FRACTURE ANALYSIS OF COMPOSITE LAMINATE WITH DELAMINATION USING X-FEM

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ABSTRACT

The extended finite element method (X-FEM), which can simplify the modeling of continua containing several cracks and delamination, is applied to the stress analysis to clarify the damage mechanism of composite laminates for structural design using composite materials. Though the two-dimensional near-tip asymptotic displacement function in homogeneous isotropic materials has been utilized in X-FEM analyses for the linear elastic fracture mechanics problems, it is not clear that such a near-tip function may be appropriate for orthotropic materials. Therefore the near-tip functions for homogeneous isotropic crack are examined in the analyses of composite materials with orthotropic material property. In this study stress analysis and evaluation of the energy release rate for a composite laminate are performed by X-FEM. In the finite element models by X-FEM analysis, the geometry of the crack front does not always match the element boundary. Therefore the virtual crack extension method and the virtual crack closure method cannot be applied to evaluate the *J*-integral or the energy release rate for linear elastic problems. The domain form of the *J*-integral can be utilized to compute the *J*-integral in conjunction with X-FEM. As numerical examples, the three-dimensional elastostatic analyses for the double cantilever beam (DCB) test specimen are performed by X-FEM and the obtained results examined.

1 INTRODUCTION

Carbon fiber-reinforced plastic (CFRP) composite materials are used extensively in a number of engineering applications. These materials are used in the form of laminates for aerospace structures. The composite laminate structures may sustain impact damage, including delamination and transverse cracks. Therefore, in the field of structural design, understanding the damage mechanism of composite laminates is very important. Several experimental and analytical studies have been performed (Suemasu[1-2]). A number of analytical studies have used finite element models that consider cracks and delamination in composite laminates.

Recently, the extended finite element method (X-FEM) proposed by Belytschko [3], Moës [4], Sukumar [5] has been applied to the evaluation of stress intensity factors and crack extension simulations in conjunction with the level set method (Moës [6], Gravouil [7]). X-FEM can simplify the modeling of continua containing several cracks and hence can be used to perform effective stress analyses related to fracture mechanics. We have applied X-FEM to model interlaminar delaminations, which may grow in composite materials. In our previous study, elastostatic and linear buckling analyses for composite laminates were performed (Nagashima [8-9]). The present paper describes the calculated results for the energy release rate at the front tip of the delamination in a composite laminate. In the present study, the enrichment function obtained from the asymptotic solution near the crack tip in homogeneous isotropic materials is used even though the material property of the composite laminate is orthotropic. Stress analysis and evaluation of the energy release rate for the double cantilever beam (DCB) specimen are performed by X-FEM. The numerical results obtained by X-FEM using such an enrichment function are demonstrated and examined for three-dimensional elastostatic analyses.

2 NUMERICAL METHOD

In this section, the numerical procedure utilized in this study is outlined.

2.1 Interpolation function

In this study, the interlaminar planar delamination in the composite laminate is assumed. As shown in **Figure 1** the analyzed domain is defined in the Cartesian coordinate x, y, z and the planar delamination is assumed to place on the x-y plane. In X-FEM, the approximate displacement function $\boldsymbol{u}^h$ of the distributed displacement $\boldsymbol{u}$ near a delamination is expressed as:

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I=1}^{m} N_I(\mathbf{x})\mathbf{u}_I + \sum_{I\in\mathbf{C}} N_I(\mathbf{x}) \sum_{k=1}^{n} \gamma_k(\bar{f}(\mathbf{x}), z)\mathbf{a}_I^k + \sum_{I\in\mathbf{J}} N_I(\mathbf{x})H(z)\mathbf{b}_I \tag{1.1}$$

$$\bar{f}(\mathbf{x}) = \sum_{I=1}^{m} N_I(\mathbf{x}) f(\mathbf{x}_I) \tag{1.2}$$

where N_I is the interpolation function as used in the formulation of the conventional FEM, m is the number of nodes in the finite element, **C** and **J** denote the node set considering the asymptotic solution and the discontinuity of displacement near a crack, respectively, and $\mathbf{u}_I$, $\mathbf{a}_I^k$, and $\mathbf{b}_I$ denote the vector of freedoms assigned to each node. Here, $\boldsymbol{C} \cap \boldsymbol{J} = \phi$ is satisfied. γ_i (i=1,n) are the near-tip functions, which can consider the discontinuity near crack tip and n is the number of near-tip functions. And H(x) is the *Heaviside* function used to express the discontinuity of displacement on a delamination and far from crack front. In addition $f(\mathbf{x})$ is a level set function introduced to express the shape of the crack front of the delamination using nodal information. The level set function $f(\mathbf{x})$ is described below.

