NON-LINEAR FRACTURE MECHANICS APPLIED TO WOOD IN MODE I

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ABSTRACT
The study presented in this document aims at using non-linear fracture mechanics in order to study the propagation of a longitudinal crack in wood in mode I. Finite-elements simulation are proposed to model crack. They are based on the using of joint-elements located on the crack path. The possibility to implement a constitutive damage law is investigated. Different shape and size of the local damage law are tested. Numerical results show that the local behaviour of the failure interface has an important influence on the Force-COD of DCB specimen. The peak load of the specimen is strongly dependent of the constitutive law form and the propagation of the crack can be expressed by the critical energy released rate defined in the frame of the Linear Elastic Fracture Mechanics. R-curves are determined from NLFM in order to study the influence of the constitutive damage law parameters on their shape. Study shows that new simulations should be performed; they would take into account a damaged volume by considering a non zero thickness failure interface.

1 INTRODUCTION

Due to the anisotropy of the material, wood exhibits low strength perpendicular to grain. The analysis of wooden elements failure is a crucial point for designer in Timber Engineering. Generally, stresses perpendicular to grain induce cracks which propagate longitudinally. Researches to characterise ultimate load of beams are undertaken by the way of Linear Elastic Fracture Mechanics (LEFM).

Due to the fact that the crack path is generally known because it corresponds to grain direction, we propose to simulate crack propagation by using a non-linear fictitious crack model (Hillerborg, [1]). The numerical investigations are compared to LEFM ones.

2 MODELLING CRACK BY USING JOINT ELEMENTS

Finite elements simulations focuses on failure of timber along the grain in mode I. Based on fictitious crack model, the crack path is considered well known. This area is meshed by using an interface composed of joint elements (Beer [2] and Allix et al. [3]). It takes advantage in considering the continuity of the stresses and displacements fields between each opposite nodes composing the crack. This method generates also a significant stability of calculation iterations and requires fewer numerical resources in comparison with classical analysis requiring located forces. In the linear elastic field, stresses perpendicular to the interface σ_n, (i.e. perpendicular to grain) are expressed by the transverse displacement δ_n and a stiffness K_n in N/mm^3 (equation 1):

$$\sigma_n = K_n \cdot \delta_n \tag{1}$$

To reproduce linear elastic behaviour of the material, K_n is characterised by the following expression:

$$K_n = \frac{E}{e} \tag{2}$$

Where E is the Young’s modulus and e is the thickness of the interface.

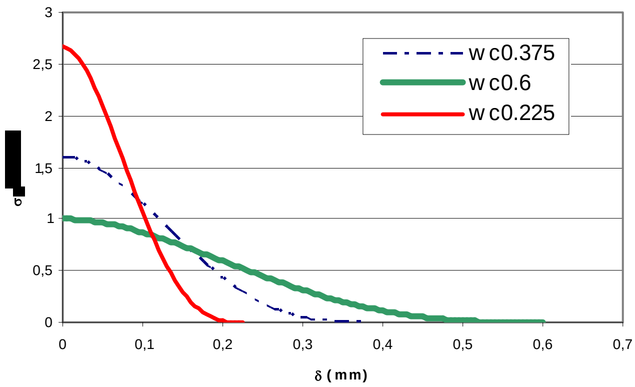


Figure 1 : Constitutive damage laws (polynomial functions)

Two choices are possible to model crack path. First, we may consider that crack will be located in a longitudinal area within a given *e*. This assumption corresponds to a Crack Band Model: the macro-crack resulting of micro-cracks dislocations is created in this specific volume (Bazant [4]).
Second, it may possible also to take a zero thickness interface considering only *e=0 mm*. In this approach, the simulation represents a Cohesive Crack Model (Bazant [5]). In this condition, (according to equation 2), K_n tends toward infinity. This second approach is explored in this document and numerical method is proposed further to calibrate K_n.

3 COHESIVE CRACK MODEL USING JOINT ELEMENTS WITH NON-LINEAR CONSTITUTIVE LAW

Cohesive Crack Model needs to determine a relationship between σ_n and δ_n in order to characterise the progressive damage of wood in mode I. Generally, two approaches are used to model constitutive law of the cohesive crack in others materials as concrete for instance. A linear function describes that timber damages progressively before failing, a bilinear relationship takes into account the crack bridging mechanism at the crack tip. The slopes of these two representations can be determined from w_c (local ultimate crack opening displacement), f_t (ultimate stress of the interface) and G^* (critical damage energy release rate of the interface in mode I).
We propose in this document to use a polynomial function to describe the non-linear constitutive damage law which is more representative of the experimental fracture behaviour. Polynomial function P_n is deduced from G^*, w_c. The choice of the function degree n modifies the form of the law.The objective of this research is to study the influence of the non-linear model on the peak load of an element.

