A PROBABILISTIC MODEL TO SIMULATE THE ORIGIN AND INCEPTION OF FATIGUE FAILURE IN METALS

SIDNEY A. GURALNICK[1] AND JAMSHID MOHAMMADI[1]
[1]Department of Civil and Architectural Engineering, Illinois Institute of Technology, Chicago, Illinois

ABSTRACT

This paper presents a probabilistic model that can be used to simulate the process of fatigue damage accumulation and failure in metals. The underlying concept in the model is to simulate the material behavior with a series of mechanical springs with randomized behavior. Depending upon the applied stress level, which is resisted by these springs, a certain number of springs are expected to rupture with every stress application. The pattern at which rupture occurs among the springs is assumed to be random. The behavior of each spring is modeled with a nonlinear stress-deformation curve, which is also random. Damage which occurs after each stress cycle is simulated by the number of springs which have ruptured upon the stress application; and the damage accumulation is modeled by the total number of springs that have ruptured during the entire stress application cycle. When a sufficiently large number of springs rupture, the entire system is considered to have failed. This constitutes fatigue failure. The model is demonstrated by simulating the fatigue failure of simple metal specimens, and it results in a Wöhler-like S-N curve.

1 INTRODUCTION

Crack formation in materials due to repeated load application is a complex phenomenon. The material behavior, the macrostructure arrangement in the material, applied stress level, size of the component, existence of prior cracks are among factors that are known to affect the fatigue process. Uncertainties associated with these parameters are the main source of large scatter often observed in fatigue failure behavior of materials. Although there have been efforts in the past to develop models for describing fatigue behavior (e.g., Jackel [1] and Martin. et al [2]), experimental investigations have remained as the only reliable method to establish fatigue failure behavior in materials. Previous mechanical models developed to explain the interlinked set of phenomena needed to describe fatigue failure have been unsatisfactory to a greater or lesser degree because these models simulated only one or two of the many gross macroscopic aspects of metal fatigue. The evolution of fatigue failure is assumed to proceed through four stages prior to fracture or rupture (see Fig. 1). These are: (1) the inception of isolated microscopic zones of plasticity; (2) the "organization" of the microscopic zones of plasticity into macroscopic plastic regions; (3) the initiation of cracks; and (4) complete separation or rupture. Stage 1 occurs at very low stress levels and, if stresses remain low, fatigue failure in metals with a definite endurance limit does not occur. At stress levels higher than those needed to initiate microscopic yielding, stage 2 occurs (curve marked P in Fig. 2). If stresses are raised to still higher levels, then stage 3, or crack initiation occurs (curve F in Fig. 2). This curve is sometimes called “French’s line of damage.” When the stresses are raised to even higher levels, the cracks, which first appeared in stage 3 propagate and eventually cause rupture as illustrated by the uppermost curve in Fig. 2. This last curve is what is known as the S-N (or Wöhler) curve in fatigue behavior of metals. Guralnick [3] describes fatigue phenomena based on incremental collapse of a simple portal frame composed of an elastic perfectly plastic material. This model, at best, can only simulate the first two stages of the progression leading to fatigue failure depicted in Fig. 1. To overcome this shortcoming and to develop a model to simulate the fatigue behavior through the S-N curve, a model must be capable of describing the non-linear behavior of the material as well as the many sources of uncertainties that result in the large scatter observed in the S-N behavior. This means that the model, more appropriately, requires a probabilistic formulation. Accordingly, Guralnick and Mohammadi [4] developed a simple model to demonstrate the capability of probabilistic formulations in providing a pathway through all four stages culminating in rupture or fatigue. The model consisted of only a limited number of random variables. It was developed as an extension of the model originally proposed by Jenkin [5] to simulate the elastic-plastic mechanical behavior of materials. This model is based on the behavior of a system of parallel springs, which undergo a random stress cycle. Failures among the springs occur at random and can be used as a means to simulate fatigue damage and fatigue behavior. When a sufficient number of springs rupture, fatigue failure is assumed to have occurred. The model used only a small number of random variables and was intended as an initial step to verify its capability in simulating fatigue damage and failure behavior. This paper further expands the model proposed by Guralnick and Mohammadi by enhancing the random characteristics of the material behavior.

