Dynamic behavior of shear deformable laminated composites with delaminations and edge cracks

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In this paper dynamics of shear deformable laminated plates with delaminations is studied. Laminated composite plates are used in a multitude of applications for their superior specific strength and specific stiffness in addition to their tailorable properties. It is also known that for laminated composites, shear deformation assumes importance in studying their characteristics. The formulation is based on Rayleigh Ritz method with beam characteristic functions used as admissible functions. The related integrals are evaluated using symbolic mathematics where integrable and numerical integrations are used otherwise. Delaminations are simulated by discarding the load carrying capacity of the lamina and recomputing the frequencies. Results are presented for various aspect ratios and side to thickness ratios. Analysis is also performed to compute the change in dynamic characteristics of edge cracks and thereby for detecting and monitoring damage in laminated composites. Finite element simulation is based on the principle of virtual displacement using Lagrangian elements including hygrothermal effects.

Salient Features:
The displacement functions are approximated as follows:

$$\begin{aligned} u(x,y,z) &= u_o(x,y) + z\overline{\alpha}(x,y) \\ v(x,y,z) &= v_o(x,y) + z\overline{\beta}(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned}$$

The potential energy expression may be expressed as

$$V = \frac{1}{2}\sum_{k=1}^{N}\int\int_A\int_{h_{k-1}}^{h_k}\{\sigma_{xx}\epsilon_{xx} + \sigma_{yy}\epsilon_{yy}$$
$$+\sigma_{yz}(2\epsilon_{yz}) + \sigma_{zx}(2\epsilon_{zx}) + \sigma_{xy}(2\epsilon_{xy})\}dz\,dA$$

The potential energy further takes the following after introducing the stress-strain relations for laminated plates by means of integrating across the thickness,

$$
\begin{aligned}
V &= \int_0^a \int_0^b \left[\frac{D_{11}}{2}\left(\frac{\partial\overline{\alpha}}{\partial x}\right)^2 + D_{12}\frac{\partial\overline{\alpha}}{\partial x}\frac{\partial\overline{\beta}}{\partial y} \right.\\
&+ D_{16}\frac{\partial\overline{\alpha}}{\partial x}\left(\frac{\partial\overline{\alpha}}{\partial y}+\frac{\partial\overline{\beta}}{\partial x}\right) + \frac{D_{22}}{2}\left(\frac{\partial\overline{\beta}}{\partial y}\right)^2\\
&+ D_{26}\frac{\partial\overline{\beta}}{\partial y}\left(\frac{\partial\overline{\alpha}}{\partial y}+\frac{\partial\overline{\beta}}{\partial x}\right)\\
&+ \left. \frac{D_{66}}{2}\left(\frac{\partial\overline{\alpha}}{\partial y}+\frac{\partial\overline{\beta}}{\partial x}\right)^2 \right] \mathrm{dx\ dy}
\end{aligned}
$$

where D’s are the material stiffness coefficients.

Some sample results are presented in the following figures:

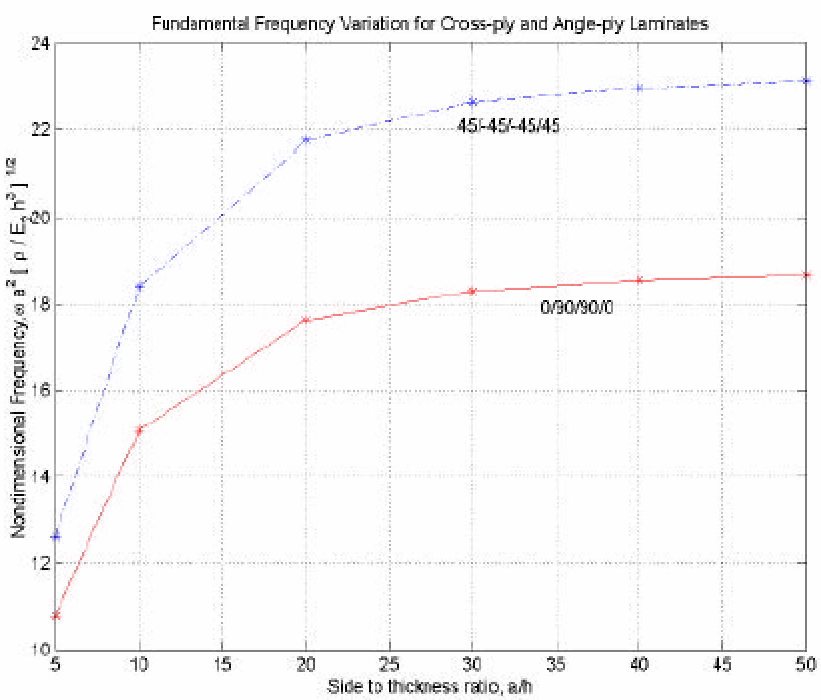

Figure 1 Fundamental Frequency Variation with Side to Thickness Ratio for Angle-ply and Cross-ply Laminates

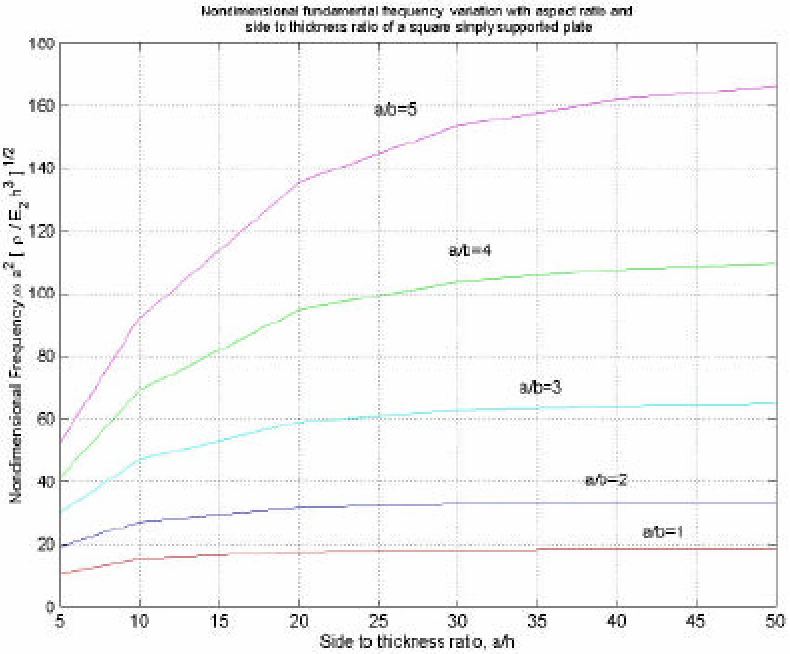

Figure 2 Fundamental Frequency Variation for Different Aspect Ratios