SOIL CRACKING DUE TO MOISTURE DIFFUSION AND CRACKING PREVENTION

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ABSTRACT

A simple analytical and design oriented procedure for crack analysis in the soil is presented in this paper. The cracks appear as a result of the moisture decrease and the shrinkage of the soil. All relevant parameters, such are Stress Intensity Factors (SIF) at the crack tip and the displacements of the crack faces, are found using Fracture Mechanics approach. The problem of the crack reinforcement is also considered. The geotextile is used as reinforcement. The force in the geotextile during the time is obtained for the different values of the geotextile stiffness. This parameter is sufficient to determine the geotextile characteristics.
Cracking and its prevention is very important for the pavement, or highway design. For the purpose of the engineering application the simple, analytical solution, will be presented. The paper considers infinite half space with uniform initial distribution of moisture. The diffusion of moisture will commence due to sudden change of boundary condition at the free surface when the value of moisture becomes zero. Mathematically the problem is defined with the well known diffusion equation. The solution of this equation, for prescribed initial conditions, is given by the error function. Once the moisture distribution is determined the stresses in the soil can be obtained. Due to stresses the soil, like clay will crack. According to Linear Fracture Mechanics, the problem of the crack can be solved as the superposition of the two governing problems. First problem is the half-plane without crack and the second is soil with the crack loaded by far field stresses at crack faces. Also for the sake of practical application the nonlinear distribution of stresses is replaced by linear distribution. This approximation is obviously on safe side. From handbooks of stress intensity factors K_I can be easily obtained. In literature results of soil fracture toughness can be found. Soil fracture toughness K_I^C is the critical stress intensity factor. Using Griffith criterion $K_I = K_I^C$ it is possible to determine whether the crack will propagate or not, or the size of the crack can be determined for prescribed far field stress.
In the pavement or highway design the geotextile reinforcement is usually used to prevent cracking. In this paper the same procedure as in Carpinteri's and Sumarac and Krajcinovic's papers will be used. By using Fracture Mechanics it is possible to determine the geotextile characteristics needed to prevent further cracking.

1 INTRODUCTION

It is well known that some sort of soil, like clay, show quite different behavior when they are saturated with moisture (water) than when they are dry. Due to moisture exchange (decrease) these types of soil shrink, causing the tensile stresses, and then crack. Cracking and its prevention is very important for the pavement, or highway design. This paper intends to analyze this problem, using Fracture Mechanics. For the purpose of the engineering application the simple, analytical solution, will be presented.

2 STRESSES IN THE SOIL DUE TO MOISTURE DIFFUSION

Consider the infinite half space, shown in Figure 1, with the uniform initial distribution of moisture $C(x,t=0)=C_0$.

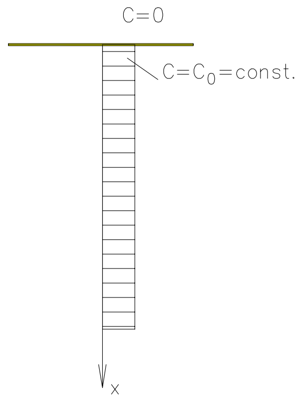

Figure 1: Infinite half-space with the initial moisture distribution

The diffusion of moisture will commence due to sudden change of moisture at the free surface to the value $C=0$. The problem of moisture distribution is then defined with the diffusion equation:

$$D_m \nabla^2 C = \frac{\partial C}{\partial t} \ . \tag{1}$$

The initial and boundary conditions are:

$$C = C_0, \ \text{for } t=0; \ x \geq 0 \text{ and } C = 0, \text{ for } t>0; \ x = 0. \tag{2}$$

