VIRTUAL TESTING FOR ESTIMATING MATERIAL FRACTURE PROPERTIES
(REDUCTING TIME & COST OF TESTING)

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ABSTRACT
When estimating the life of a space structure, material fracture toughness, $K_I$, $K_e$, and $K_C$, and fatigue crack growth rate data must be available through the ASTM Standards. These tests are costly and time consuming. The proposed virtual testing approach can estimate material fracture toughness ($K_{IC}$ & $K_C$) and can generate fatigue crack growth rate data for the aircraft and aerospace metallic alloys by using only the static parameters. An empirical equation has been established that can define the critical crack length as a function of fracture stress. Having these two parameters available, the plane strain and plane stress fracture toughness can be calculated. This quantity is used to establish the region III of the $da/dn$ curve. The threshold region of fatigue crack growth curve (region I of the $da/dn$ curve) can be estimated through the Kitagawa diagram. Two additional data points were estimated in the region II of the fatigue crack growth curve and were used to establish the Paris constants describing the $da/dn$ versus $ΔK$ equation. Two aluminum alloys were selected from NASGRO material database and the $da/dn$ versus $ΔK$ variations were compared with the proposed technique. Excellent agreement between NASGRO $da/dn$ data and analytical method were found.

1 INTRODUCTION
To generate the fracture properties through the virtual testing approach, information on the material full stress-strain curve must be available, which for most aerospace alloys are accessible through the MIL-HDBK. The virtual testing methodology for establishing material fracture properties is based on the energy balance approach, which first was proposed by the Griffith theory of brittle fracture. With the extended Griffith theory approach, the energy absorption rates for plastically deforming material at the crack tip and near crack tip are derived and used to extend the Griffith concept of brittle fracture to Fracture Mechanics of Ductile Metals (FMDM) theory [1,2]. An empirical equation has been established that can define the critical crack length as a function of fracture stress. Having these two parameters available, the plane strain and plane stress fracture toughness (through the stress intensity factor equation) can be calculated.

The fracture toughness is used to establish the region III of the $da/dn$ curve (accelerated region). Note that the region III of the $da/dn$ curve is thickness dependent and fracture toughness data as a function of thickness must be available for life estimation of structural components. The threshold region of fatigue crack growth curve (region I of the $da/dn$ curve) can be estimated through the Kitagawa diagram. The fundamental assumption used to establish the threshold value is based on the behavior of large cracks within the linear elastic fracture mechanics regime and the boundary between small and large cracks where linear elastic fracture mechanics collapse [3]. Two additional data points were estimated in the region II of the fatigue crack growth curve and were used to establish the Paris constants describing the $da/dn$ versus $ΔK$ equation [4]. Experimental observation based on several aluminum alloys showed that the fatigue crack growth rate corresponding to two quantities, $K_c/K$ and $K/Kth$ for two points in the Paris region are nearly a constant. One data point is situated just before reaching the critical value of stress intensity factor, $K_c$ ($K_c/K$ of region III) and one data point before the threshold region, $K/Kth$. These two points have found to have almost the same amount of crack growth rate, $da/dn$, for many aluminum alloys.
6061-T6 and 2219-T87 aluminum alloys were selected from NASGRO material database and the da/dn versus \( \Delta \)K variations were compared with the proposed analytical technique [5]. Excellent agreement between NASGRO da/dn data and analytical method were found. The application of this concept will reduce the amount of physical testing at the expense of virtual testing and will result in significant cost reduction to aircraft and space industry. Currently, under the Collaborative Virtual Testing (CVT) program more work is underway to extend the above-mentioned analytical work to other aerospace alloys. Two computer codes called Fatigue Crack Growth (FCG) and Fracture toughness Determination (FTD) are available for establishing fracture properties of aerospace material when physical testing is not feasible. In addition to the FTD and FCG computer codes, variation of fracture toughness and fatigue crack growth rate data due to material variation was assessed through the probabilistic approach and upper and lower bounds fracture properties were established [4] (Figure 1).

![Road map to the FTD & FCG & life estimation](image)

**Figure 1: Road map to the FTD & FCG & life estimation**

### 2 FRACTURE TOUGHNESS DETERMINATION

Material residual strength capability curve, a plot of fracture stress versus half a crack length, can be generated through the extended Griffith theory [1,2]. Energy absorption rate for plastic deformation at the crack tip is calculated and used to establish a relationship between fracture stress and half critical crack length. The total energy per unit thickness absorbed in plastic straining of the material around the crack tip, \( U_p \), can be written as:

\[
U_p = U_F + U_U
\]  

(1)

where \( U_F \) and \( U_U \) are the energy absorbed per unit thickness in plastic straining of the material beyond the ultimate stress at the crack tip and below the ultimate stress near the crack tip, respectively.

