LIBRARY OF GEOMETRIC INFLUENCES FOR SIF WEIGHT FUNCTIONS

F. P. BRENNAN, L. S. TEH and A. J. LOVE
NDE Centre, Department of Mechanical Engineering, University College London, UK

ABSTRACT

Most researchers agree that the SIF is an important if not vital parameter in the assessment of defects prone to linear elastic fracture behaviour. Unfortunately the SIF is difficult to compute or measure, particularly if the crack is situated in a complex three-dimensional geometry or subjected to a non-simple stress state. Enormous developments in computing processing ability in the past few years has led to exciting Finite Element and Boundary Element approaches to the solution of SIFs in complex situations. These however, are likely to always remain the preserve of specialists.

The need to provide quick robust and accurate SIF solutions for engineering defect assessment led to the development of a weight function approach to compose complex SIF solutions from simple constituent parts. These simple constituents make up a library that can be used with an appropriate composition algorithm to quickly build solutions for cracks in real engineering geometries.

This paper describes the form of the "Library of Geometric Influences" for two dimensional symmetrical notch SIF Weight Functions. Future development of the approach to extend it to asymmetrical and three dimensional geometries is also discussed.

1 INTRODUCTION

The SIF is in effect the currency of Linear Elastic Fracture Mechanics and is generally agreed to be the key parameter in the assessment of defects prone to linear elastic fracture behaviour. SIF solutions for many geometries and load conditions exist to varying degrees of validity and accuracy. Some are presented in the form of parametric equations others as figures and tables. Invariably solutions are coded into software combining some quite accurate and reliable solutions with others which are not so useful. The current practice for determination of SIFs is generally to use numerical or experimental techniques, both of which are time consuming and require great care and attention. Exciting developments in Finite Element and Boundary Element approaches have allowed extremely complex aspects of crack behaviour to be studied. The use of these methods are however, likely to always remain the preserve of specialists. It also means that available solutions have generally been developed for specific applications and therefore are generally applicable within often quite restrictive limits of validity. Engineering optimisation and defect assessment of components in service however, often require broad ranging solutions which can be rapidly calculated.

Although the approach reported here still requires such reference constituent solutions, their number is limited and can be used to generate an unprecedented number of new solutions. The real value of this approach is that it allows the rapid formulation of SIF solutions in complex geometries under any stress field (including residual and applied stresses).

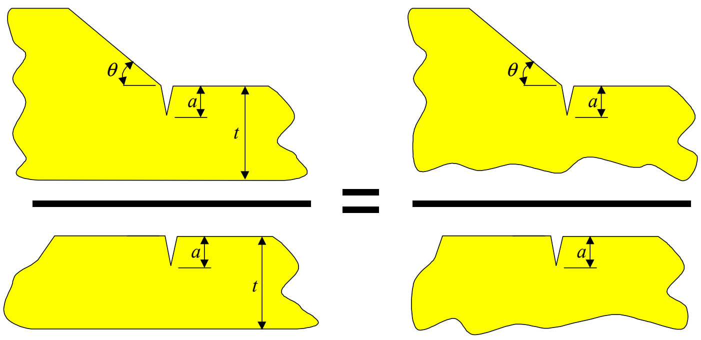
This paper describes the successful development of a universal approach to the generation of SIFs for a wide range of geometries and load conditions [1, 2]. The publication of simple coefficients allows the calculation of the SIF weight function for any mode I edge crack in a finite-strip (any thickness) from the notches analysed. This in turn allows the calculation of the SIF under any load system. The authors believe that this approach has tremendous potential in allowing rapid calculation of SIFs for use with in-service monitoring systems. In addition, SIFs can easily be calculated to model crack propagation through residual stress fields and other complex stress systems. Widespread use of the solutions could mean better understood and safer structures and allow the use of advanced manufacturing process to provide beneficial residual stresses for crack retardation and arrest (e.g. laser peening) [3].

2 THE WEIGHT FUNCTION COMPOSITION APPROACH

By virtue of the definition of the weight function as a unique property of geometry, the influences it represents can be isolated and combined. This characteristic is unique to the weight function. Niu and Glinka [4, 5] and Mattheck *et al.* [6] made similar composition approaches after Impellizzeri and Rich [7] used what they called Geometry Correction Factors for the influence of geometric anomalies on the SIF. These studies represented the beginning of the idea that geometric influences could be analytically separated and subsequently many engineering solutions were developed from this work (e.g. [8], [9]).