In this study, the level set functions $f(\mathbf{x})$ used in Eq. (1) is defined as follows:

$$f(\mathbf{x}) = \min_{\bar{\mathbf{x}}\in\Gamma} \|\mathbf{x} - \bar{\mathbf{x}}\| sign(\mathbf{n}(\bar{\mathbf{x}})^T(\mathbf{x} - \bar{\mathbf{x}})) \tag{2}$$

where Γ represent the curved crack front line and $\bar{\mathbf{x}}$ is a point on the curved line Γ, $\mathbf{n}(\bar{\mathbf{x}})$ denotes the vector orthogonal to the curved line Γ at point $\bar{\mathbf{x}}$.

This function is called the signed distance function and the absolute value of f means the distance between the point and Γ.

In this study, the near-tip function γ_i, which is determined from the asymptotic solution of a crack in homogeneous isotropic material, is defined as follows:

$$\gamma_1 = \sqrt{r}\cos\left(\frac{\theta}{2}\right), \gamma_2 = \sqrt{r}\sin\left(\frac{\theta}{2}\right), \gamma_3 = \sqrt{r}\sin\left(\frac{\theta}{2}\right)\sin\theta, \gamma_4 = \sqrt{r}\cos\left(\frac{\theta}{2}\right)\sin\theta \tag{3}$$

and

$$r = \sqrt{f^2 + z^2} \tag{4.1}$$

$$\theta = \arctan(z / f) \tag{4.2}$$

where r and θ are polar coordinate in a plane defined near the crack tip.

2.2 Nodal property

The distribution of the enriched node utilized for two-dimensional X-FEM analyses in this study is illustrated in **Figure 2**. In the figure, the node entitled “J” has an adding freedom for the *Heaviside* function as shown in the third term in the right hand side of the Eq.(1.1). Moreover, the node entitled “C” has an adding freedom for the near-tip function as shown in the second term in the right hand side of the Eq.(1.1). If the front tip of the delamination matches the element boundary, the pattern *3D-J* can be utilized. Otherwise the enrichment with near-tip functions as shown in the pattern *3D-JC* is required. In both cases the enrichment with near-tip functions is available when the crack tip is included in finite element.

2. 3 Evaluation of energy release rate

In the finite element models by X-FEM analysis, the geometry of the crack front does not always match the element boundary. Therefore the virtual crack extension method and the virtual crack closure method, which are utilized in conjunction with conventional FEM analysis, cannot be applied to evaluate the *J*-integral or the energy release rate for linear elastic problems. The domain form of the *J*-integral (Moran [10]) can be utilized to compute the *J*-integral in conjunction with X-FEM.

The domain form of the *J*-integral in three dimensional problems can be expressed as follows:

$$\bar{J} = -\iiint_V \left(w\delta_{1i} - \sigma_{ij}u_{j,1}\right)q_{,i}\,dV \quad (i = 1, 2, 3) \tag{5.1}$$

$$J = \frac{\bar{J}}{L/2} \tag{5.2}$$

where q is the weight function and V is the volume including the front tip of the delamination

The cylindrical region, the radius and the length of which is R and L, respectively, as shown in **Figure 3(a)** is utilized as an integrated domain for conventional FEM and it can be used for special cases of X-FEM analysis, where the geometry of the front tip matches the element boundary. But in X-FEM analysis the geometry of the front tip does not always match the element boundary. In such a case, a rectangular parallelepiped region defined in the local Cartesian coordinate $\tilde{x}, \tilde{y}, z$ as shown in **Figure 3(b)**, the center of which is correspond to the evaluation point at the crack tip, is utilized (Sukumar [5], Moës [6]). The *J*-integral is calculated using the stress, strain and displacement which are evaluated at integration points in the domain. The weight function q takes the value of one at the center of the rectangular parallelepiped and vanishes at the surface.