2 DESCRIPTION OF THE MODEL

Figure 3 illustrates the mechanical model used in this study to simulate fatigue behavior. The model consists of *m* parallel springs, which resist the applied stress *F*. Each spring is a non-linear element with the stress-deformation behavior curve shown in Fig. 4. Although the springs are considered to be identical, their behavior will differ from one another if one defines one or more of the spring parameters as random variables. The uncertainty in the spring behavior can specifically be considered from two distinct sources. One is related to the randomness in the variables that describe the stress-deformation curve in the elastic region. The other is related to those variables that describe the stress-deformation in the post-yield region and, specifically, in the stress-hardening portion of the curve as depicted by variables u'_y and u_L and the slope of the curve in the post-yield region (k_e). In the current formulation of the model, in addition to the spring behavior curve, the applied stress (*F*), the number of springs (*m*) making up the model and the percentage of ruptured springs (N_L) that constitute fatigue failure are also treated as random variables. All of the random variables in the model are described with their respective means and standard deviations and a prescribed distribution model (e.g., normal).

3 BASIC FORMULATION

The applied stress (*F*) is distributed evenly among the *m* springs (see Fig. 3); and as such, the stress in any spring *i* is $s_i = F/m$. Rupture in a spring will occur when the stress s_i exceeds the resistance R_L (see Fig. 4). After the first stress cycle, failure probability p_1 in any one spring is

$$p_1 = P(R_L \le s_i) \tag{1}$$

let $Z_1 = (R_L - s_i)$, then $p_i = P(Z_1 \le 0)$.

From Fig. 4, R_L and consequently Z_1 can be written as

$$R_L = R_y + \Delta R = R_y + k_e(u_L - u'_y) \tag{2}$$

$$Z_1 = R_y + k_e(u_L - u'_y) - F/m \tag{3}$$

where $k_e = kk'/(k+k')$. Parameters F, R_y, k_e, u_L and u'_y are all treated as random variables. Using the first order reliability method (FORM), the probability term in Eq. 1 can be written in the following form $p_1 = \Phi(-\bar{Z}_1 / S_1)$ in which $\bar{Z}_1$ and S_1 are the mean and standard deviation of Z_1, respectively, and $\Phi(\cdot)$ represents the normal probability distribution function. They can be estimated using the first order approximation in terms of the mean and standard deviations of all random variables describing the spring behavior as depicted in Fig. 4. Upon the application of the first stress cycle, the estimated number of springs that survive the load is $(1-p_1)m$. After the second cycle, the probability of failure of a given spring is p_2, where $p_2 = P(Z_2 \le 0) = \Phi(-\bar{Z}_2 / S_2)$ and $Z_2 = R_y + k_e(u_L - u'_y) - F / m(1-p_1)$. Again $\bar{Z}_2$ and S_2 are the mean and standard deviation of Z_2, respectively. The number of springs that survive after the second stress cycle is $(1-p_1)(1-p_2)m$. In general, after the nth stress cycle, if p_n is the failure probability of a given spring, then,

$$p_n = P(Z_n \le 0) = \Phi(-\bar{Z}_n / S_n) \tag{4}$$

$$Z_n = R_y + k_e(u_L - u'_y) - F / [m(1-p_1)(1-p_2)...(1-p_n)] \tag{5}$$

Again $\bar{Z}_n$ and S_n are the mean and standard deviation of Z_n, respectively. Note that the product $(1-p_1)(1-p_2)...(1-p_n)m$ is the number of springs surviving after the nth stress cycle. With the first order approximation, general equations for the mean and standard deviation of Z_n are

$$\bar{Z}_n = \bar{R}_y + \bar{k}_e(\bar{u}_L - \bar{u}'_y) - \bar{F} / [m(1-p_1)(1-p_2)...(1-p_n)] \tag{6}$$

$$S_n^2 = S_{R_y}^2 + S_{k_e}^2(\bar{u}_L - \bar{u}'_y)^2 + \bar{k}_e^2(S_{u_L}^2 + S_{u'_y}^2) + \frac{S_F^2}{m(1-p_1)(1-p_2)...(1-p_n)} \tag{7}$$

in which a bar on each symbol indicates the mean value of the respective random variable; whereas, the term such as S_X indicates the standard deviation of a random variable X.