4 FINITE ELEMENT SIMULATION REALISED ON DCB SPECIMEN

4. 1. LEFM simulations

The first step of the simulation consists in determining Force-COD (Crack Opening Displacement) curve of the DCB specimen by using a linear elastic theory. The estimation is based on compliance method and the load P is evaluated from G (equation 3) for each crack length a , as :

$$G = \frac{P^2}{2b} \cdot \frac{dC}{da} \qquad (3)$$

Where C is the compliance of the specimen for a crack length equal to a.

Computation required successive meshes according to crack length, in order to plot each point of the Force-COD curve (Figure 2). The progressive decrease observed after the peak load corresponds to the crack propagation at constant resistance to crack growth, i.e. G is constant which is synonymous with the definition of the critical energy release rate G_c.

4. 2. NLFM simulations

To compare linear elastic approach with non-linear model integrated in joint elements, the same DCB specimen is meshed with Beer elements on the crack path. When K_n is calibrated to 10000N/mm^3, the difference of compliance is close to 0.5%, which is considered as representative of the pure linear elastic behaviour.

Afterwards, several polynomial functions are tested to characterise their influence on the crack propagation mechanism. Calculations are performed with a damage energy G^* equal to 0.24 N/mm, whose the value will be explained further. So three critical crack opening displacements w_c are chosen arbitrary for simulations (0.225, 0.375 and 0.6 mm), and the degree n of P_n is fixed to 4. Results are compared to those issued from LEFM.

For a same G^*, constitutive law produces a deviation from the initial elastic compliance of the Force-COD curve, translating the progressive damage of the interface as observed in experimental investigations. Figure 2 shows the impact of w_c concerning the progressive inflexion of the diagram. After several displacements steps, non linearity starts due to the progressive damage of the interface. The ultimate stress f_t in the interface generates the beginning of the deviation from the initial compliance. When w_c is reached, crack propagates for a constant G^* whose the value seems to correspond to G_c value defined in LEFM.

Others sensitivity tests are undertaken by the assumption of non-linear and linear constitutive damage law (w_c and G_c are fixed). Results show clearly that the ultimate load of the specimen is strongly dependent of the constitutive law form integrated in the joint elements model. This observation reveals the crucial point concerning the choice of the appropriate damage law in order to estimate the peak load in the Force-COD diagram.

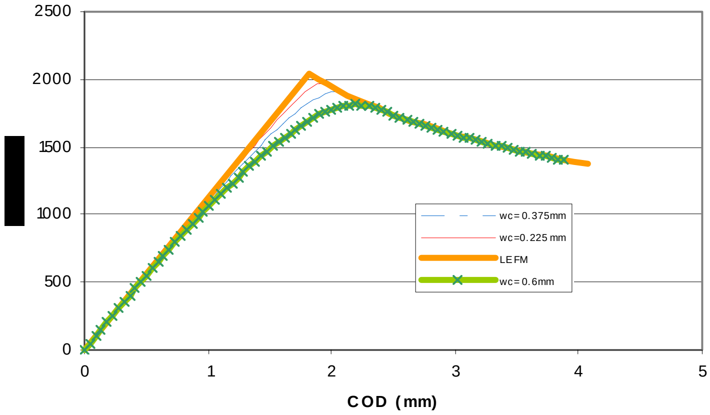
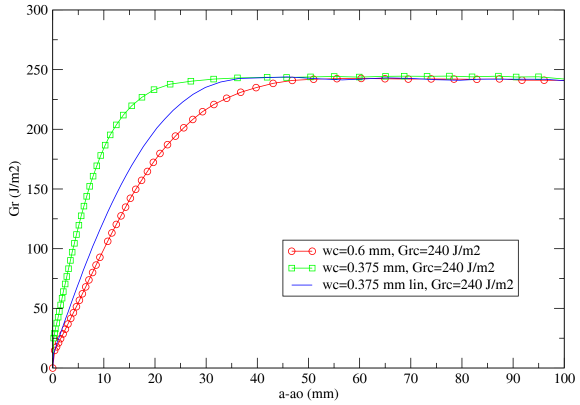