In the eqn (1) D_m is the overall diffusion coefficient. In this paper we will replace very complex three phase medium (soil, water-moisture and air) with a continuum characterized with the overall constants. Also the stresses in the paper will represent effective stresses (stresses in the soil). In the eqn (2) C_0 represents, and hereafter will be referred to as, the increment of the moisture decrease (ΔC) because in the soil it is impossible to have zero moisture. The solution of the eqn (1) is (Carslaw and Jeger [2]):

$$C = C_0 erf \frac{x}{2\sqrt{D_m t}} \tag{3}$$

where erf, stands for the error function. Internal stresses, induced in the soil by the moisture distribution given by eqn (3) in the case when the changes of temperature are negligible are, (Sih et al. [3]):

$$\sigma_p = \frac{EC_0 \beta}{1-\nu} \left[1 - erf \frac{x}{2\sqrt{D_m t}} \right] \tag{4}$$

The stresses due to moisture exchange are shown in Figure 2. In the above formula β is moisture shrinkage coefficient. It should be mentioned that for infinite time ($t \to \infty$) stress is everywhere $\sigma_p = EC_0\beta/(1-\nu) = const$ and it represents its maximal value.

3 SOIL CRACKING

Soil like clay, will crack due to stresses given by eqn (4) and shown in Figure 2. Let us consider the crack of size *a*. According to Linear Fracture Mechanics, the problem of crack under the stresses shown in Figure 2 can be obtained as the superposition of the two governing problems.

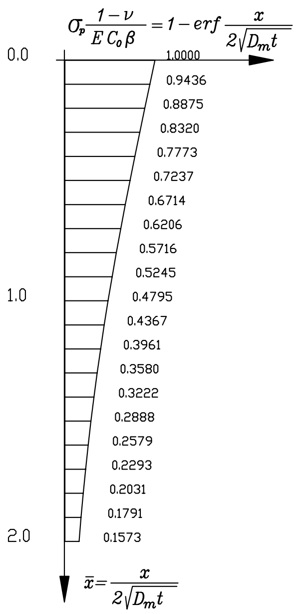

Figure 2: The stress distribution due to moisture change

The first problem is the half plane without crack, and the second is the soil with the stresses shown in Figure 2 applied at the crack faces. For the sake of engineering application , we are replacing the exact distribution of the stresses (given with the error function represented by eqn (4)) with the linear distribution. This approximation is on the conservative side because the area of the linear distribution of stresses is larger if compared with exact value shown in Figure 2. This approximation is easier because it can be integrated analytically. In that case, using Tada [7] it is obtained:

$$K_I = \frac{EC_0\beta}{1-\nu}\sqrt{\pi a}\left(1.1215 - 0.6825 erf \frac{a}{2\sqrt{D_m t}}\right) \tag{5}$$

In the expression (5), K_I is the Stress Intensity Factor (SIF), for the Mode I of the crack deformation. To obtain formula (5), the superposition is also taken into account for constant and linear distribution of stresses. Once the SIF is known, using Griffith criterion:

$$K_I = K_I^C \tag{6}$$

it is possible to determine whether the crack will propagate or not. In the above formulas K_I^C is the critical stress intensity factor. There is the way to obtain critical stress intensity factor for Mode I K_I^C, see for example Siriwardane and Layne [4].

4 REINFORCED CRACKS

In the pavement or highway design the geotextile reinforcement is usually used to prevent cracking. In this paper the same procedure as in Carpinteri [5] and Sumarac and Krajcinovic [6] will be used. By using Fracture Mechanics it is possible to determine the geotextile characteristics needed to prevent further cracking. In the following approximate analytical procedure, the geotextile (bond) will be taken with the finite width (*a-d*) to avoid infinite displacement at the place of the geotextile. Also we will assume the perfect bonding between the reinforcement and the soil, i.e. we will neglect possible geotextile-soil interface debonding. Taking Bueckner [1] procedure, for the partially loaded crack at the place of geotextile, it is obtained:

$$K_P = P\sqrt{\pi a}\left[1.1215 - 0.900316\sqrt{\frac{d}{a}}\left(1 + 0.2049\frac{d}{a} + 0.05004\frac{d^2}{a^2}\right)\right] \tag{7}$$

It is easy to check the expression (7). For $d/a = 1.0$ (there is no loading) $K_P = 0.008$ which is very close to the exact value $K_P = 0$, and for $d/a = 0$ (fully loaded crack) $K_P = 1.1215$, which is the exact value. In the above formula, P is still the unknown stress in geotextile.