3 NUMERICAL EXAMPLES

As numerical examples the elastostatic analysis of the DCB (Double Cantilever Beam) specimen as shown in **Figure 4** was performed. The DCB specimen has an initial delamination, the length of which is 60 *mm*, and the enforced opening displacement 1 *mm* is given at the edges of the specimen. The energy release rate at the front tip of the delamination and the reaction forces at the enforced edge were evaluated. The material of the DCB specimen is assumed to the fiber-reinforced material. The elastic properties of the unidirectional ply of the fiber-reinforced material are E_L=142 *GPa*, E_T =10.8 *GPa*, G_{LT} =5.49 *GPa*, G_{TT}=3.71 *GPa*, ν_{LT} =0.3, ν_{TT}=0.45. In the present calculation an eight-node blick element was used for three-dimensional analysis. The 6th-order Gauss integration are adopted to evaluate the local stiffness containing any enriched elements.

The employed finite element mesh has 150 divisions in the length direction, 25 divisions in the width direction and 4 divisions in thickness wise. Both of the pattern *3D-J and 3D-JC* as shown in **Figure 2** were utilized for modeling the delamination. The geometry of the front tip matches the element boundary. Both the cylindrical and parallelepiped region as shown in **Figure 3** were used to evaluate the distribution of energy release rate at the front tip. The radius and length of the cylindrical region to perform domain integral are 3mm and 2 mm, respectively. 6mm x 2 mm x 4mm box is utilized for the integration parallelepiped region.

The distribution of the energy release rate at the crack tip was calculated by three-dimensional X-FEM analyses and compared with that obtained by the beam theory [11] in **Figure 5**. Consistency between present numerical analyses using X-FEM and the theoretical result was observed. In case of using cylindrical region, the difference of the energy release rate between pattern *3D-J* and *3D-JC* is small as shown in **Figure 5 (a)**. However, the difference between pattern *3D-J* and *3D-JC* cannot be negligible in case of using parallelepiped region as shown in **Figure 5 (b)** Enrichment with adding freedom for the near-tip function seems to improve the numerical accuracy for evaluated energy release rate in this calculation.

4 CONCLUDING REMARKS

This paper described the basic study for application of X-FEM to stress analyses of the composite laminates contains the delamination and demonstrated the numerical results of elastostatic analyses by X-FEM for the DCB test specimen. Three-dimensional analyses can evaluate the detail distribution of the energy release rate along the crack tip. In X-FEM analysis, delamination modeling can be performed by only modifying the nodal freedom near the delamination, therefore the modeling of a delamination is more easier than the conventional FEM. Specially the analysis model using the near-tip enrichment functions seems to give more accuracy of the analysis than the conventional FEM using the double nodes. In future studies, delamination with complex geometry in composite laminate will be modeled and the distribution of energy release rate at the front tip will be investigated in detail.

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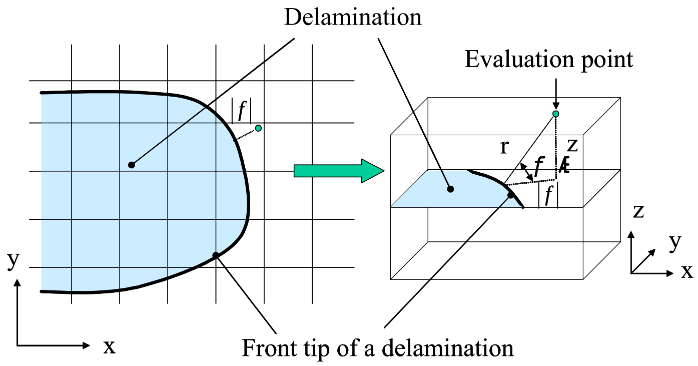
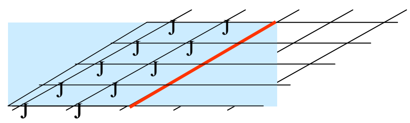
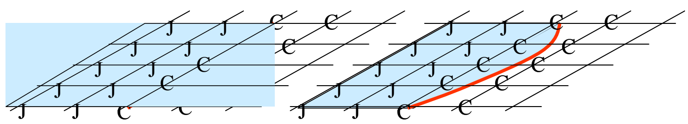