Figure 2 : Force-COD curves obtained by NLFM and LEFM FE-Simulations



Resistance curves (R-curves) have been also determined from NLFM simulations. Classical form of R-curves is obtained with a saturation of *G* (Figure 3). Numerical data show that the gradual transition of G_r to G_{rc} is generated by the non-linear constitutive law P_n. The degree of P_n and w_c influence the estimation of a_c value characterising the equivalent crack length in LEFM when G_c is reached. Consequently, observation made illustrate that R-curve would be a representation of the non-linearity damage law of timber in mode I in the frame of linear elastic theory. a_c depends on the degree *n* and w_c, and we obtain $G^*=G_c=G_{rc}$ when the equivalent crack length is superior to a_c.
Following these remarks, fitting tests are realised on experimental curve. Polynomial and linear constitutive damage law are formulated to obtain the best compromise with experiments. Post treatment shows that a polynomial function of degree equal to 3 and w_c equal to 0.8 gives the best results.

5 DISCUSSION AND CONCLUSION

Numerical investigations reveal that non-linear fracture mechanics using joint-elements in the crack path gives good results in order to obtain Force-COD curve according to experimental tests on DCB. Work demonstrates the different possibilities to integrate damage law behaviour modelling the progressive damage of timber with a cohesive crack approach. Form of the constitutive law has an substantial influence concerning the ultimate load of the specimen.
Nevertheless, R-curves determined from DCB, TDCB of a same size show that specimen geometry has a strong influence (Morel et al. [5]). The shapes of the curves are different: a_c and G_c are dependant of the geometry (Figure 4).
Due to the fact that the critical resistance G_{rc} corresponds to the damage energy release rate $G*$, different constitutive law should be required. In order to describe such a difference, the first idea consists in considering a non zero thickness interface. Indeed, for a given thickness *e*, the critical damage energy release value $G*$ can be expressed (equation 4):

$$G_d = \frac{dW}{dV} = \frac{1}{eb}\cdot\frac{dW}{da} = \frac{1}{e}\cdot G^* \qquad (4)$$

Where G_d characterises a critical damage energy rate (J/m^3).

The value of G^* needed to describe the R-Curves of TDCB will be higher than the one of the DCB specimen. Such a result is in agreement with the stress intensity factor function shown in Figure 5. Indeed, for the same load and the same crack length, the SIF is higher in TDCB than the one in DCB, and hence, fracture process zone is expected to be larger in TDCB. That is consistent with different thickness interface $e_{TDCB} > e_{DCB}$.
Numerical investigations will be performed by considering Crack Band Model, in order to integrated damage volume in the calculation.

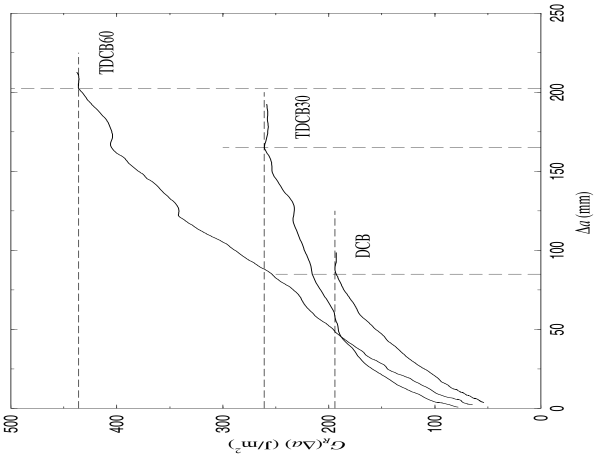


Figure 4 : Experimental R-Curves (TDCB, DCB)

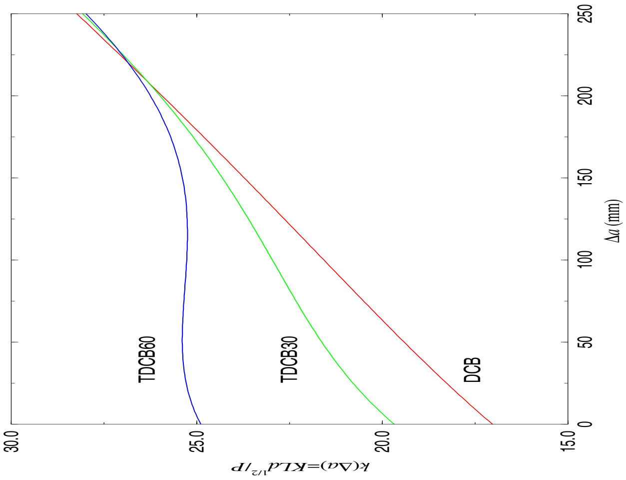


Figure 5 : SIF evolution in TCD and DCB

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