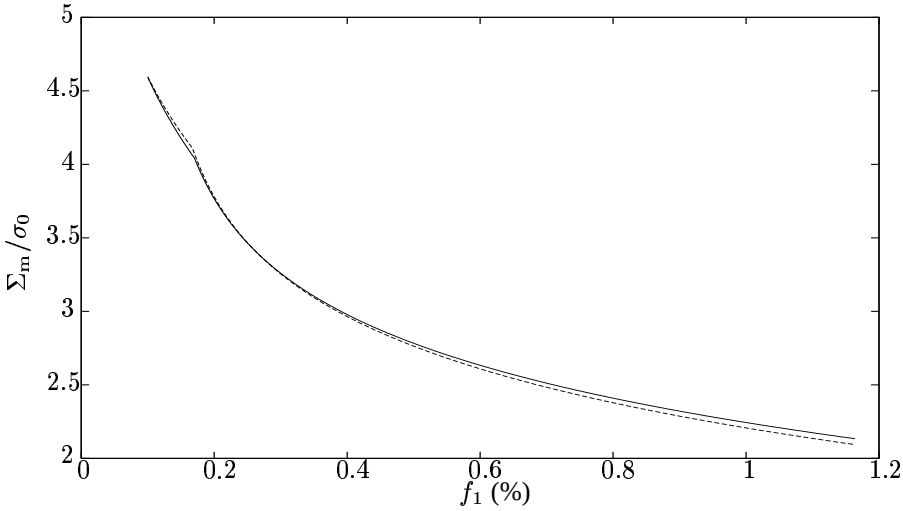

Figure 2: Normalized macroscopic stress versus true porosity of the central void plus ruined zone - Full line: numerical calculation, dashed line: model

the local porosity f_2 reaches some critical value f_2^c. Thus, at each instant, the distribution of porosity is first calculated without accounting for evolution of the ruined zone, then its present radius a^* is determined from the condition $f_2(r = a^*) = f_2^c$, and the region $r \leq a^*$ is "eliminated" by shifting the boundary condition $\sigma_{rr} = 0$ to $r = a^*$ for the next time step.

Figures 2, 3 and 4 show, for $f_1^0 = 10^{-3}$, $f_2^0 = 10^{-5}$, $A = 0.05$ and $f_2^c = 0.03$, Σ_m/σ_0, $\bar{f}_2$ and f_1^* as functions of f_1. Here $\bar{f}_2$ denotes the average porosity of small voids in the not-yet-ruined region $r > a^*$; f_1^* denotes the "effective" porosity of the zone $r \leq a^*$, defined by an equation similar to (1) with $\bar{f}_2$, a^* and b instead of f_2^0, a_0 and b_0, that is by artificially considering this zone as empty; and f_1 denotes its real porosity, that is accounting for the fact that some metal is enclosed in it, although it can no longer sustain any mechanical load. It is clear from Fig. 2 that the stress-bearing capacity of the material decreases more quickly when coalescence of small voids starts. Also, Fig. 4 shows that the effective porosity f_1^* of the zone $r \leq a^*$ (which governs, together with $\bar{f}_2$, this stress-bearing capacity) then starts to depart from its real porosity f_1 and ultimately becomes quite larger. In contrast, Fig. 3 shows that the growth rate of the average porosity $\bar{f}_2$ in the not-yet-ruined region is not significantly affected by coalescence of small cavities.

6 SIMPLIFIED MODEL

Using the analytical solution at the initial instant and elements of such a solution at later instants, one can define a simplified model for porous ductile metals with two populations of cavities reproducing the main features of the model problem, while containing only 3 internal parameters, f_1, f_1^* and $\bar{f}_2$, which are numbers rather than function of r as in the original model, which obey simple evolution laws. The model first involves a Gurson-like criterion:

$$\frac{\Sigma_{eq}^2}{\sigma_0^2} + 2q\left(f_1^* + \bar{f}_2\right)\cosh\left(\frac{3}{2}\frac{\Sigma_m}{\sigma_0}\right) - 1 - q^2\left(f_1^* + \bar{f}_2\right)^2 = 0, \tag{9}$$

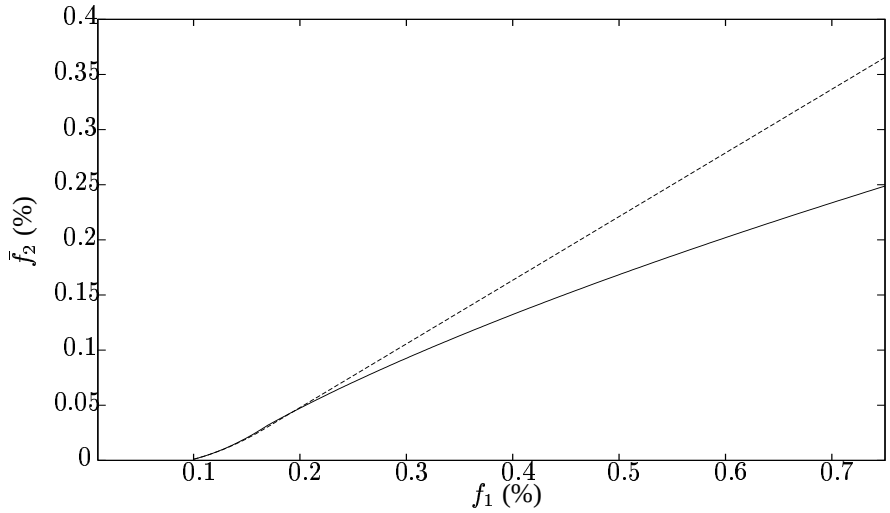

Figure 3: Average porosity in the not-yet-ruined region versus true porosity of the central void plus ruined zone - Full line: numerical calculation, dashed line: model

