Variance of crack growth enhancement and impediment in piezoelectric materials complicated by difference in fracture criteria

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The path independent J-integral has been used in fracture mechanics based on the premise that it is connected with the energy released by a unit extension of a line crack. It is limited to nonlinear elastic materials and serves a useful purpose for evaluating the local stress field of a “conservative” system. When energy dissipation occurs, the concept serves little or no purpose. Use of the energy dissipation function becomes necessary. This work attempts to better understand whether the path independent integral and energy release rate tool could be applied without controversy to multiscale energy transfer rate crack problems. A particular situation would be the simultaneous application of electrical and mechanical energy. The theory of linear piezoelasticity is thus applied to test the validity of the so referred to \( J_k \) \((k=1,2)\) and J-integral in the literature beyond isotropic elasticity. It is disturbing to find that both \( J_k \) (or \( J_1 \)) and \( J \) for a crack can switch sign for different value of the applied electric field \( E_\infty \) and electric displacement \( D_\infty \) in relation to the applied mechanical stress \( \sigma_\infty \) and strain \( \varepsilon_\infty \). It depends on the prevailing boundary conditions. Previous works related to J-integrals were limited to simple materials and boundary conditions. The simple formulation adopted in this work provides closed form solutions to complicated boundary value problems for testing the validity of \( J \) or \( J_k \). It is important to know whether they will remain valid when they are applied to piezoelectric materials. The objective is to test whether \( J \) or \( J_k \) could be used as a fracture criterion other than idealized isotropic elastic systems where the energy release rate is limited to a crack segment the length of which is assumed to vanish in the limit. To this end, it has been shown conclusively that \( J \) or \( J_k \) deny positive energy release for the specification of the applied electric field \( E_\infty \) and electric displacement \( D_\infty \) when no mechanical stress \( \sigma_\infty \) and strain \( \varepsilon_\infty \) are applied. The validity or invalidity could then be decided by an experiment where only \( E_\infty \) is applied with \( \sigma_\infty \) (and \( \varepsilon_\infty \)) equal to zero. If a crack could be extended (corresponding to positive energy release), then the experiment would contradict the validity of \( J \) and \( J_k \) where they are predicted to be negative.

For the eight different boundary conditions examined, J integral tends to increase in the negative direction as the applied electric field is increased. This implies that there is less chance of fracturing a precracked piezoelectric specimen as the intensity of the applied electric field is increased. This conclusion does not seem to make physical sense. The same type of tests could be done to show whether a crack in piezoceramics would extend for values of \( E_\infty \) where \( J_M \) is predicted to be negative.
It also should be reiterated that regardless of whether the original J-integral (referred to as \( J_M \) in this work) is modified to \( J \) (or \( J_1 \)) to include piezoelectricity or not they both could become negative. This is discomforting from the viewpoint of physics. Whether these proposed theoretical ideas could be used in fracture mechanics remain to be seen. Prior to using them in practical applications, they should pass the fundamental tests of not yielding contradictory results that are not permitted in mathematics. By the same token, the results could also not be validated by experiments.