FATIGUE CRACK GROWTH IN STRUCTURAL ALLOY AS PREDICTED BY THE STRAIN ENERGY DENSITY THEORY

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ABSTRACT

In this paper the fatigue behavior of a edge cracked plate is considered. The strain energy density theory is applied . The fatigue crack growth model adopted is the one due to Sih and Barthelemy, which assumes that the fatigue crack growth is controlled by the fluctuation ΔS of the Strain Energy Density Factor S.

A simple method for obtaining approximate stress intensity factors is also applied. It takes into account the elastic crack tip stress singularity while using the elementary beam theory.

1 INTRODUCTION

The classical linear elastic solution for the stress field around a crack is

$$\sigma_{ij} = \frac{K_i}{\sqrt{2\pi r}} f_{ij}\left(\vartheta\right) + \quad O(r^{\frac{1}{2}}) \tag{1}$$

where σ_{ij} are the stresses acting at a distance r from the crack tip. The angle ϑ is measured from the crack plane, while K_i are the "stress intensity factors" with $f_{ij}\left(\vartheta\right)$ being known functions of ϑ. For small r the higher order terms are negligible compared to the first term . Hence, the use of the first term only is perfectly adequate for the analysis of crack behavior. The stress intensity factors K_I, K_{II} and K_{III} are associated with the three basic modes of deformation : mode I or "opening mode", mode II or "sliding mode", mode III or "tearing mode". They are functions of the loading on the cracked configuration , of the size and shape of the crack, and of other geometrical boundaries. It is significant to note that K_i have the dimensions of stress x $\sqrt{\text{length}}$. The elastic solution predicts infinite stresses at the crack tip. In reality this cannot occur since there is plastic flow in this highly stressed region. If this region is small compared to that over which the $r^{-\frac{1}{2}}$ term dominates, the stress intensity domain should provide an adequate description of the crack tip stress field.

Linear Elastic Fracture Mechanics is widely used to describe many aspects of crack behavior. Knowledge of the stress intensity factors plays an important role in fracture control.

Stress intensity factors for many configurations are available . In most cases the results were obtained by means of analytical and numerical methods. In many cases the results were obtained by finite element methods and boundary element methods. Experimental methods have been applied to simple cases in order to determine the fracture toughness K_{IC} of engineering materials. Solutions for many structural configurations are not available in the handbooks.

Simple engineering methods which allow a fast but approximate determination of the stress intensity factors are highly valued to a design engineering.

One of the purposes of this note is to employ a new simple method [1,2] for approximate evaluation of stress intensity factors in cracked plates. The method takes into account the elastic singularity and is derived by the equilibrium condition for internal forces evaluated in the cross section passing through the crack tip. Simple formulas for SIFs are derived. The results show a good approximation when compared to known solutions.

2 APPROXIMATE EVALUATION OF STRESS INTENSITY FACTORS: SINGLE EDGE CRACKED PLATE TENSION SPECIMEN

Consider the single edge cracked plate tension specimen (Fig 1).

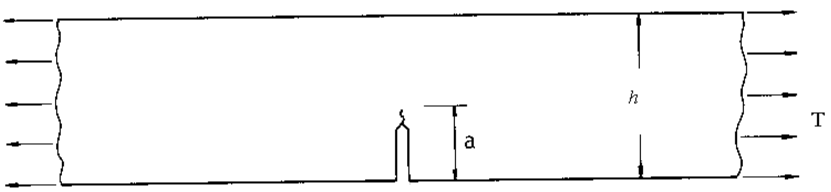

Fig.1. Schematic view of a plate tension specimen

According to Sain-Venant principle, the analysis can be performed in reference to N=Tht, being t the thickness. Since the neutral axis of the reduced cross section passing through the crack is shifted by $e=a/2$,the distribution of normal stresses on this cross section is:

$$\sigma_z = \frac{N}{A^*} \pm \frac{N\frac{a}{2}}{I_x^*} y \,, \tag{2}$$

where a is the crack length, A^* and I_x^* are the reduced cross-sectional area and the moment of inertia for the reduced part of the cross section , respectively. The singular stress component is related to the Mode-I stress intensity factor as in Eq. (3). The procedure by which the approximate stress intensity factor K_I can be obtained is reported in [2].
The stress intensity factor can be written as:

$$K_I = \mathrm{T}\sqrt{\pi a} \quad f_1\left(\frac{a}{h}\right), \tag{3}$$

where

$$f_1\left(\frac{a}{h}\right) = \frac{\sqrt{2}}{9}\sqrt{\frac{\left[1+2\left(\frac{a}{h}\right)\right]^3}{\left(\frac{a}{h}\right)^2\left(1-\frac{a}{h}\right)^3}} \;. \tag{4}$$

The function $f_1(a/h)$ can be compared with the expression by Brown and Srawley [3] :

$$f_2\left(\frac{a}{h}\right) = 1.12 - 0.23\frac{a}{h} + 10.55\left(\frac{a}{h}\right)^2 - 21.72\left(\frac{a}{h}\right)^3 + 30.39\left(\frac{a}{h}\right)^4 . \tag{5}$$

The comparison is shown in Fig. 2. The agreement between the proposed method and the other approach appears good.

