ESTIMATION OF STRENGTH FOR THREE-DIMENSIONAL BONDED STRUCTURES CONSIDERING STRESS SINGULARITY

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ABSTRACT

Stress singularity at a vertex in three-dimensional joints is closely related the strength of joints. It is very important to analyze the stress fields at the vertex for evaluating the reliability in three-dimensional joints. There are several methods for joining two different materials. Bonding method using adhesives is widely applied to making electronic devices, panels in air planes and so on. In joints using adhesives, the thickness of adhesive will influence on the stress intensity at the vertex. In the present study, we investigate the stress singularity fields using a three-dimensional BEM with the fundamental solution for two-phase materials and eigen analysis by FEM. Using the solution for two-phase materials as the fundamental solution in BEM, displacements and stresses at any points in the domain are determined accurately. The order of stress singularity is deduced from the eigen analysis by FEM. The stress fields at the vertex in three-dimensional joints have a power-law singularity and a logarithmic singularity. The stress intensity factor for power-law singularity is determined using a least square method considering the logarithmic singularity. It was found that the intensity of stress at the vertex reduces with decreasing the thickness of adhesive.

1 INTRODUCTION

In the previous study on three-dimensional joints, we have analyzed the stress singularity occurring at the vertex in two-phase materials. Power-law singularity and logarithmic singularity occur at the vertex in three-dimensional joints. It is supposed that the intensity of power-law singularity and logarithmic singularity depends on the structure of joints. In the present study, the joints bonded two materials using adhesive are analyzed. The thickness of adhesive will influence on the characteristic of stress distribution near the vertex. Hence, we investigate the influence of the adhesive thickness on the stress distribution at the vertex using BEM with the fundamental solution for two-phase materials and eigen analysis by FEM.

2 STRESS SINGULARITY ANALYSIS OF THREE-DIMENSIONAL DISSIMILAR MATERIALS

In the present study, the stress distribution in three-dimensional joints is obtained by BEM, and a boundary integral equation is described as follows.

$$c_{ij}(P)u_j(P)=\int_\Gamma U^*_{ij}(P,Q)t_j dS(Q)-\int_\Gamma T^*_{ij}(P,Q)u_j dS(Q) \qquad (1)$$

where U_{ij}^* and T_{ij}^* are fundamental solutions for displacements and tractions. Observation point, P, and source point, Q, are located on the boundary. t_j and u_j are traction and displacement vectors, respectively. Using the fundamental solution of two-phase materials, mesh division on the interface is unnecessary, and displacements and stresses at any points in the domain are determined accurately. In this analysis, the order of stress singularity is deduced from an eigen analysis of FEM. The characteristic solution, p, has the relationship of $\lambda=1-p$ with the order of stress singularity λ. p is determined from the following eigen equation.

$$\left(p^2[A]+p[B]+[C]\right)\{u\}=0 \qquad (2)$$

Where A, B and C are matrices composed of elastic moduli, and $\{u\}$ represents the displacement vector. Model for analysis is shown in Fig.1, and the size of the model is shown. The thicknesses of the adhesive, H, are varied from 0.1mm to 3.2mm. Tensile load, P=1GPa, is applied on the upper and lower surfaces in the model. The minimum length of mesh near the vertex is 0.005mm. Materials used in the analysis and elastic moduli are shown in Table 1. In this analysis, Material 1

Table 1 Material properties in the analysis

		Young's modulus (GPa)	Poisson's ratio
Material 1	Si	169.1	0.26
Material 2	Resin	2.97	0.38
Material 3	Si	169.1	0.26

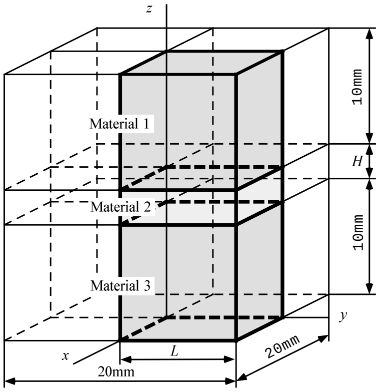

Fig.1 Joint model for analysis

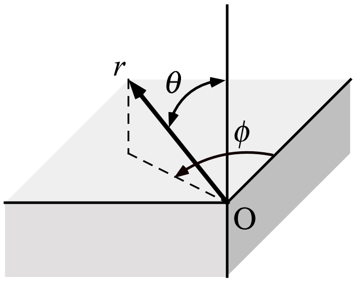

Fig.2 Spherical coodinate system with an origin at the vertex

and Material 3 have the same property, and the stress distribution near the vertex of Material 1 and Material 2 is investigated.

3 RESULTS OF ANALYSIS

Result of eigen analysis of FEM is shown in Table 2. The minimum value of eigen value is p=0.606 and the next larger eigen value is a 3-ple root of p=1. Then, the order of stress singularity, λ, is 0.393.

