STRESS INTENSITY FACTORS FOR COMBINED TEMPERATURE GRADIENT AND INTERNAL PRESSURE FOR DAMAGED CYLINDERS

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ABSTRACT

It is not unusual for cylindrical parts used in energy generation equipments to suffer damage due to thermal shock. Cracks originated in this process are usually shallow and located nearby stress concentration area. If the thermal shock happens to occur during the start up process, cracks may show at the outer face, but if the part is suddenly depressurized or receive cold fluid, cracks are bound to show up at cylinder's inner face. The resulting stresses (due to internal pressure and temperature gradients) will determine if the cracks will propagate or not. Positive net stresses may act catastrophically over damaged pressure vessels. This work explores the described situation and proposes controlling crack propagation by means of altering temperature difference between cylinder's walls. Weight functions approach is used to expresses acting stress intensity factors.

1 INTRODUCTION

It is not unusual for cylindrical parts used in energy generation equipments to suffer damage due to thermal shock. Unexpected or unpredicted occurrences during operation may produce some forms of damage that in the long run will affect safety and production (Megvesy [1]). One of the most dangerous forms of damage is the crack opening due to thermal shock, a single episode or repeated ones (Lu [2]). Cracks originated in this process are usually shallow and located nearby stress concentration area (Kim [3]). If the thermal shock happens to occur during the start up process, cracks may show at the outer face, but if the cylinder is suddenly depressurized or received a relatively cold fluid, cracks are bound to show up at cylinder's inner face (Xu [4]). Two possible situations may also arise for some parts, which are submitted to different temperature distribution, namely the outside temperature higher or lower than the internal one. If the outer temperature is higher the internal one, the cylinder inner face will be submitted to tensile hoop stress, which is counter balanced by a compressive stress on the outside (Timoshenko [5]). Another critical situation happens when the internal temperature is higher than the external one and the crack is on the outside. Once again, the temperature distribution causes a positive hoop stress. The reverse situations (To>Ti, external crack; Ti>To, internal crack) are not critical, once hoop stresses act to close the crack. Superposed to thermal stress, the internal pressure must also be considered, as it does produce positive hoop stresses (Shigley [6]).

This work explores the described situations and briefly discusses the possibilities of controlling crack propagation by means of altering temperature gradients associated with convective heat transfer outside the vessel. To deal with the expressions for the Stress Intensity Factors (SIFs) the weight functions approach is used (Bueckner [7], Rice [8], Anderson [9]).

2 WEIGHT FUNCTIONS

2.1 Reference cases

Along this work three reference cases were taken from the literature (Wu[10]). They are:

$$\frac{\sigma(x)}{\sigma}=1 \tag{1}$$

$$\frac{\sigma(x)}{\sigma}=\ln\left(\frac{r}{R_o}\right) \tag{2}$$

$$\frac{\sigma(x)}{\sigma}=\left(\frac{r}{R_o}\right)^{-2} \tag{3}$$

where r is a desired radius and R_o is the outer diameter.

3 HOOP STRESSES FOR A CYLINDER

In this study the main concern is related to hoop stress, as it is the one that may cause a crack to become unstable. A cylinder working as part of energy generation system may work under two different conditions, the first where the outer face temperature is higher than the inner one, and the second the opposite happens. Due to the very geometrical nature of the cylinder, the two cases present different temperature distributions when steady state is reached. This is the reason why two separate expressions are used to describe hoop stresses. Item 3.3 displays the hoop stress component associated with an internal pressure and Figure 1 brings the cylindrical geometry and crack origin for the studied cases.

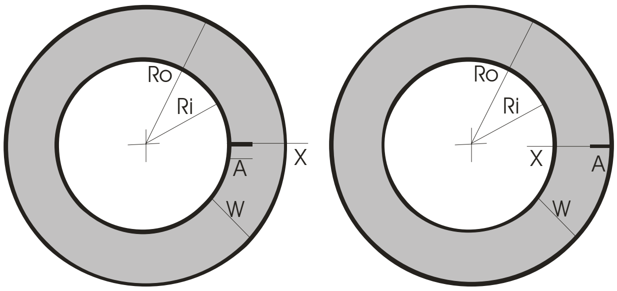

Figure 1 – Geometrical characteristics of the cracked cylinder, the left side represents situation where $T_o>T_i$ and the right hand side $T_i>T_o$.

3.1 $T_o > T_i$, p

If the temperature on the outside of the cylinder is higher than the inside, hoop stresses are (Timoshenko [5]):

$$\sigma_\theta^{To}(r)=\frac{E\alpha\Delta T}{2(1-\nu)\ln(\Phi)}\left[1+\ln\left(\frac{r}{Ro}\right)+\frac{\Phi^2\ln(\Phi)}{1-\Phi^2}\left[1+\left(\frac{r}{R_o}\right)^{-2}\right]\right]+p\frac{\Phi^2}{1-\Phi^2}\left[1+\left(\frac{r}{R_o}\right)^{-2}\right] \tag{4}$$

where E is the Young´s Modulus, α is the thermal expansion coefficient, ΔT is the difference between the external and internal temperatures, Φ is R_i/R_o (R_i is the internal radius), and p is the internal pressure.