To determine the displacement at the place of geotextile, the Castigliano's theorem will be used. Fictitious force Q would be applied at the same point. Then the total potential energy is calculated. After that, the governing displacement by differentiating of the potential energy with respect to fictitious force is obtained. From Tada [7] the SIF due to fictitious force Q is:

$$K_Q = \frac{2Q}{\sqrt{\pi a}}\frac{1.3 - 0.106(1 - d/a)^{3/2}}{\sqrt{1 - 0.25(1 - d/a)^2}} \tag{8}$$

For $d/a = 0.9$ from the expression (7) and (8) it is obtained:

$$K_P = 0.0753P\sqrt{\pi a}\ ,\ K_Q = 2.6Q\big/\sqrt{\pi a} \tag{9}$$

The total energy is then:

$$\Pi = \left(1 - \nu^2\right)\int_{(a-d)/2}^{a}\frac{K^2}{E}da = \left(1 - \nu^2\right)\int_{(a-d)/2}^{a}\frac{(K_P + K_Q)^2}{E}da \tag{10}$$

By differentiating (10) with respect to Q, using (9) and substituting into obtained result $Q=0$, it follows:

$$\delta_A = -\left.\frac{\partial \pi}{\partial Q}\right|_{Q=0} = 2\left(1 - \nu^2\right)\int_{(a-d)/2}^{a}\frac{K_P}{E}\frac{\partial K_Q}{\partial Q}da = 0.372\left(1 - \nu^2\right)\frac{Pa}{E} \tag{11}$$

Once the displacement (11) is known, due to stress P, the procedure to determine the reinforcement dimensions is obvious. Using compatibility conditions, as in Sumarac and Ktajcinovic [6], it is obtained:

$$X_1 = \frac{EC_0\beta}{1 - \nu}\frac{5.832 - 3.552 erf\dfrac{a}{2\sqrt{D_m t}}}{0.372 + E/ka\left(1 - \nu^2\right)} \tag{12}$$

From eqn (12) it can be seen that the stress in the geotextile is increasing function in time. The stress of geotextile increases with its rigidity (k) and reaches its maximum value if $k \to \infty$ i.e. totally rigid bond, and vice versa. Once the stress in the bond given by (12) is determined the total SIF is:

$$K_I = K_I^0 - K_I^1 X_1 \tag{13}$$

where K_I^0 is given by (5) and K_I^1 by the expression (7) for *P=1*. Substituting (5), (7) and (12) into (13) for *d/a=0.9*, it follows:

$$K_I = \frac{EC_0\beta}{1-\nu}\sqrt{\pi a}\left(1.1215 - 0.682 erf\frac{a}{2\sqrt{D_m t}} - \frac{0.439 - 0.267 erf\left(a/2\sqrt{D_m t}\right)}{0.372 + E/ka\left(1-\nu^2\right)}\right) \tag{14}$$

Te expression (14) represents SIF of the reinforced crack. Its values are plotted in Figure 3 during the time, for *d/a=0.9* and for the different values of the stiffness ratio $E/ka(1-\nu^2)$.