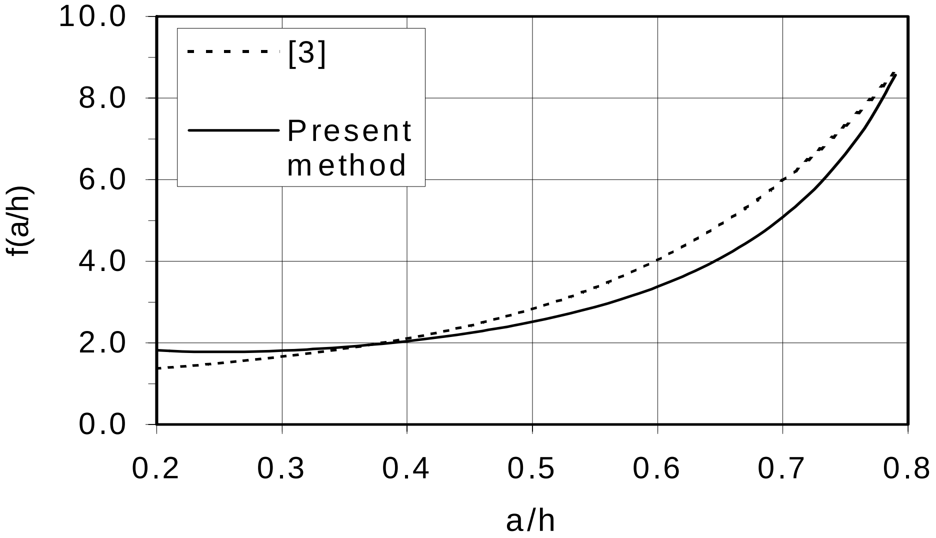

Fig. 2. Stress intensity factors for tension specimen.

3 STRAIN ENERGY DENSITY THEORY

Referring to the problem of fracture mechanics, the strain energy per unit of volume can be written as [4-6]:

$$\frac{dW}{dV} = \frac{S}{r} \tag{6}$$

where S is the *strain energy density factor* and it is related to the stress intensity factors as follows:

$$S = a_{11}\mathrm{K}_{\mathrm{I}}^2 + 2a_{12}\mathrm{K}_{\mathrm{I}}\mathrm{K}_{\mathrm{II}} + a_{22}\mathrm{K}_{\mathrm{II}}^2 \tag{7}$$

where the coefficients a_{ij} are defined by:

$$a_{11} = \frac{1}{16\pi\mu}[(3\text{-}4\nu\text{-}\cos\vartheta)(1+\cos\vartheta)] \tag{8a}$$

$$a_{12} = \frac{1}{16\pi\mu}(2\sin\vartheta)[\cos\vartheta\text{-}(1\text{-}2\nu)] \tag{8b}$$

$$a_{22} = \frac{1}{16\pi\mu}[4(1\text{-}\nu)(1\text{-}\cos\vartheta)+(1+\cos\vartheta)(3\cos\vartheta\text{-}1)] \tag{8c}$$

and μ is the second Lamè constant of elasticity.

Note that the strain energy density allows to consider all the three Modes of Fracture together and so it can be used to predict crack initiation in spatial problems.

The fundamental hypotheses of crack extension following the Strain Energy Density Theory can be summarized as follows. The crack will spread in the direction of the minimum strain energy density and the critical value of *S* (say S_{cr}) governs the onset of the crack propagation.
Summarizing, the crack begins to propagate in the ϑ_0 direction, when the following conditions are satisfied:

$$\left.\frac{\partial}{\partial \vartheta}(S)\right|_{\vartheta=\vartheta_0} = 0 \tag{9}$$

$$\left.\frac{\partial^2}{\partial \vartheta^2}(S)\right|_{\vartheta=\vartheta_0} > 0 \tag{10}$$

$$\left.S\right|_{\vartheta=\vartheta_0} \geq S_{cr} \tag{11}$$

The critical value of *S* is a material parameter and for the isotropic case is related to K_{IC}.

4 FATIGUE BEHAVIOUR

In order to analyze the fatigue behavior of a notched beam under a cyclic load, the fatigue crack growth model adopted is the one due to Sih and Barthelemy [7]. The model assumes that the fatigue crack growth is controlled by the fluctuation ΔS of the Strain Energy Density Factor *S*. After a finite number of loading cycles ΔN, the crack advances by an amount Δa as a consequence of the accumulation of a critical amount of the Strain Energy Density:

$$\frac{\Delta W}{\Delta V}\Delta N \tag{12}$$

The fatigue crack growth relation can be written as:

$$\frac{\Delta a}{\Delta N} = C\left(\Delta S\right)^n \tag{13}$$

where *n* and *C* are material parameters. Note that the quantity ΔS has to be calculated in the direction of the fatigue crack growth that it is still assumed to be defined by the angle ϑ_0, obtained minimizing the strain energy density.
Eq. (13) has to be replaced by:

$$\frac{\Delta a}{\Delta N} = C\left(\Delta S_{\min}\right)^n \tag{14}$$

$\Delta S_{\min}$ is defined by:

$$\Delta S_{\min} = S\left(\vartheta_0, \mathrm{T}_{\max}\right) - S\left(\vartheta_0, \mathrm{T}_{\min}\right) \tag{15}$$