3.2 $T_i > T_o$, p

For the cases where the temperature on the inside of the cylinder is higher than the outside, thermal hoop stress is [5]:

$$\sigma_\theta^{Ti}(r) = \frac{E\alpha\Delta T}{2(1-\nu)\ln(\Psi)}\left[1-\ln\left(\frac{Ro}{r}\right)-\frac{\ln(\Psi)}{\Psi^2-1}\left[1+\left(\frac{Ro}{r}\right)^2\right]\right]+p\frac{\Phi^2}{1-\Phi^2}\left[1+\left(\frac{r}{R_o}\right)^{-2}\right] \quad (5)$$

where Ψ is R_o/R_i

4 STRESS INTENSITY FACTORS FOR THERMAL AND PRESSURE STRESSES ACTING IN A CYLINDER

Taking the SIFs as:

$$K = \sigma\sqrt{\pi a W}.f(a/W) \quad (6)$$

The SIFs for the critical situations are obtained using the combination of the known reference cases (1) to (3) and expression (6) (Wu [10]).

4.1 $T_o>T_i$

In this situation the internal crack is critical. When $r = R_i$ expression (6) becomes:

$$K_{tot}^{To,i} = \frac{E\alpha\Delta T}{2(1-\nu)\ln(\Phi)}\left(1+\frac{2\ln(\Phi)}{1-\Phi^2}\right)\sqrt{\pi a W}f_T^{To,I} + p\frac{1+\Phi^2}{1-\Phi^2}\sqrt{\pi a W}f_p^i \quad (7)$$

and

$$f_{tot}^{To,i} = \frac{\sigma_p f_p^i + \sigma_{\Delta T} f_T^{To,i}}{\sigma_{tot}} \quad (8)$$

4.2 $T_i>T_o$

In this situation the external crack is critical. When $r = R_o$ expression (6) becomes:

$$K_{tot}^{Ti,e} = \frac{E\alpha\Delta T}{2(1-\nu)\ln(\Psi)}\left(1-\frac{2}{\Psi^2-1}\ln(\Psi)\right)\sqrt{\pi a W}f_T^{Ti,E} + p\frac{2\Phi^2}{1-\Phi^2}\sqrt{\pi a W}f_p^E \quad (9)$$

and

$$f_{tot}^{Ti,e} = \frac{\sigma_p f_p^e + \sigma_{\Delta T} f_T^{To,e}}{\sigma_{tot}} \quad (10)$$

5 PHYSICAL PROPERTIES OF THE A-333 STEEL

The studied case used material properties from an ASTM A-335 P12 steel. This material is used for high temperature parts and is a ferritic, seamless alloy (ASM [11]):

Table 1 – Some of the steel physical properties.

Property	E	ν	α	Sut	K_{IC}
value	210 GPa	0.29	12.2e-6 1/ºC	415 MPa	110 MPa√m

6 RESULTS

For this study an internal pressure was assumed to be 7.355 MPa (75 kgf/cm^2), as presented in the charts offered by the manufacturer.

6.1 SIFs for the studied case ($T_o > T_i$)

Taking results from reference [4] and calculating stresses using expressions (7) to (12) and (14), it is possible to determine the general expression defining the non-dimensional SIFs for a situation where a single internal crack, found in a cylindrical region ($R_o/R_i = 1.5$) where $T_o > T_i$ and $\Delta T = 50$ to 250 K:

$$f_{tot-i}^{\Delta T} = 1.119 - g_1(\Delta T)\left(\frac{a}{W}\right) + g_2(\Delta T)\left(\frac{a}{W}\right)^2 - g_3(\Delta T)\left(\frac{a}{W}\right)^3 + g_4(\Delta T)\left(\frac{a}{W}\right)^4 \quad (15)$$

where:

$$g_1(\Delta T) = 1.661 + 2.182\mathrm{x}10^{-3}(\Delta T) - 5.000\mathrm{x}10^{-6}(\Delta T)^2 \quad (16)$$

$$g_2(\Delta T) = 5.232 - 1.067\mathrm{x}10^{-3}(\Delta T) + 2.171\mathrm{x}10^{-6}(\Delta T)^2 \quad (17)$$

$$g_3(\Delta T) = 7.360 - 4.151\mathrm{x}10^{-4}(\Delta T) + 6.571\mathrm{x}10^{-7}(\Delta T)^2 \quad (18)$$

$$g_4(\Delta T) = 3.671 - 1.343\mathrm{x}10^{-3}(\Delta T) + 2.971\mathrm{x}10^{-6}(\Delta T)^2 \quad (19)$$