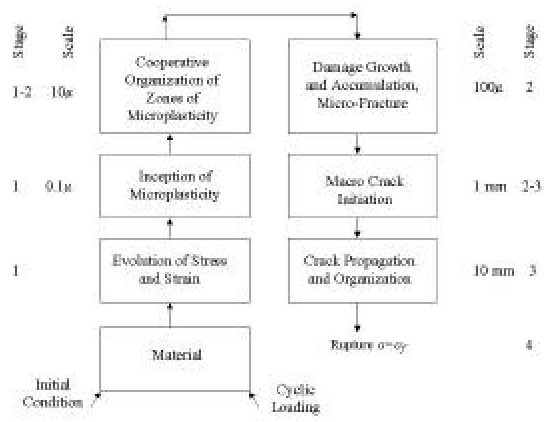

Fig. 1 Stages in progression leading to fatigue fracture or rupture

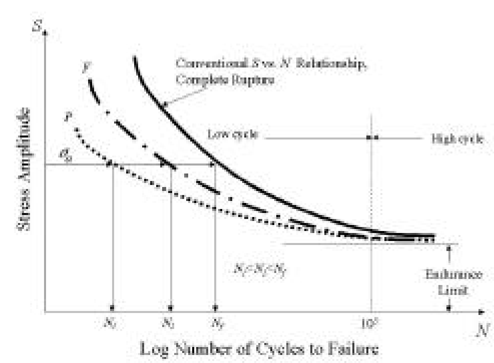

Fig. 2 Conventional representation of fatigue process

3.1 Fatigue Failure

Fatigue failure occurs when a significantly large number of springs have failed. Depending on the type of material, this number of failures is also random. In a given case of applied stress, a random value for N_L is selected based on the mean and standard deviation of N_L.

3.2 Solution Process

Equations 4 and 5 can be solved in a step-by-step manner starting with n=1 and continuing the computations until system failure occurs. Before each computation run, the mean and standard deviation of F as well as those of the other parameters describing the spring behavior are selected. A random number is then generated for the required percentage of ruptured springs, that constitutes fatigue failure (i.e., N_L) and the total number of springs that make up the entire model (i.e, the parameter m). The computation of the terms p_n is then achieved using Equations 4-7 after each stress application. Upon each stress cycle, the percentage of ruptured springs is computed and compared against N_L to determine whether fatigue failure has occurred. Once the percentage of ruptured springs is equal to or exceeds N_L, then the cycle number corresponding to this stress application is recorded. This is the number of stress cycles causing failure.