The extended energy balance equation, in terms of \( U_F \) and \( U_U \), described by equation 1, can be rewritten as:

\[
\partial( U_E - U_S - U_F - U_U) / \partial c = 0
\]  

(2)

where \( U_E \) and \( U_S \) are the total available energy and energy necessary to create two new crack surfaces. The terms \( g1=\partial U_F / \partial c \) and \( g2=\partial U_U / \partial c \) are the rates at which energy is absorbed in plastic
straining beyond the ultimate stress at the crack tip and below the ultimate stress near the crack tip, respectively. The extended Griffith theory in terms of $g_1$ and $g_2$ can be rewritten as:

$$\frac{\partial U_E}{\partial c} = 2T + \frac{\partial U_F}{\partial c} + \frac{\partial U_U}{\partial c}$$

(3)

where $\frac{\partial U_E}{\partial c} = \frac{\pi \sigma^2 c}{E}$ and $\frac{\partial U_F}{\partial c} = 2T$, is the work done in creating two new crack surfaces. The derivation of the two terms $g_1$ and $g_2$ can be calculated from the energy per unit volume under the full stress-strain curves as shown in Figure 2 for 6061-T6 and 2219-T87 aluminum alloys [1,2]. The residual strength relationship derived from the extended Griffith theory is shown by equation 4.

\[ c = \frac{E}{\pi \sigma^2 \mu \sigma} \left[ 2T + \sigma_{UF} \epsilon_{PN} h_F k + \frac{n}{n-1} \sigma_{TU} \epsilon_{TU} \left[ 1 - \left( \frac{\sigma_T}{\sigma_{TU}} \right)^{n+1} \right] \right] \frac{\epsilon_{TF} \epsilon_{TL}}{\epsilon_{TU} \epsilon_{T}} \]

(4)

where true stress and strain quantities embedded in equation 4 are associated with points on the full stress-strain curve as shown in Figure 2. Having fracture stress, $\sigma$, and half critical crack length, $c$, on hand (Equation 4), material fracture toughness can be calculated. This quantity can be useful in the region three of the da/dn curve as was described in detail in references [4,6]. Figure 3 is a plot of fracture toughness versus material thickness for 6061-T6 and 2219-T87 aluminum, where test data extracted from [5] is compared with analysis. Excellent agreement with test data can be seen.

### 3 FATIGUE CRACK GROWTH ESTIMATION

Fatigue crack growth diagrams are used extensively in linear elastic fracture mechanics for safe-life analysis of structural components subjected to load varying environments. Material fatigue crack growth data are generated in the laboratory based on ASTM–E647 procedures. There are several empirical equations that are currently available in the literature, which can be used to describe fatigue crack growth curve. The constants associated with these equations are obtainable through the da/dn data generated in the laboratory. These equations must be able to describe fatigue crack growth behavior in all the regions of the curve. One of the most acceptable equations
that defines material crack growth behavior, which is used in the NASGRO computer code, can be written as:

$$\frac{da}{dN} = \frac{c(1 - R)^n \Delta K^n (1 - \frac{\Delta K_{th}}{\Delta K})^p}{(1 - R)^q (1 - \frac{\Delta K}{\Delta K_c})^q}$$

(5)

By the analytical approach proposed here, constants in the Equation 5 can be obtained without using the costly and time-consuming ASTM testing. Material fracture toughness for the thickness in consideration can be obtained through the FMDM approach. From experimental data available for aluminum alloys [5], the two quantities p and q are estimated to be 0.5 and 1.0, respectively. The threshold stress intensity factor, $\Delta K_{th}$, is calculated based on the Kitagawa diagram concept. A surface crack with the total length $2c=0.005$ inch (the boundary between small and large cracks)

Figure 3: Fracture Toughness Versus Plate Thickness (Compared with NASGRO data [5])

![Fracture Toughness Versus Thickness (6061-T6) vs (2219-T87)](image)

Figure 4: Fatigue crack growth curves for several aluminums (to establish two points in the Paris region, see Figure 5 below)
was selected for calculating the threshold value [7]. The function \( f \) shown in the Equation 5, describes crack closure phenomenon. It can be obtained by using the Newman crack closure concept (fully described in reference 5). The remaining constants (C and \( n \)) can be found by applying the following two assumptions: 1. In the region I, where crack growth rate is slow, the stress intensity factor range of \( \Delta K=1.1\Delta K_{th} \) corresponds to crack growth rate of \( da/dn=1E^{-8} \) in./cycle, and 2. For the region III, where crack growth accelerates, the \( da/dn =5E^{-2} \) in./cycle, and the corresponding \( \Delta K=0.99 \) \( K_c \) (Figures 4 & 5). Figure 6 show \( da/dn \) curves generated for 6061-T6 and 2219-T87 based on analysis described above.

Figure 5: Two points in the Paris region can be estimated for many aluminum alloys

Figure 6: The \( da/dn \) versus \( \Delta K \) curve generated by the virtual testing approach for the 6061-T6 & 2219-T87 aluminums
The proposed approach can generate fracture toughness and fatigue crack growth data for classical metal alloys used in the aerospace industry. Good correlation between the estimated fracture toughness by the virtual testing approach and test data were obtained for the 6061-T6 and 2219-T87 aluminum alloys. The work was extended to generate the da/dn curve by estimating all the regions of the curve. Results of analyses were checked against the existing test data and they both were in good agreement with each other. In addition, the integration of the probabilistic method has allowed a better understanding of the effect of material variation and life prediction. Sensitivity study was conducted on several parameters:

1. Material fatigue crack growth curve is sensitive to parameters that contribute to the threshold, Paris, and accelerated regions.
2. Probabilistic study has shown that both fracture toughness versus material thickness and fatigue crack growth curves will shift depending on material variations observed through static tests.

5 References


