CONDITIONS ON COHESIVE LAWS IN COHESIVE ZONE FRACTURE MODELS

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ABSTRACT
Some basic issues regarding the cohesive zone modeling of fracture in homogeneous materials as well as at interfaces are studied. It is shown that in order to remove the stress singularity at the tip of a cohesive zone in a homogeneous elastic medium the cohesive law must have a finite traction at the initial zero opening displacement. As to interface fracture, the stress singularities in tension and shear may not be simultaneously cancelled at the cohesive zone tip if a single cohesive zone length is adopted for both tensile and shear fracture modes. The dependence of the cohesive energy density on the phase angle for interface fracture is also discussed under small scale cohesive zone conditions. Finally, the energy dissipation at the tip of a prescribed cohesive zone is examined using a bilinear cohesive zone model under the uncoupled tension/shear conditions.

1 INTRODUCTION

In recent years, the cohesive zone modeling approach has emerged as a popular tool for simulating fracture processes in materials and structures due to the computational convenience. In the cohesive zone approach, it is assumed that a cohesive zone exists ahead of the crack tip. The cohesive zone consists of the upper and lower surfaces (cohesive surfaces) which are held by the cohesive traction. The cohesive traction is related to the separation displacement between the cohesive surfaces by a 'cohesive law', or cohesive zone model. Upon the application of external loading, the two cohesive surfaces separate gradually, leading to the physical crack growth when the displacement at the tail of the cohesive zone (physical crack tip) reaches a critical value.

Since Needleman [1] introduced the cohesive surface concept in the finite element framework for fracture study, the cohesive zone models together with interface-cohesive elements have been used to study various fracture problems, for example, fracture along a bi-material interface [2], dynamic crack growth in brittle materials [3], impact damage in brittle materials [4], numerical aspects of cohesive zone models [5], and so on.

In spite of the extensive use of cohesive zone models in the study of material and structural fracture, it seems that some fundamental aspects of cohesive zone models have not been fully understood. For example, how are cohesive zone models directly related to failure mechanisms at the atomic, micro-, or macro-level? are there physical restrictions on the functional form of cohesive laws? The cohesive zone length for Mode I fracture can generally be determined from the condition that no energy dissipation occurs at the cohesive zone tip. For the mixed mode fracture case, however, there are two independent cohesive tractions (normal and shear) and a single cohesive zone length may not be able to satisfy the condition that stress singularity at the cohesive zone tip be cancelled and, as a result, energy would be dissipated at the singular cohesive zone tip in addition to that dissipated inside the cohesive zone. It is well known that interfacial fracture toughness is not a constant but is a function of mode mixity. Such a characteristic of interfacial cracks must also be accounted for by the cohesive zone model. To provide some answers to the above questions, Nguyen and Ortiz [6] tried to derive a universal macroscopic cohesive law based on the coarse-graining and renormalization of atomistic binding relations.

The present work aims to study some basic issues in the application of cohesive zone models especially in the fracture of an interface between two dissimilar elastic media. The focus of this study is on the constraint conditions on the formulation of cohesive laws in single mode as well as mixed mode fractures. The dependence of the cohesive energy density on the loading phase angle in bi-material interfacial cracks is discussed. Issues arising from the determination of cohesive zone length by removing the stress singularity at the tip of the cohesive zone are examined. Examples for possible energy dissipation at a cohesive zone tip are illustrated using a bilinear cohesive model.

2. LINEAR HARDENING COHESIVE ZONE MODEL

The linear hardening cohesive zone model is a basic element in several widely used cohesive zone models. In numerical simulations, a linear hardening model is usually used as the first step in a nonlinear model. The linear hardening model itself has also been used in some studies. This kind of models has a common feature that the cohesive traction takes a zero value at the start of cohesive surface separation (zero separation). In other words, these models have an initial (or asymptotically initial) linear elastic response. Physically speaking, a cohesive zone model describes the fracture process in a material and the material separation occurs only after it deforms significantly. It is thus expected that cohesive models should have a finite traction at the start of separation, or an initial rigid response. In this section, we discuss this problem by considering the mathematical and physical restrictions on the cohesive zone development in an elastic background material.

Consider a two-dimensional elastic medium of infinite extent with a crack of length $2a$ subjected to remote tension σ_∞, as shown in Fig. 1 (homogeneous material case). A cohesive zone of length, $\rho = c - a$, develops ahead of each crack tip upon external loading. The cohesive traction σ and the separation δ follow the linear hardening model

$$\sigma = \sigma_c \left(\delta / \delta_c \right), \qquad 0 \le \delta \le \delta_c \tag{1}$$

Where σ is the cohesive traction, δ the opening displacement of the cohesive surfaces, σ_c the peak cohesive traction, and δ_c a characteristic opening at which the cohesive traction suddenly drops to zero.