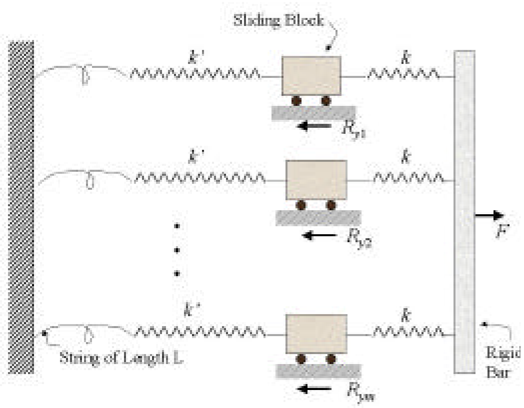

Fig. 3 Model schematics

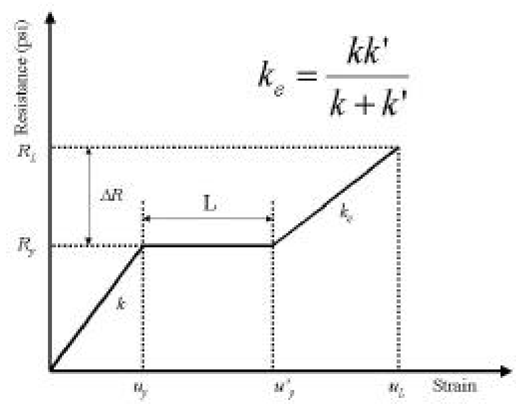

Fig. 4 Response of a single element of model

3.3 Determination of Spring Parameters

Spring parameters depicted in Fig. 4 can be determined from simple tests of materials. The standard deviations of individual parameters are the measures of uncertainty in materials behavior and can be estimated from published data on variations observed in metal yield and ultimate strain values. The stress hardening behavior shown in Fig. 4 appears to play a crucial role in the overall behavior of the material in producing random fatigue failure results (as described later). This stress-hardening behavior can also be investigated in a limited number of simple tension tests. The limited number of tests needed to calibrate the spring behavior would require, by far, much less labor and cost compared with the numerous tests needed to investigate the fatigue failure behavior using conventional laboratory investigations. As is evident from the formulation, the parameter m plays an important role in the model and in the simulation process. Generally, m will be proportional to the inverse of the probability of failure p_n. One can begin with a large m as a starting value and revise it later if such revision becomes necessary.

4 NUMERICAL ILLUSTRATION

The following example is intended for illustration purposes only. In a given material, the estimated value for *m* is selected as 10,000. However, this will be subject to a 5% coefficient of variation, which is incorporated in the analysis. Twelve random values for *m* are selected using a normal distribution, a mean of 10,000 and a coefficient of variation of 0.05. These random values range from 9,545 to 10,640. The mean value of the parameter N_L is selected as 25 percent with a coefficient of variation equal to 0.20. Several random numbers for this parameter are also generated using a normal distribution. The random values range from 20.45 to 31.40 percent. The selection of *m* and N_L as random numbers further increases the variations to be expected in typical fatigue failures. The applied stress *F*, as well as all parameters describing the behavior of the spring are also treated as random variables. In this example, their corresponding means and coefficients of variation are summarized in Table 1. Within each stress value (say 9000 psi), the results of the total number of cycles to failure are random and show a rather wide range. Table 2 summarizes the total number of stress cycles obtained for various stress values applied to the system. Figure 5, shows the results in graphic form using a semi-log scale. This figure clearly depicts the scatter in results, which is a distinct characteristic of the usual S-N (or Wöhler-type) curve.

Table 1 Variability in Stress and Resistance Behavior

Variable	Unit	Mean	Coefficient of Variation
F	psi	3,000-9,000	0.20
R_y	psi	1.00	0.20
R_L	psi	2.00	0.20
u_y	inches	0.067	0.05
u'_y	inches	0.183	0.05
u_L	inches	0.25	0.05
k_e	Lbs/in^3	15	0.20

Table 2 Number of Stress Cycles to Failure

Applied Stress (psi)	Range for Number of Cycles to Failure
9,000	21 - 56
8,500	33 - 89
8,000	55 - 145
7,500	92 - 245
7,000	162 - 425
6,500	294 - 763
6,000	555 - 1,419
5,500	1,086 - 2,681
5,000	2,197 - 5,250
4,500	5,597 - 10,569
4,000	11,527 - 21,851
3,500	22,091 - 46,340
3,000	Over 65,000