6.2 SIFs for the studied case ($T_i > T_o$)

For a situation where a single internal crack, found in a cylindrical region ($R_o/R_i = 1.5$) where $T_o > T_i$ and ΔT ranges from 50 to 250 K, it is found that:

$$f_{tot-e}^{\Delta T=50} = 1.122 - h_1(\Delta T)\left(\frac{a}{W}\right) + h_2(\Delta T)\left(\frac{a}{W}\right)^2 - h_3(\Delta T)\left(\frac{a}{W}\right)^3 + h_4(\Delta T)\left(\frac{a}{W}\right)^4 \quad (20)$$

where:

$$h_1(\Delta T) = 0.353 + 2.601\mathrm{x}10^{-3}(\Delta T) - 5.571\mathrm{x}10^{-6}(\Delta T)^2 \quad (21)$$

$$h_2(\Delta T) = 2.575 - 2.613\mathrm{x}10^{-3}(\Delta T) - 7.743\mathrm{x}10^{-6}(\Delta T)^2 \quad (22)$$

$$h_3(\Delta T) = 1.638 + 1.146\mathrm{x}10^{-3}(\Delta T) + 1.800\mathrm{x}10^{-6}(\Delta T)^2 \quad (23)$$

$$h_4(\Delta T) = 0.247 - 2.532\mathrm{x}10^{-3}(\Delta T) - 8.400\mathrm{x}10^{-6}(\Delta T)^2 \quad (24)$$

6.3 Total Stress Limits for a Cracked Cylinder ($T_o > T_i$)

Taking the fracture strength from Table 1 and using together with expressions (15) to (19), it is possible to determine total stress limits. Figure 2 shows results for the studied case. One must realize that no safety factor was taken into account, so if this is to be done, total stress limits are much lower than the presented ones.

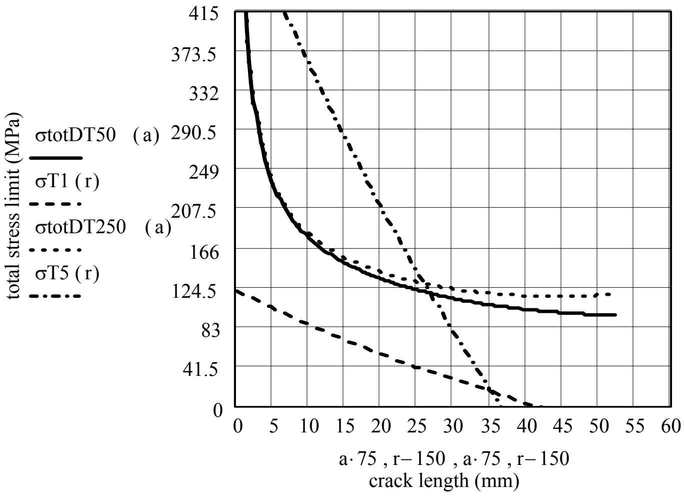

Figure 2 – Total limit stress (solid lines) and total stress controlled by ΔT. For $T_o>T_i$ and single internal crack.

6.4 Total Stress Limits for a Cracked Cylinder ($T_i>T_o$)

Figure 3 shows results for the studied case, using expressions (20) to (24).

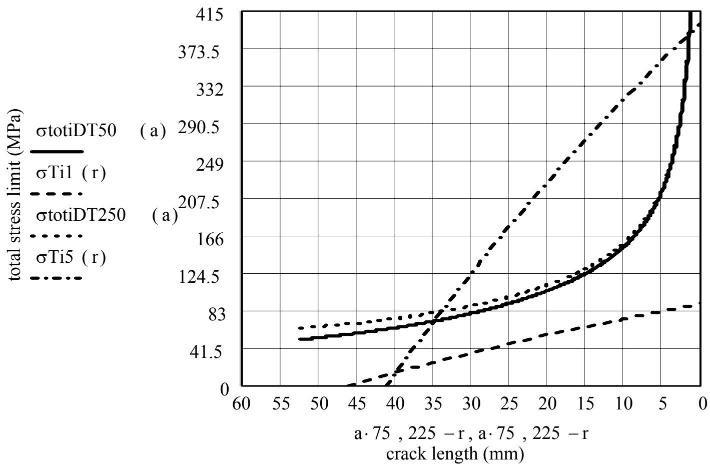

Figure 3 – Total limit stress (solid lines) and total stress controlled by ΔT. For $T_i>T_o$ and single external crack.

8 CONCLUSIONS

- It was possible to devise a set of expressions that describe the total stress state as well as total SIF.
- Reducing ΔT, allows the equipment to work even in the presence of relatively deeper cracks.
- A part bearing an internal crack ($T_o > T_i$) and ΔT = 100 K may stand up to 223.7 MPa total hoop stress. For ΔT larger than 150 K, cracks cannot be deeper than 3 mm if the part is to work at full pressure.
- A part bearing an external crack ($T_i > T_o$) and ΔT = 100 K may stand up to 166 MPa total hoop stress. For ΔT larger than 150 K, cracks cannot be deeper than 5 mm if the part is to work at full pressure.
- If a ΔT larger than 120 K is present in a part bearing an internal crack ($T_o > T_i$), total hoop stress must be reduced according to Figure 2.
- If a ΔT larger than 105 K is present in a part bearing an internal crack ($T_i > T_o$), total hoop stress must be reduced according to Figure 3.
- As a safety factor or other complex values were not taken into account, results must be considered theoretical.

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