Figure 1 Definition of a delamination in a finite element mesh



(a) Pattern 3D-J



(b) Pattern 3D-JC

Figure 2 Modeling of a delamination in three-dimensional X-FEM analysis

(a) Cylindrical domain integration (b) Parallelepiped domain integration

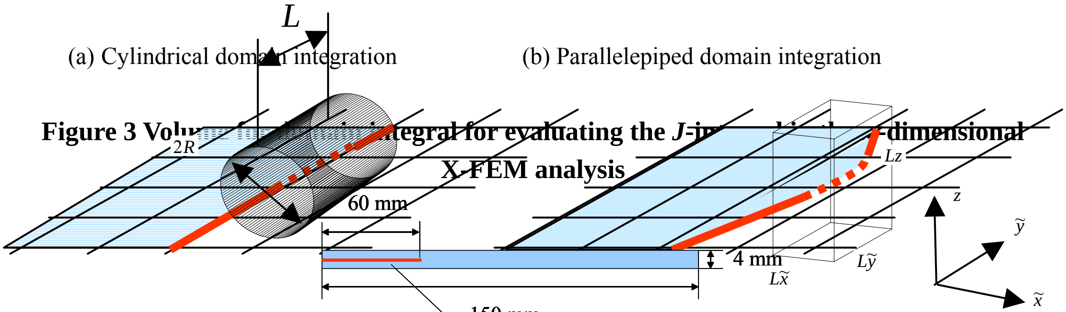


Figure 3 Volume domain integral for evaluating the *J*-integral in three-dimensional X-FEM analysis

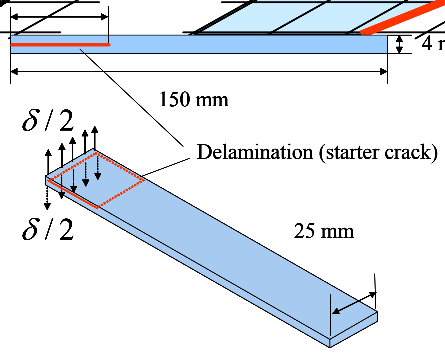
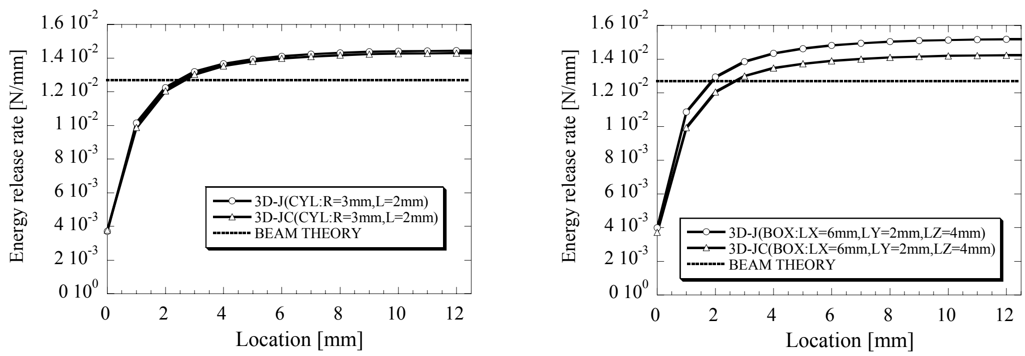


Figure 4 Double Cantilever Beam (DCB) test specimen



(a) Cylindrical domain integration (b) Parallelepiped domain integration

Figure 5 Distribution of energy release rate at the crack tip of the delamination in the DCB specimen (Length of delamination: 60 mm)