The stress distribution near the vertex obtained by BEM is shown in Fig.3. Stress component, $\sigma_{\theta\theta}$, is derived by transforming stress components in Cartesian coordinate system to those in a spherical coordinate system shown in Fig.2. Figure 3 shows the distribution of normal stress, $\sigma_{\theta\theta}$, at the angle of $\phi = 45$ degree on the interface ($\theta = 90$ degree) for various thickness of adhesive. Plots are almost straight in a log-log scale for several different thickness of adhesive, and its slope almost agrees in all stress components. In this case, the stress fields at the vetex is governed by the power-law singularity. The stress distribution near the vertex can be expressed as follows.

$$\sigma_{\theta\theta} / P = C_1 \bar{r}^{-\lambda} + C_2 + C_3 \log \bar{r} + C_4 \left(\log \bar{r}\right)^2 \tag{3}$$

Table 2 Eigen values in Si-Resin joint

	Real	Imaginary	λ
1	0.6065744	0.0000000	0.3934256
2	1.0000070	0.0000000	-0.0000070
3	1.0000056	0.0000000	-0.0000056
4	1.0000004	0.0000000	-0.0000004
5	1.0000240	0.5675051	-0.0000240

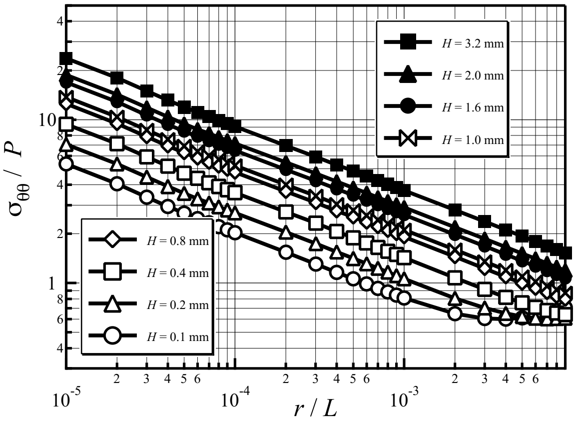

Fig.3 Distribution of stress $\sigma_{\theta\theta}$ against r/L for thickness of Resin (ϕ=45 deg., θ=90 deg.)

where, $\bar{r}$, represents r/L. The average slope of lines, λ, is about 0.396 in Fig.3. This value agrees well with the result of eigen analysis of FEM.

Coefficients, C_i, for terms in eqn.(3) are determined using a least square method. Then, the order of power-low singularity, λ, is fixed to 0.393, that is the result of FEM. The variation of the coefficient, C_1, for various thickness of adhesives is shown in Fig.4, and other coefficients, C_2, C_3 and C_4, are shown in Fig.5. You can see from Fig.4 that the plot is almost straight in a log-log scale. The plots are almost straight except a small value of H in Fig.5. The absolute value of coefficient C_1 is maximum in all coefficients, and the value of C_2, C_3, and C_4, become small in turn. From this result, coefficient, C_i, can be expressed as follows.

$$C_i = KH^{\alpha} \tag{4}$$

where K=0.146 and α=0.464 for C_1.

4 CONCLUSION

In the present paper, the influence of the adhesive thickness on the stress distribution at the vertex was investigated using BEM with the fundamental solution for two-phase materials and eigen analysis by FEM. It was found that the intensity of stresse at the vartex reduce as the thickness of adhesives decreases. On other hand, the coefficients, C_i, for terms in eqn.(3) are proportional to H^{α}.

References

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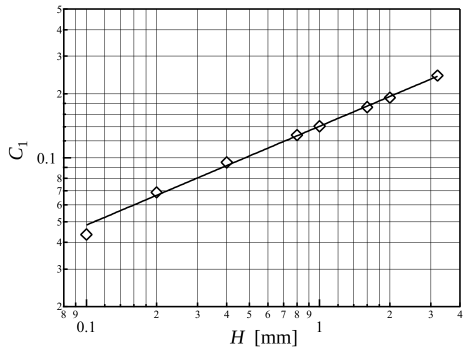

Fig.4 Variation of coefficient C_1 for the thickness of adhesive H

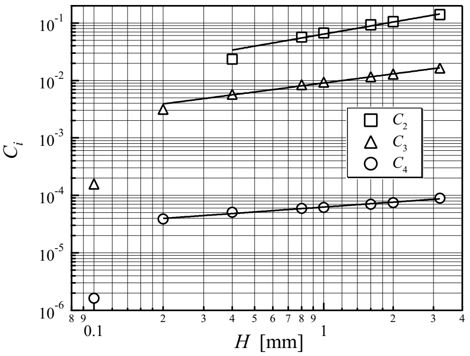

Fig.5 Variation of coefficient C_2-C_4, for the thickness of adhesive H