# FINE DESCRIPTION OF FRACTURE BY USING DISCRETE PARTICLE MODEL

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ABSTRACT

A discrete model for analysing response of concrete under mechanical loading is presented. Material is described by an assembly of Voronoi particles, and nonlinear response is obtained by introducing an elastic brittle behavior for beams that join the particles. After a brief presentation of the model and details about the numerical implementation, few examples are presented and illustrate the model performance. The force of such a model is the ability to give a realistic representation of the cracking process. Our study focuses on this particular aspect, and an original crack process analysis is performed, with a comparison with experimental results.

## 1. INTRODUCTION

Rupture of quasi-brittle materials like concrete has been extensively studied with satisfactory results by using continuum models, in terms of global response and description of damage area. However, needs in numerical simulations are always increasing, and description of cracked areas, knowledge of heterogeneity influence on the response have to be improved. This goal could not be reached easily with continuum models because of their formulation, *i.e.* a continuum description of the media.

On the other hand, discrete models are naturally able to describe a cracked media (see [1], [2], [3], [4] and references herein). We propose in this study to show the reliability of a particle model to represent evolution of cracking in brittle materials. More precisely, apparition of microcracks in the beginning of the loading, localisation and microcrack coalescence, propagation of a macrocrack will be studied. In a first part, we present the general framework of our model. Material is described by an assembly of Voronoi particles, linked together by elastic beams. Damage response of material is obtained by introducing a perfectly elastic brittle behavior for beams, with random rupture thresholds. Note that numerical implementation is effective in 2D as well as in 3D, due to the simple ingredients of the model.

In a second part, a study of the cracking process is proposed. Two aspects are emphasized: coalescence of microcracks into a single macrocrak during the loading process, and main macrocrack properties. We show that the particle model is able to capture the crack geometry, in terms of spacing, roughness and opening. Knowledge of these properties is very important when dealing with flow through a cracked wall, or sliding dissipation under a cyclic loading.

## 2. DISCRETE MODEL

Material is described as an assembly of Voronoi particles. All the neighboring particles are joined together by cohesive forces. These forces are represented by beams that are strictly equivalent to simple cohesive forces as shown in [5]. At this point, the model is nothing else than a lattice model. Introduction of particles allows dealing deal with contact forces. Contact occurs when two particles that are not joined together overlap. In that case, a contact force is computed, with a response in terms of force-displacement equals to the response obtained with a cohesive force. Hence, stiffness restitution due to crack closure is guaranteed.

Nonlinear behavior of the material is obtained by allocating a random breaking threshold for each beam during the mesh generation. If strain in a beam is greater than its threshold, the beam breaks irreversibly [6]. Despite the simplicity of this behavior, damage behavior of material like concrete is well described. Furthermore, 3D analysis could be performed without difficulties, Voronoi polygons are just changed to Voronoi polyhedra, and six degrees of freedom per nodes are taken into account. Figure 1 shows the response of a tension test on a concrete cube.

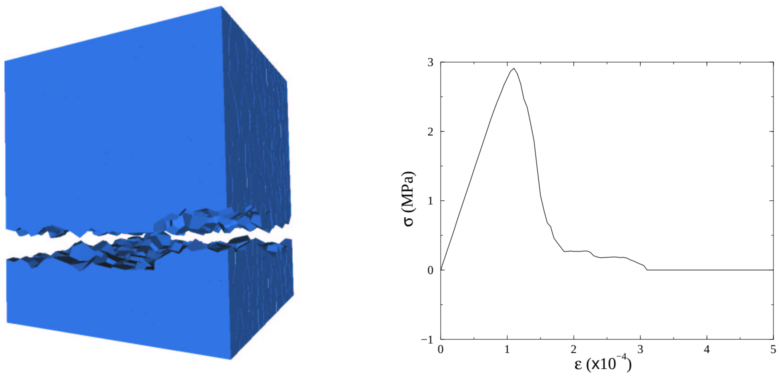


Figure 1: Traction test on a concrete cube of 0.1×0.1×0.1m (3375 particles are used)

### 2.1. Quasi-static analysis

One of the advantages of such a model is the possibility to perform an elastic prediction loading (see for instance [7], [8]): it is well known that negative stiffness due to softening response causes some difficulties in numerical resolution. An alternative consists to break at each step just one link, with the following procedure:

Iteration $k$

1. apply an elastic load $\mathbf{f}^{el}$
2. compute $\mathbf{u}^{el}$ with an iterative solver
3. calculate $\alpha_{\min}$ as:

$$\alpha_{\min} = \min_{\substack{i,j \in (1,..,n_b) \\ i \neq j}} \left( \frac{1}{P_{ij}} \right)$$

   where $P_{ij}$ is the breaking threshold of beam $ij$, and $n_b$ the total number of beams.

4. extract $(\alpha_{\min}\mathbf{u}^{el}, \alpha_{\min}\mathbf{f}^{el})$ for the global load-displacement curve
5. change the global stiffness matrix with

$$\mathbf{K}^{k+1} = \mathbf{K}^{k} - \mathbf{L_{ij}}^{T}\mathbf{K_{ij}}\mathbf{L_{ij}}$$

   where $\mathbf{L_{ij}}$ denoting the connectivity matrix of beam $ij$.

End of iteration $k$.

Using this algorithm ensures a unique response without numerical difficulties, and captures naturally snap-back. Figure 2 shows a four point bend test on a notched beam with such a procedure.