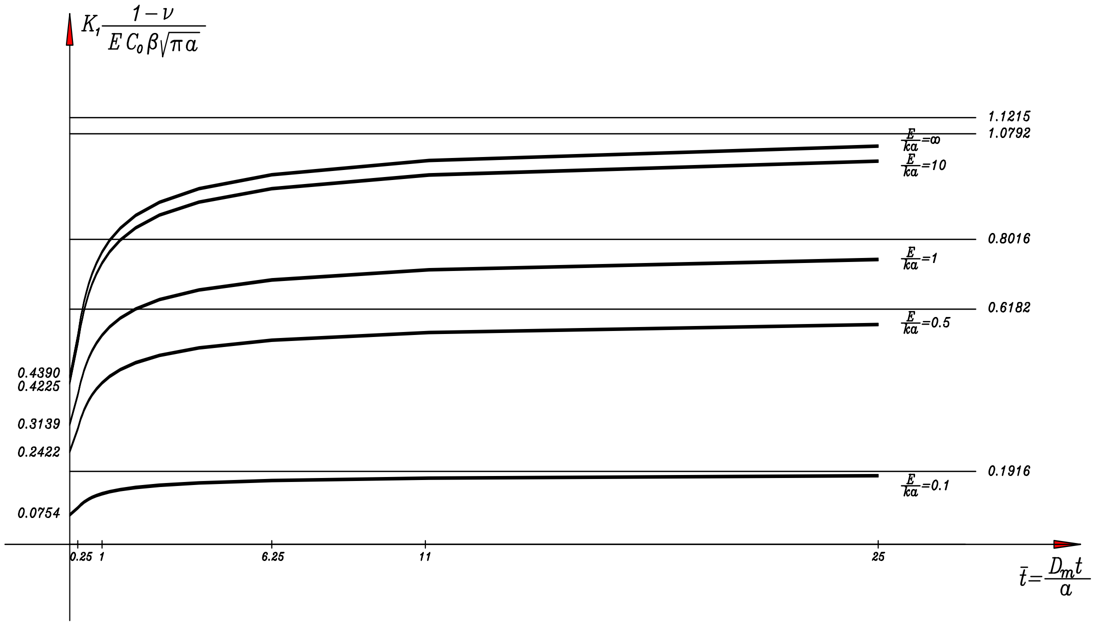

Figure 3: SIF for the reinforced crack during the time for the different values of E/ka

It is seen that SIF is increasing function in time. Also the decrease of stiffness of geotextile causes the increase of SIF. Taking into account (6), one can choose characteristics of geotextile (ratio $E/ka(1-\nu^2)$) and then from (14) determine what would be the size of crack *a* for $t\to\infty$. On the other hand, by choosing the size of the maximum crack length *a*, the characteristics of the geotextile would be determined. This approach leads to the optimization of the whole structure.

5 NUMERICAL RESULTS AND CONCLUSIONS

The expression (12) can be written in the dimensionless form:

$$\bar{X}_1 = \frac{5.382 - 3.552 erf\sqrt{\left(\bar{t}/4\right)}}{0.372 + E/ka\left(1-\nu^2\right)} \tag{15}$$

where are:

$$\bar{X}_1 = \frac{X_1(1-\nu)}{EC_0\beta}, \; \bar{t} = \frac{tD_m}{a^2} \tag{16}$$

For $\bar{t} = t = \infty$, especially interesting in the application, for $E/ka(1-v^2)=0$ (rigid bond) and for $E/ka(1-v^2)=1.0$, from (15) it is obtained:

$$\bar{X}_1 = 15.68, \bar{X}_1 = 4.25 \tag{17}$$

respectively. In the case of the clay, governing constants are:

$$\beta = 0.001\ (1\%);\ v = 0.03;\ E = 30000\ KN/m^2 \text{ and for } C_0 = 10\% \tag{18}$$

where, as stated earlier, β is shrinkage coefficient of the soil, v is the Poisson's ratio, E - Young's modulus and $C_0=\Delta C$ is the moisture decrease. From (18), (17) and (16) the stress obtained in the geotextile for the infinite time is:

$$X_1 = 6720\ kN/m2,\ X1 = 1821\ kN/m^2 \tag{19}$$

for the infinitely rigid bond ($E/ka(1-v^2)=0$) and for $E/ka(1-v^2)=1.0$ respectively. The infinitely rigid bond is unreal but the other one with characteristics $E/ka(1-v^2)=1.0$ is within the usual range for the often used geotextile. In that case, the force in the geotextile is, for the crack size $a=0.5m$, $X_1*0.05 = 91\ kN/m'$, while the displacement at the place of the geotextile is $X_1/k = 0.03m$. From the above explained numerical calculations it is seen that the results are in the expected domain. They are obtained by a very simple approximate and analytical procedure suitable for everyday engineering application.

6 ACKNOWLEDGEMENTS

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