SCALING OF QUASI-BRITTLE FRACTURE: A BOUNDARY EFFECT MODEL

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ABSTRACT

The common size effect on quasi-brittle fracture of concrete-like materials is analysed by a simple asymptotic analysis based on a boundary effect model originally proposed for a large plate with a small edge crack. The large plate analysis considers exclusively the interaction of the crack-tip fracture process zone with the specimen front face and its influence on the fracture conditions. The new asymptotic model considers both the specimen boundary and size, and thus extends the boundary effect model to the size effect study on finite-sized specimens. The new boundary effect model shows that the specimen size alone is not sufficient in determination of the size effect on quasi-brittle fracture behaviour, the specimen boundary conditions have to be considered as well. It is shown that even very large specimens that normally do not show any size effect can still experience quasi-brittle fracture if they contain very shallow or very deep cracks. The new boundary effect model uses the two well-defined fracture criteria, the tensile strength and fracture toughness, as its two asymptotic limits, which allows the determination of the two important material constants from the quasi-brittle fracture results. The size effect issue in concrete specimens without initial notches is also studied by the asymptotic model after assuming the natural pre-existing defects in concrete as very shallow notches. The geometrically similar specimen condition can be satisfied by those assumed shallow notched specimens.

1 INTRODUCTION

Size effect, or the quasi-brittle fracture transition behaviour occurring with the variation in specimen size, has been studied for many years as neither the traditional strength nor fracture toughness criterion applies (Bažant [1], Carpinteri & Chiaia [2], Karihalloo [3], Hu & Wittmann [4], Duan & Hu [5-7], Duan et al [8]). Most commonly, the specimen size, W, is taken as the sole variable and the geometrical similarity in specimens has been taken as the prerequisite condition in size effect modelling. Under those conditions, typically two unknown parameters need to be determined experimentally from the quasi-brittle fracture results through curve-fitting. It is known that those two experimentally-determined parameters may vary even for a single material if different specimen geometry and loading conditions are involved.

Clearly, it would be preferred if the two experimental parameters were constants and independent of specimen geometry and loading condition. Furthermore, it would make experiments lots easier if the geometrical similarity condition could be removed. In fact, when the ligament or size effect on the specific fracture energy G_F is considered, specimens of a given size but different notch (and then ligament) lengths are often preferred instead of geometrically similar specimens (e.g. Hu & Wittmann [9]).

We have studied the quasi-brittle fracture behaviour of a large plate with a small edge crack (Hu [10,11], Hu & Wittmann [4], Duan & Hu [5-7], Duan et al [8]). Indeed, two material constants, the tensile strength f_t and fracture toughness K_{IC} have been used in the asymptotic model. Although the specimen size is large enough, the large plate can still experience quasi-brittle fracture or even pure strength controlled failure depending on the crack length and the crack-tip fracture process zone (FPZ) size. As expected, the fracture toughness K_{IC} criterion applies if the edge crack is long enough. Actually, the fracture transition of the large plate from the f_t to K_{IC} criterion is identical to the size effect problem commonly studied using geometrically similar specimens of different sizes. The only difference is the specimen size of the large plate does not vary, but the crack length varies, which shows how the specimen front face and FPZ influence the quasi-brittle fracture behaviour. However, it is noted that the large plate assumption is not as practical as the geometrically similar specimen condition.

Therefore, the objective of the present study is to extend the previous asymptotic model for the special case of a large plate to more common finite-sized specimens, and to show that two material constants, f_t and K_{IC}, can be used as two scaling constants instead of adopting two experimental parameters. It is also going to be shown that geometrical similarity in specimens is not necessary for the size effect study. Therefore, for a given material, finite-sized specimens of different size and geometry under different loading conditions will follow a unique asymptotic curve specified by f_t and K_{IC}.

2 ASYMPTOTIC SOLUTION FOR QUASI-BRITTLE FRACTURE

2.1 Boundary effect model for large plate

Here, the definition of a large plate is that the geometry factor $Y = 1.12$ for the range of the edge crack length under investigation and the specimen size is large enough so that K_{IC} is applicable if the initial short edge crack is increased while Y remains as 1.12. Therefore, the specimen size needs not to be considered here or can be taken as a constant.