Due to the restrictive mathematics of the SIF weight function at the time, these studies were unable to respect the critical fact that only the pure weight function can be separated in order to obtain a true (load mode independent) weight function solution. Solutions for the weight function were developed by Ojdrovic and Petroski in the early nineteen-nineties allowing far more robust SIF weight function solutions. A full description of the significance of these can be found in reference [10].

Considering that the weight function is a unique property of geometry, these geometric influences can been isolated and combined. A relationship that forms the basis of the composition principle is shown pictorially below.

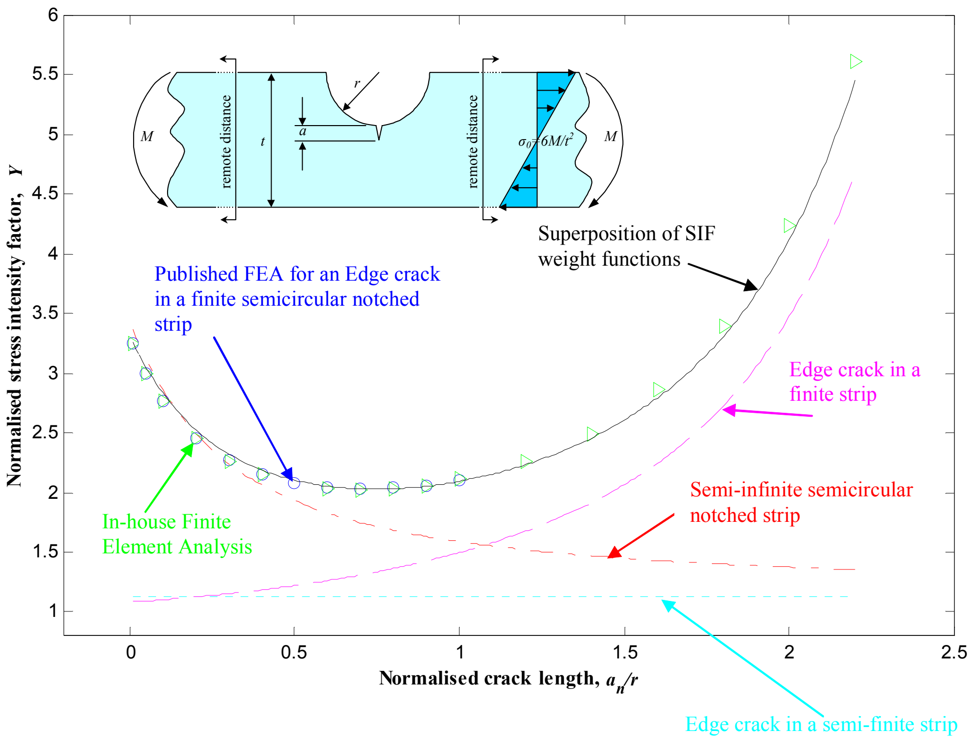


i.e. The ratio of SIF weight function of a cracked body having a projection and finite thickness to that of a cracked body without a projection having the same thickness is equal to the ratio of SIF weight function of a cracked body having a projection and infinite thickness to that of a cracked body without a projection having infinite thickness.

On the face of it this appears simple and should be equally applicable to symmetric, asymmetric and three dimensional geometries. This will be discussed further in the following section. In order to prove the hypothesis work was carried out considering symmetrical notches only. This is described in detail in reference [1] and some results presented here for convenience.