The cohesive fracture modeling approach employs the cohesive crack assumption that the cohesive zone is treated as an extended part of the crack with the total stress intensity factor being vanished at the tip of the cohesive zone thereby canceling the stress singularity. Hence,

$$\sigma_\infty \sqrt{\pi c} - \frac{2}{\sqrt{\pi c}} \int_a^c \frac{\sigma(\xi) d\xi}{\sqrt{1 - \xi^2 / c^2}} = 0 \tag{2}$$

where the first and second terms on the left-hand side are the stress intensity factors due to the external load σ_∞ and the cohesive traction σ, respectively. The total crack opening displacement δ is [7]

$$\delta(x) = \frac{4c\sigma_\infty}{E^*} \sqrt{1 - x^2 / c^2} - \frac{4}{\pi E'} \int_a^c G(x,\xi) \sigma(\xi)\, d\xi \tag{3}$$

where $E' = E$ for plane stress and $E' = E/(1-\nu^2)$ for plane strain, and $G(x, \xi)$ is a known function. The integral equation for the opening displacement δ can be obtained using Eqs. (1) – (3) as

$$\delta(x) + \frac{4\sigma_c}{\pi E' \delta_c} \int_a^c K(x,\xi)\, \delta(\xi) d\xi = 0, \qquad a \le x \le c \tag{4}$$

where $K(x,\xi)$ is a Fredholm kernel. Eq. (4) is a homogeneous linear integral equation for δ and has only a trivial solution. This becomes evident by considering the corresponding crack bridging problem. For the bridging problem, a nonhomogeneous term due to the crack tip energy dissipation appears on the right-hand side in Eq. (4), which leads to a unique nontrivial solution [8]. This, in turn, requires that the homogeneous equation Eq. (4) have only a trivial solution. The nonexistence of a nontrivial solution for Eq. (4) implies that the crack tip singularity can not be cancelled. As a result, the cohesive zone model can not assume a linear hardening law with an initial zero cohesive traction, if the stress singularity is to be removed, or if the energy dissipation is not allowed, at the tip of the cohesive zone.

In view of the foregoing, it is concluded that an appropriate cohesive law must have a finite cohesive traction at the onset of separation. In fact, softening cohesive zone models with a finite initial cohesive traction have been widely adopted for concrete fracture applications. The cohesive zone model derived by Jin and Sun [9] based on crack front necking also has a finite initial cohesive traction.

3. COHESIVE ENERGY DENSITY FOR INTERFACE CRACKS

The cohesive energy density has been regarded as a material constant. For a Mode I crack with a small cohesive zone so that an elastic K-dominance zone exists around the crack tip, this can be proved by using the J-integral technique and the cohesive energy density is the critical energy release rate. For a crack at the interface between two dissimilar elastic materials, however, it may not be appropriate to assume a constant cohesive energy density.

Consider a crack at the interface between two dissimilar elastic materials with the shear modulus and Poisson's ratio for the upper and lower media denoted by μ_i and ν_i (i = 1, 2), respectively. It is assumed that the cohesive zone is surrounded by the elastic interface crack tip oscillation field with the complex stress intensity factor $K_1 + iK_2$. For the two integration paths with one along the boundary of the cohesive zone and the other within the dominance zone of the elastic oscillatory field, application of the J-integral [10] at crack initiation yields

$$\Gamma_c = G_c = \left(1 - b^2\right)\left(K_1^2 + K_2^2\right)/E^* \tag{5}$$

where Γ_c is the cohesive energy density, G_c the critical energy release rate, b Dundurs' constant

$$b = \left[m_1(k_2 - 1) - m_2(k_1 - 1)\right]/\left[m_1(k_2 + 1) + m_2(k_1 + 1)\right] \tag{6}$$

$\kappa_i = 3 - 4\nu_i$ for plane strain and $\kappa_i = (3 - \nu_i)/(1 + \nu_i)$ for plane stress, and E^* given by

$$1/E^* = \left[\left(1 + k_1\right)/m_1 + \left(1 + k_2\right)/m_2\right]/16 \tag{7}$$

It is known from the interface fracture mechanics (see, for example, [11]) that G_c depends on the phase angle, i.e.,

$$G_c = G_c(\Psi) \tag{8}$$

where Ψ is an appropriately defined phase angle. Eqs. (5) and (8) indicate that

$$\Gamma_c = \Gamma_c(\Psi) \tag{9}$$

Hence, the cohesive energy density for interface cracks depends on the phase angle.