The asymptotic solution for the nominal strength of the large plate with a small edge crack shown in Figure 1(a) has been derived (Hu [10], Hu & Wittmann [4]),

$$\sigma_N = \sigma_n = \frac{f_t}{\sqrt{1 + a/a^*_\infty}} \qquad (1a)$$

$$a^*_\infty = \frac{1}{\pi \cdot Y^2} \cdot \left(\frac{K_{IC}}{f_t}\right)^2 \approx 0.25 \cdot \left(\frac{K_{IC}}{f_t}\right)^2 \qquad (1b).$$

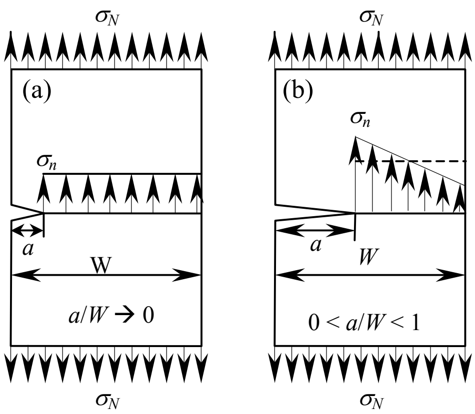

Figure 1: Two nominal strengths for (a) large plate and (b) SENT specimen: σ_N without consideration of the crack and σ_n with consideration of the crack.

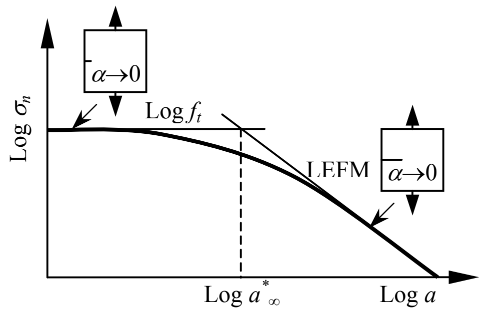

Figure 2: Boundary-effect asymptotic curve, eqn (1) for a large plate with a small edge crack where $Y = 1.12$ and $a^*_\infty = 0.25 \cdot (K_{IC}/f_t)^2$.

The nominal strength σ_N does not consider the presence of the edge crack while σ_n does. They are identical for a large plate, but different for finite-sized specimens, e.g. as shown in Figure 1(b).

The condition Y = 1.12 has been used in eqn (1), and the subscript ∞ indicates the large plate condition. The reference crack a^*_∞ is a material constant, and is a measurement of the crack-tip FPZ for a quasi-brittle material, or the crack-tip plastic zone for a ductile material since it is proportional to $(K_{IC}/f_t)^2$. The reference a^*_∞ is also illustrated in Figure 2.

2.2 Boundary effect model for finite-sized specimens

The common single-edge-notch-tension (SENT) specimens illustrated in Figure 1(b) provide the most direct comparison to the large plate discussed in the previous section. In this case, the geometry factor Y is not equal to 1.12, but depends on the crack size, or the α-ratio (= a/W).

As shown in Figure 1(b), the two nominal strengths are related as follows:

$$\sigma_N = \mathrm{A}(\alpha)\cdot\sigma_n \tag{2a}$$

$$\mathrm{A}(\alpha) = \frac{(1-\alpha)^2}{1+2\alpha} \tag{2b}$$

The two nominal strengths are identical when α = 0, which is the case for the large plate. σ_N is used for the stress intensity factor formulae, and has been commonly used for various size effect models. Here, σ_n is preferred for the following reasons. It is expected that the tensile strength criterion f_t will apply if $\alpha \to 0$ and $\to 1$. If σ_N is used in a model, it will become zero and the asymptotic limit for $\alpha \to 1$ will not be satisfied. The nominal strength σ_n considering the presence of the crack is clearly a better choice.

For any given material, a suitable specimen size can always be found so that the K_{IC} criterion applies for a moderate α-ratio around 0.5. In this case, the following classic relationship is valid.

$$K_{IC} = \sigma_N \cdot Y(\alpha)\cdot\sqrt{\pi\cdot a} \tag{3a}$$

$$Y(\alpha) = \sqrt{\frac{2}{\pi\cdot\alpha}\cdot\tan\left(\frac{\pi\cdot\alpha}{2}\right)}\cdot\frac{0.752+2.02\alpha+0.37\cdot(1-\sin(\pi\cdot\alpha/2))^3}{\cos(\pi\cdot\alpha/2)} \tag{3b}$$

The geometry factor $Y(\alpha)$ for SENT specimens is found in [12]. Note that σ_N instead of σ_n is used in eqn (3). Now, we can solve the nominal strength σ_n from eqn (3), i.e.