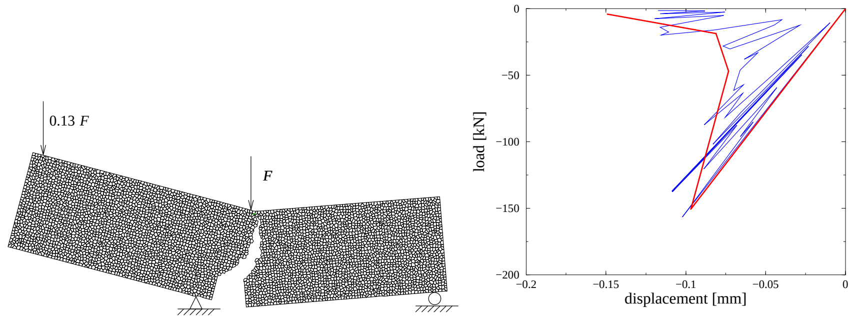


Figure 2: Four point bend test on a notched beam, with the particular rotating crack. Right graph shows the good agreement between numerical (blue line) and experimental (red line) responses.

2.2. Dynamic analysis

Two time integration schemes is proposed for solving problems with dynamic loadings [9]. For high rate loading, an explicit scheme (central difference scheme) is used. On the other hand, an implicit scheme is used for low rate loading. Unfortunately, a classic Newmark implicit scheme could not be used, due to the elastic brittle response of the beams: this simple behavior, allowing large computations, generates unphysical high frequencies in the response. We use a specific scheme that damps these high frequencies, based on the introduction of two dissipative terms [10]. The first one is introduced during the computation of inertia terms,

$$\mathbf{d}_{t+\Delta t} = \mathbf{d}_t + \frac{\Delta t}{2}(\mathbf{v}_{t+\Delta t} + \mathbf{v}_t) + \alpha_2 \Delta t(\mathbf{v}_{t+\Delta t} - \mathbf{v}_t) \tag{1}$$

the second one is introduced during the computations of internal forces

$$\begin{aligned} \mathbf{K}(\mathbf{d}_{t+\frac{\Delta t}{2}})\mathbf{d}_{t+\frac{\Delta t}{2}} = \; & \tfrac{1}{2}(\mathbf{K}(\mathbf{d}_{t+\Delta t})\mathbf{d}_{t+\Delta t} + \mathbf{K}(\mathbf{d}_t)\mathbf{d}_t) + \\ & \alpha_1(\mathbf{K}(\mathbf{d}_{t+\Delta t})\mathbf{d}_{t+\Delta t} - \mathbf{K}(\mathbf{d}_t)\mathbf{d}_t) \end{aligned} \tag{2}$$

where $\alpha_1$ and $\alpha_2$ are the two parameters that control the dissipation. Hence, just unphysical high frequencies are damped, and the full response could be obtained without numerical artefacts.

## 2. CRACK DESCRIPTION

Particle models give realistic description of cracks. But a more extensive study should be performed to compare crack properties with real ones issue from breaking samples. Different results on crack roughness have been obtained during the last years ([11], [12], [13]). In particular, long range height correlations on crack profile have been found on concrete and mortar material. This property is useful when dealing with friction laws or flow through crack. Similarly to experimental process, we perform correlations analysis on different cracked samples obtained with our model. Crack profile is extracted from a sample (Figure 3), and a

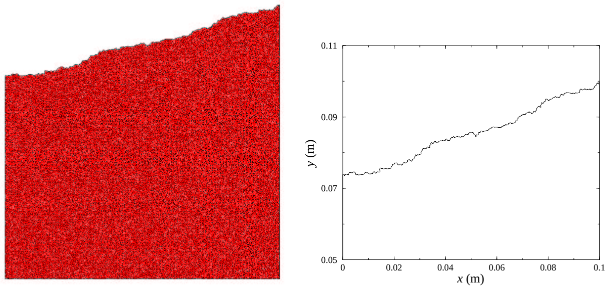


Figure 3: A breaking sample (32400 particles) with the corresponding crack profile

wavelet analysis is performed [14]. A linear response on a log-log plot reveals the self-affine properties. Figure 4 shows the response for three different samples. As expected, a linear regime is obtained, showing that the computed profiles manifest scaling properties as for the real cracks, with a similar value for roughness exponent. This first analysis is very promising and is carried on for different loadings and different samples.

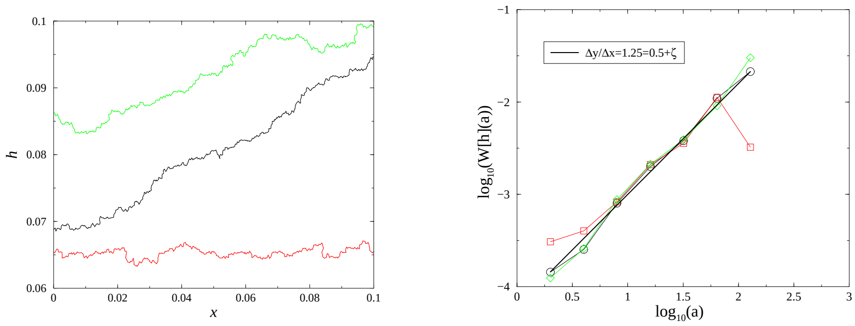


Figure 4: Wavelet analysis of three different crack profiles. Best fitted line gives a roughness exponent of 0.6

3. CONCLUSION

A particle model is introduced for describing concrete behavior. The general framework of the model is presented, with details on numerical resolution for static and dynamic loadings. The second part of the paper focuses on the ability of the model to represent material cracking. Roughness of crack profiles obtained from numerical breaking samples is calculated with a wavelet analysis and compared with experimental result, showing self-affine properties of cracks.

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