4. INTERFACE STRESS SINGULARITY AT A COHESIVE ZONE TIP

In the cohesive zone modeling approach, the energy released in the formation of new crack surfaces (unit area) is assumed to be the cohesive energy density. That is, there should be no energy dissipation at the tip of the cohesive zone. It is from this condition that the cohesive zone length is determined in Mode I case. For mixed mode fracture, however, both opening and sliding separations contribute to the energy dissipation. Null energy dissipation at the tip of the cohesive zone thus requires cancellation of singularities in both normal and shear stresses at the cohesive zone tip.

Consider a crack of length $2a$ at the interface between two semi-infinite dissimilar elastic media subjected to remote tension s_∞ and shear t_∞, as shown in Fig. 1. A cohesive zone of length $r = c - a$ develops ahead of each crack tip upon external loading. It is assumed that the cohesive tractions are constant. Consider the small scale cohesive zone case, i.e., $r << a$. Hence, $a \approx a + r$. The complex stress intensity factor (SIF) at the cohesive zone tip due to the applied loads is [12]

$$K_{app} = (s_{\infty} + it_{\infty})(1+2ie)\sqrt{pa} / \cosh(pe) \tag{10}$$

and the SIF due to the cohesive traction can be obtained from the Green solution of Rice and Sih [12] as follows

$$K_{coh} = -\sqrt{\frac{2}{p}}(s_n + is_s)(2a)^{ie}\int_0^r x^{-1/2-ie}\,dx = -\sqrt{\frac{2}{p}}(s_n + is_s)(\frac{2a}{r})^{ie}\,\frac{2\sqrt{r}}{1-2ie} \tag{11}$$

Here, the definition of SIFs of Sun and Jih [13] is adopted and e is the oscillatory index given by

$$e = \ln[(1-b)/(1+b)]/(2p) \tag{12}$$

The cancellation of stress singularity at the cohesive zone tip means

$$K_{app} + K_{coh} = 0 \tag{13}$$

or

$$Te^{iy}\,\frac{1+2ie}{\cosh(pe)}\sqrt{pa} = \sqrt{\frac{2}{p}}s_0 e^{ij}\,(\frac{2a}{r})^{ie}\,\frac{2\sqrt{r}}{1-2ie} \tag{13'}$$

where φ is the phase angle of the cohesive traction and ψ is the loading phase angle, i.e.,

$$s_n + is_s = s_0 e^{ij}, \qquad s_{\infty} + it_{\infty} = Te^{iy} \tag{14}$$

Eq. (13)' can be satisfied only when

$$j = y + e\ln(r/2a) \tag{15}$$

In the cohesive zone model, σ_n and σ_s are assumed to be material-dependent constants. While they may depend on the phase angle ψ according to Eq. (9), Eq. (15) can not be satisfied in general, which implies that the stress singularity at the cohesive zone tip may not be cancelled.

5. ENERGY RELEASE AT THE TIP OF A COHESIVE ZONE

As stated in the above section, there should be no energy dissipation (or equivalently no stress singularity) at the tip of the cohesive zone and a single cohesive zone size may not suffice this requirement under mixed mode fracture conditions. In Section 2, it has been shown that for Mode I fracture in homogeneous materials, the cohesive zone model can not assume a cohesive law with an initial hardening segment having an initial zero cohesive traction, if the stress singularity is to be removed at the tip of the cohesive zone. In other words, energy will be released at the tip of a cohesive zone described by a linear hardening law with an initial zero traction.

When the stress singularity can not be removed at the tip of a cohesive zone, the size of the cohesive zone will not be precisely determined. In this case, the length of the cohesive zone does not represent the actual cohesive zone size. To make a distinction, we refer this kind cohesive zone as the *prescribed cohesive zone*. In this section, we will discuss the energy dissipation at the tip of a prescribed cohesive zone described by the following bilinear model

$$\begin{aligned} s_n &= s_n^c\left(d_n / d_n^1\right), && 0 \le d_n \le d_n^1 \\ &= s_n^c\left(d_n^c - d_n\right)/(d_n^c - d_n^1), && d_n^1 \le d_n \le d_n^c \end{aligned} \tag{16}$$

where s_n^c is the peak normal cohesive traction, d_n^c the critical opening separation at which the normal cohesive traction vanishes, and d_n^1 the characteristic separation corresponding to the peak normal tractions, as shown in Fig. 2. For simplicity and without compromising our fundamental argument, the normal and shear components are assumed to be uncoupled in the cohesive zone model and the cohesive law in shear has the same form of Eq. (16). The oscillatory index e is also taken to be zero. Hence, the opening and shear modes are uncoupled in the interface crack problem.

Again consider a two-dimensional bi-material elastic medium of infinite extent with an interface crack of length $2a$ subjected to remote tension s_{∞} and shear t_{∞}, as shown in Fig. 1. It is assumed that a cohesive zone of length, $r = c - a$, is prescribed ahead of each crack tip upon external loading. The integral equation method described in Section 2 is used to calculate the energy release rate G_I^{tip} at the cohesive zone tip (stress singularity now can not be cancelled there).