$$\sigma_n = \frac{K_{IC}}{\mathrm{A}(\alpha)\cdot Y(\alpha)\cdot\sqrt{\pi a}} = \frac{f_t}{\sqrt{\dfrac{\left(\dfrac{\mathrm{A}(\alpha)\cdot Y(\alpha)}{1.12}\right)^2\cdot a}{\dfrac{1}{\pi\cdot(1.12)^2}\cdot\left(\dfrac{K_{IC}}{f_t}\right)^2}}} = \frac{f_t}{\sqrt{\dfrac{\mathrm{B}(\alpha)\cdot a}{a^*_\infty}}} \tag{4a}$$

or

$$\sigma_n = \frac{f_t}{\sqrt{a_e/a^*_\infty}} \tag{4b}$$

Eqn (4) can be taken as the asymptotic limit when the crack ratio $a_e/a^*_\infty >> 1$ or when the K_{IC} criterion applies. If $a/a^*_\infty >> 1$, eqn (1) becomes:

$$\sigma_N = \sigma_n = \frac{f_t}{\sqrt{a/a_\infty^*}} \tag{5}$$

Comparing eqns (4) and (5), the general asymptotic solution for finite-sized specimens can be written as:

$$\sigma_n = \frac{f_t}{\sqrt{1 + a_e/a_\infty^*}} \tag{6}$$

Eqn (6) holds even if neither K_{IC} nor f_t applies. B(α) and a_e used in eqn (4) are as follows:

$$a_e = \mathrm{B}(\alpha) \cdot a \tag{7a}$$

$$\mathrm{B}(\alpha) = \left(\frac{\mathrm{A}(\alpha) \cdot Y(\alpha)}{1.12} \right)^2 \tag{7b}$$

Equations for other geometries such as three-point-bend (3-p-b) or compact tension (CT) are the same. Only A(α) and $Y(\alpha)$ need to be worked out for individual specimen geometry.

Eqn (6) for SENT is shown in Figure 3(a) as scaled by a/a^*_∞ with the non-dimensional specimen size specified by W/a^*_∞. For very small specimens ($W/a^*_\infty = 1$), $\sigma_n \approx f_t$ and the crack size has no influence on fracture. Therefore, the strength criterion applies. Increasing the size to $W/a^*_\infty = 100$, the quasi-brittle fracture behaviour is observed. When the crack reaches the specimen back face ($a = W$), $\sigma_n \approx f_t$ and the strength criterion is again valid. For very big specimens ($W/a^*_\infty = 10{,}000$), we have pure strength controlled failure at both the front and back faces of the specimens, quasi-brittle fracture and K_{IC} controlled fracture regions. The large plate solution from eqn (1) is also shown in Figure 3(a).

Eqn (6) for SENT is shown in Figure 3(b) as scaled by a_e/a^*_∞, which transforms the back face asymptotic solution to that of the front face. The turning points are marked on the curve of the large plate asymptotic solution from eqn (1). Clearly, eqn (1) determines the unique fracture curve while the specimen size W determines the turning point along the master curve. Eqn (6) can be approximated by the pure strength controlled failure if $a_e/a^*_\infty \le 0.1$, and the pure toughness controlled fracture if $a_e/a^*_\infty \ge 10$, as indicated in Figure 3(b).

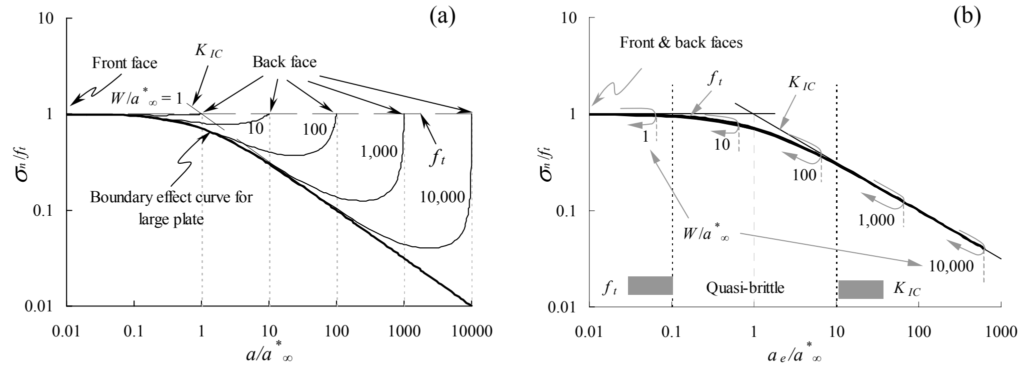

Figure 3: (a) Asymptotic fracture curve of the large plate, and quasi-brittle fracture curves of SENT specimens, with the common starting point at the front face, but different ending points at the back face depending on the specimen size W. (b) The unique asymptotic fracture curve based on eqn (6), with the common starting point for both front and back boundaries. $\alpha = 0.2 \sim 0.4$ at the turning points.