where Σ_{eq} and $\Sigma_{\rm m}$ denote the macroscopic equivalent and mean stresses. The macroscopic strain rate $\mathbf{D}$ obeys the associated flow rule. Finally, t_0 and t_c denoting the inital instant and that corresponding to the onset of coalescence of small voids, the porosity rates are given by

$$\left\{\begin{array}{rcl} \dot{f}_1+\dot{\bar{f}}_2 &=& \left(1-f_1-\bar{f}_2\right)\,\mathrm{tr}\,\mathbf{D}+A\,\dot{\bar{\epsilon}} \quad , \quad \dot{\bar{\epsilon}}=\kappa\dot{f}_1 \\ \dfrac{\dot{f}_1^*}{\dot{f}_1} &=& 1 \quad \text{if} \quad f_1\leq f_1(t_c) \quad , \quad \dfrac{9f_2^c+A\left[\left(\frac{f_1(t_c)}{f_1^0}\right)^3+2\right]}{A\left[\left(\frac{f_1(t_c)}{f_1^0}\right)^3-1\right]} \quad \text{if} \quad f_1>f_1(t_c) \\ \dfrac{\dot{\bar{f}}_2}{\dot{f}_1} &=& \gamma\dfrac{\bar{f}_2}{f_1}+A\kappa \quad \text{if} \quad f_1\leq f_1(t_c) \quad , \quad \gamma\dfrac{\bar{f}_2(t_c)}{f_1(t_c)}+A\kappa \quad \text{if} \quad f_1>f_1(t_c), \end{array}\right. \tag{10}$$

$$\left\{\begin{array}{l} \kappa\equiv\dfrac{u_b(t_0)}{v_b(t_0)} \quad , \quad \gamma\equiv\dfrac{1}{2}\left(-\chi+1+\sqrt{\chi^2+6\chi+1}\right) \quad , \quad \chi\equiv\dfrac{f_1^0}{f_2^0} \\ f_1(t_c)=f_1^0\sqrt{\dfrac{f_2^c+A/3}{f_2^0+A/3}} \quad , \quad \bar{f}_2(t_c)=\left(f_2^0+\dfrac{A\kappa}{\gamma-1}f_1^0\right)\left(\dfrac{f_1(t_c)}{f_1^0}\right)^\gamma-\dfrac{A\kappa}{\gamma-1}f_1(t_c). \end{array}\right. \tag{11}$$

Figures 2, 3, 4 show that the predictions of this model for a hydrostatic macroscopic loading ($\Sigma_{eq}=0$) are in acceptable agreement with the results of the full numerical simulation of the model problem.

The model thus defined accounts for couplings between growth of big voids and nucleation, growth and coalescence of small ones. It does *not* account for coalescence of big voids since the model problem which serves as its basis involves only one such void. Investigation of this question requires new ideas which will be expounded in another paper.

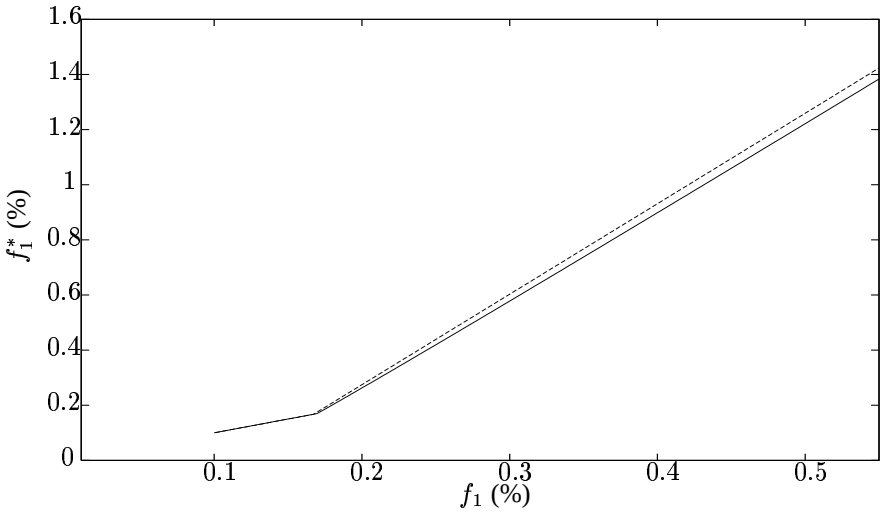

Figure 4: Effective versus true porosity of the central void plus ruined zone - Full line: numerical calculation, dashed line: model

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