Fig. 3 shows the normalized Mode I energy dissipation G_I^{tip} / Γ_n^c versus the nondimensioanl crack extension, $\Delta a / l_n^c$, for $a_0 = l_n^c$ and various ratios of d_n^1 / d_n^c, where $\Gamma_n^c = s_n^c d_n^c / 2$ is the cohesive energy density (opening mode), $l_n^c = E^* \Gamma_n^c / \left(s_n^c\right)^2$ is the characteristic cohesive length, and a_0 is the initial half crack length. The characteristic length l_n^c is usually employed to estimate the cohesive zone size. The prescribed cohesive zone length r is taken as $2 l_n^c$. It can be seen in the figure that for a given crack extension, the normalized energy dissipation increases with the increase in d_n^1 / d_n^c, a parameter measuring the initial stiffness of the cohesive zone model. The energy dissipation remains under 5% of the cohesive energy density for $d_n^1 / d_n^c = 0.1$ and when the crack extension does not exceed half of the prescribed cohesive zone length. The energy dissipation remains almost above 30% of the cohesive energy density for a linear hardening model ($d_n^1 = d_n^c$) regardless of the crack extension amount.

6. CONCLUDING REMARKS

Some conceptual issues regarding fracture in a homogeneous material as well as along a bi-material interface using cohesive zone models are examined. It is shown that the cohesive zone model can not assume a linear hardening law with an initial zero cohesive traction, if the stress singularity is to be removed. For interface fracture, a single cohesive zone length may not suffice the condition that both tensile and shear stress singularities at the cohesive zone tip be cancelled. The energy dissipation at the tip of a prescribed cohesive zone is then studied using a bilinear cohesive zone model under the uncoupled tension/shear conditions. It is concluded that the energy dissipation at the cohesive zone tip may not be neglected if the initial stiffness of the cohesive model in both opening and shear modes is low and the pre-embedded cohesive zone is not much greater than the characteristic length of the cohesive zone model in both cases of tension and shear. Finally, the cohesive energy density for interface cracks should be taken as a function of the loading phase angle rather than a constant.

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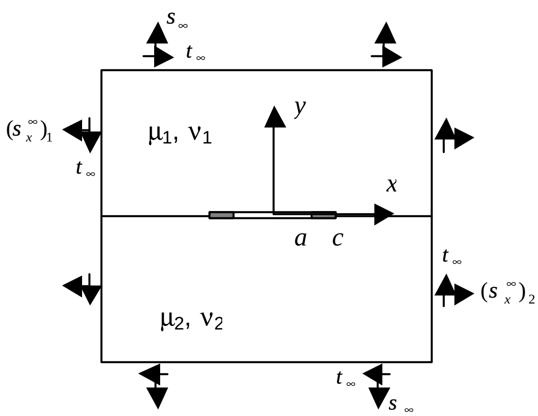

Fig. 1 A crack at the interface between two dissimilar elastic materials under remote loads

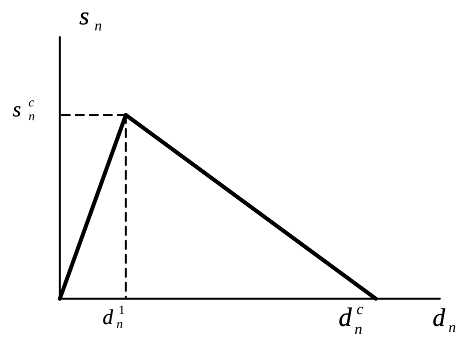

Fig. 2 A bilinear cohesive zone model

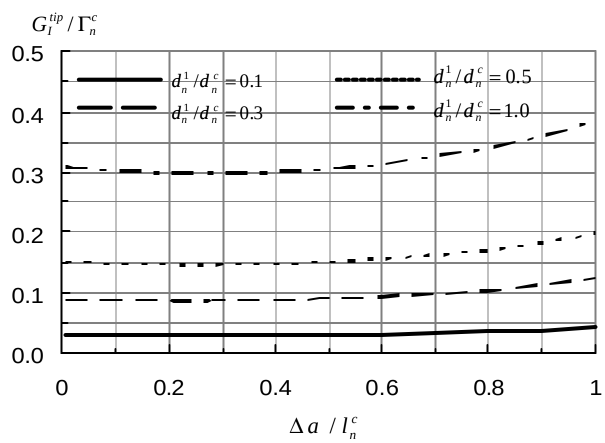

Figure 3 Normalized energy dissipation at the prescribed cohesive zone tip versus nondimensional crack extension