2.3 Comparison of boundary and size effect models

The well-known size effect model proposed by Bažant [1] for geometrically similar specimens is as follows.

$$\sigma_N = \frac{A \cdot f_t}{\sqrt{1 + W/W^*}} \tag{8}$$

Note that σ_N, rather than σ_n, is used. A and W^* need to be determined from experimental results. Comparing eqns (6) and (8), we obtain the following results.

$$A = \mathrm{A}(\alpha) \tag{9a}$$

$$W^* = \frac{a_\infty^*}{\mathrm{B}(\alpha)\cdot\alpha} \tag{9b}$$

A(α) related the two different nominal strengths as shown in eqn (2) can be easily worked out for various specimens such 3-p-b, CT and SENT. B(α) can be worked out following eqn (7) for the corresponding specimen geometry. Figure 4 shows the A and W^* results for those common specimens. Clearly, A and W^* are strongly α-ratio dependent. A and W^* as determined by the size effect model can only be treated as experimental parameters for a particular set of geometrically similar specimens. The present boundary effect model based on the large plate asymptotic provides the detailed expressions for those two parameters as shown in eqn (9).

3 DISCUSSION AND CONCLUDING REMARKS

We have studied experimental results available from the literature (Duan & Hu [5-7], Duan et al [8]). One useful feature of the present boundary effect model is that very shallow cracks (α-ratio $\rightarrow$ 0) can be studied. Therefore, un-notched concrete specimens containing natural distributed defects can be modelled by notched specimens with equivalent shallow notches. For instance, the concrete results (Karihaloo et al [13]) are shown Figure 5 (Duan & Hu [7]), the results from both notched and un-notched specimens are presented together. It is determined that the un-notched specimens have the equivalent α-ratio of around 0.012.

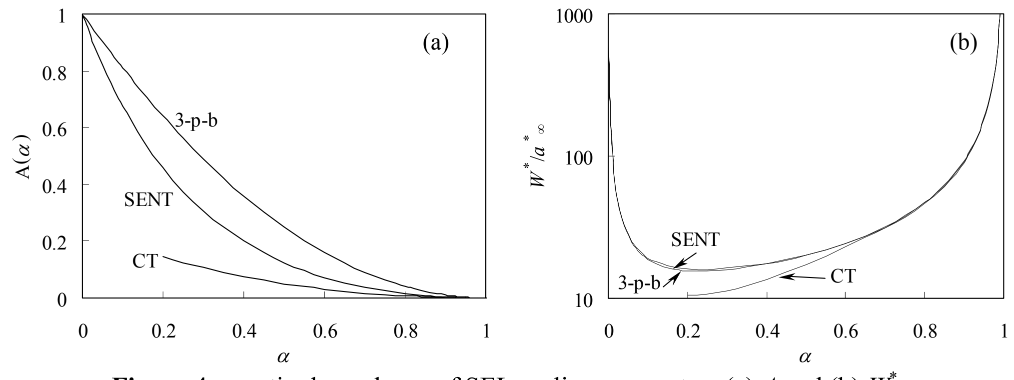

Figure 4: α-ratio dependence of SEL scaling parameters (a) A and (b) W^*.

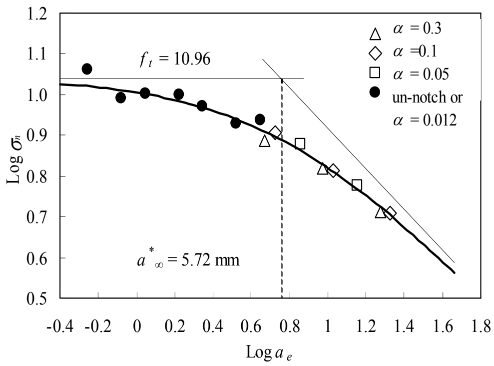

Figure 5: Comparison of the prediction using eqn (6) with the σ_n data of high strength concrete (HSC) measured on both notched and un-notched 3-p-b beams (Karihaloo et al [13]).

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