5 SENSITIVITY OF RESULTS TO VARIATION IN MATERIAL PROPERTIES

An important feature of the model presented herein is its ability to portray the scatter in fatigue life, which is often observed in experiments with real metals. To further explore the main sources of this scatter, the influence of the variation in model parameters on the final outcome for fatigue life was investigated. Three sources of parameter variations were evaluated. These were: (1) variation in the applied stress and uncertainty in the percentage of ruptured springs (i.e., parameter N_L) needed to reach fatigue failure; (2) variation in the parameters describing the elastic portion of the spring behavior curve; and (3) variation in the parameters describing the post-yield region (i.e., stress hardening region and the onset of plastic behavior) in the spring behavior curve. Figure 6 presents the significance of variability in the parameters describing the post yield region of the spring behavior curve compared with the variability in parameters describing the elastic region of the curve. As the variability in the elastic parameters increases, the change in the variability in fatigue life is only marginal. However, when the variability in post-yield parameters increases, there is a dramatic change in the variation obtained for fatigue life. These results are consistent with actual fatigue behavior in metals. As explained earlier (see Fig. 1), the inception of fatigue damage starts by the formation of microscopic zones of plasticity and the organization of these zones into macroscopic plastic region which lead to crack initiation. To a great extent the scatter inherent in fatigue life is affected by randomness in the plastic regions formation which is predominately affected by the material non-linearity.

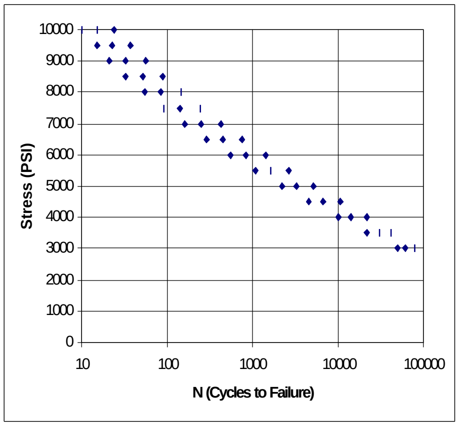

Fig. 5 S-N Curve for the Example Problem

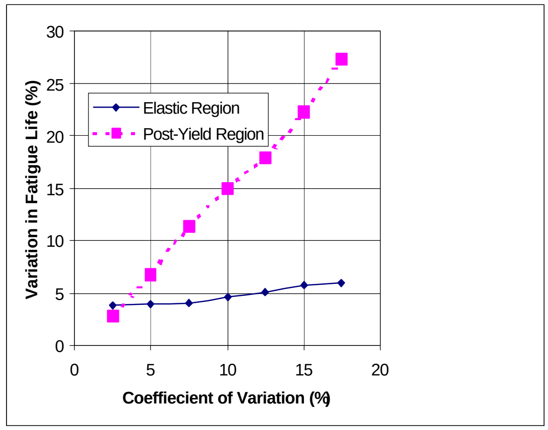

Fig. 6 Effect of Parameter Uncertainty on Variation in Fatigue Life

6 DISCUSSION AND CONCLUSIONS

The model presented in this paper can be used in developing the S-N relationship with characteristics similar to those associated with curves obtained in experiments with real metals. To apply the model to a broader fatigue study, several additional developments will be necessary. Specifically, one needs to develop an approach that can be used in calibrating the parameters describing the spring behavior for a given material. We believe that this can be achieved with a limited number of simple laboratory tests on material behavior and calibration of the spring model parameters with a few cyclic load test results (as described earlier). In conclusion: (1) A model made of nonlinear parallel springs can be used to simulate fatigue failure in metals; (2) The results from simulating fatigue behavior from this model show that most of the variation in fatigue life is inherent in the stress hardening and plastic behavior of the springs.

7 REFERENCES

1. Jackel, H.R., Simulation, duplication and synthesis of fatigue load histories. Sound and Vibration, Vol. 4, p. 18-29, March 1970.
2. Martin, J.F., Topper, T.H., and Sinclair, G.M., "Computer-based simulation of cyclic stress-strain behavior with applications to fatigue," Materials Research and Standards Journal, ASTM, Vol. 11, No. 2, Feb. 1971.
3. Guralnick, S.A., An incremental collapse model for metal fatigue, Publ. Intern. Association of Bridge and Structural Engineers, Vol. 35, Part II, Zurich, 1975
4. Guralnick, S.A., and Mohammadi, J., "The orogin and inception of fatigue in steel - a probabilistic model, " in Recent Advances in Experimental Mechanics, E.E. Gdoutos (ed.), Kluwe Academic Publishers, Netherlands, p. 187-196, 2002.
5. Jenkin, C.F., Engineering, Vol. 114, p. 603, 1922.