Introducing the stress intensity factor ranges Δk_j and the mean stress intensity factors $\bar{k}_j$ (j=1,2), Eq. (15) becomes:

$$\Delta S_{\min} = 2\begin{bmatrix} a_{11}\overline{\mathrm{k}}_{\mathrm{I}}\Delta \mathrm{k}_{\mathrm{I}} + a_{12}\overline{\mathrm{k}}_{\mathrm{II}}\Delta \mathrm{k}_{\mathrm{I}} + \\ a_{12}\overline{\mathrm{k}}_{\mathrm{I}}\Delta \mathrm{k}_{\mathrm{II}} + a_{22}\overline{\mathrm{k}}_{\mathrm{II}}\Delta \mathrm{k}_{\mathrm{II}} \end{bmatrix} \tag{16}$$

Note that Eq. (16) contains both the cyclic load range and the mean cyclic load.

5 FATIGUE LIFE FOR EDGE CRACKED PLATE

Refer to the problem reported in fig. 1. The Mode-I crack extension will occur. The strain energy density factor S will be:

$$S = a_{11} K_{\mathrm{I}}^2 \tag{17}$$

where the stress intensity factor K_I is reported in Eq. (3).
The direction of crack extension is defined by $\vartheta_0 = 0$. Hence Eq. (13) yields:

$$\frac{\Delta a}{\Delta N} = C\left[\frac{1-2\nu}{2\mu} a \overline{\mathrm{T}} \Delta \mathrm{T}\right]^n \tag{18}$$

where:

$$\Delta \mathrm{T} = \mathrm{T}_{\max} - \mathrm{T}_{\min}$$
$$\overline{\mathrm{T}} = \frac{\mathrm{T}_{\max} + \mathrm{T}_{\min}}{2}$$

are the stress range and the mean stress respectively.
Integration of Eq. (18) leads to

$$N = \frac{a_f^{1-n} - a_i^{1-n}}{C(1-n)\left[\frac{1-2\nu}{2\nu} f^2 \Delta \mathrm{T} \overline{\mathrm{T}}\right]} \tag{19}$$

with a_i and a_f the initial and final crack length.
Referring to a plate made of Aluminium Alloy 2024-T3 [8], with h=50mm the fatigue life is analysed making use of Eq. (19) . The initial crack length is assumed to be a_i=2.5mm.
Fig. 3 shows the fatigue life of the cracked plate for different values of the maximum stress, assuming:

$$R = \frac{\mathrm{T}_{\min}}{\mathrm{T}_{\max}} = 0.1$$

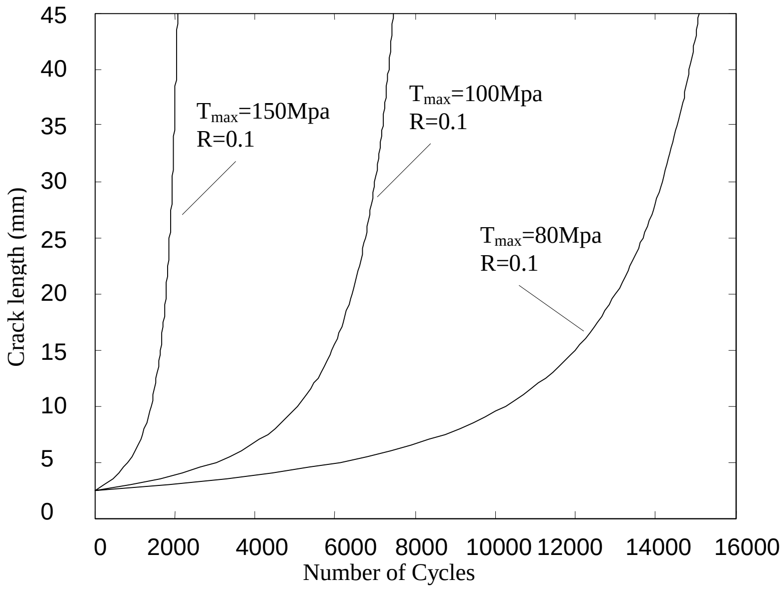

Fig. 3. Fatigue crack growth prediction for a edge cracked plate.

6 CONCLUSIONS

A simple method for obtaining approximate stress intensity factors is applied to the problem of a edge cracked plate. It takes into account the elastic crack tip stress singularity while using the elementary beam theory. The results are in reasonable agreement with the more accurate calculations.

Making use of the strain energy density theory, the fatigue behavior of the cracked plate is analyzed. The dependence of the fatigue crack growth on the value of the maximum stress is considered.

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