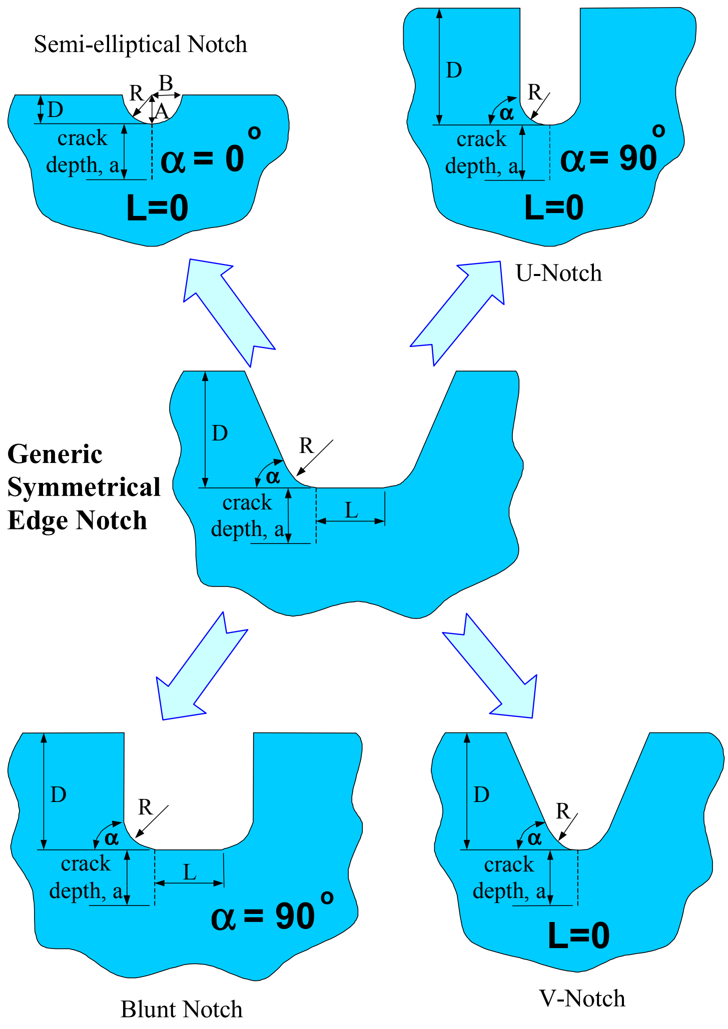
The simplicity of the above pictorial representation is somewhat deceptive; it took considerable effort to finally mathematically implement the theory however, the results are quite convincing.

One example of these is shown in the Figure below. This shows a specific case of an edge crack from a semi circular notch in a finite thickness body and independently the constituent solutions: Edge crack in a finite strip; Edge crack in a semi-finite body and an Edge crack from a semi circular notch in a semi-finite body, and the superimposed result compared with in-house and published finite element results.



This Figure also demonstrates the verification of the method where a new solution is built from its constituent parts. Readers can appreciate that in this way once the case of the notch in a semi-finite strip is solved then, a SIF solution is easily obtainable for that notch in any thickness strip from standard solutions for an edge crack in a finite strip and in a semi-finite strip. Remembering that the solution is by means of a weight function, the SIF distribution can be determined for any residual or applied stress distribution.

Once the approach was verified, project work set out to generate a library of constitutive solutions that systematically defined a wide variation of notch parameters. The generic notch definition that was arrived at is shown in the figure below. This generally split up the various notch geometries into: Semi-circular/semi-elliptical notch; U-Notch; V-Notch; and blunt notches. SIF solutions for cracks emanating from these notches in semi-finite strips were evaluated using Finite Element Analysis and uncracked models were created for the crackline stress distributions. In addition, Finite Element Analysis of finite thickness bodies was conducted for verification where published solutions did not exist.



In order to create the library, for each geometry at least twenty finite element models were created with different crack depths. Ignoring mesh optimisation and those for verification, over fifty geometries were modelled varying the geometric parameters shown in the figure above. This gives in excess of 1,500 finite element models solved to create the reference solutions. Results were fitted to parametric equations for input into the composition SIF weight function model described in reference [1]. The final results are presented as coefficients for use with the weight function to give SIF weight functions for cracks emanating from any of the notch shapes analysed in finite thickness strips. Solutions are also in the form that they can in future be used in conjunction with three-dimensional plate and tube solutions.

3 LIMITATIONS OF THE 2-D SYMMETRIC NOTCH APPROACH

The weight function composition principle presented and applied to symmetric notches proves the premise that geometric influences, described as weight functions, may be combined, or composed, via a suitable composition scheme to yield new SIF solutions. Extension of the principle to broaden the library database and incorporate symmetric notches, of extreme geometric form and asymmetric notches has exposed limitations of the composition scheme in its present form. Recent developments have sought to modify the composition scheme for application to asymmetric geometries to address these limitations and formulate a universal methodology applicable to all inter-related geometric forms shown above.

Considerable attention has been dedicated to determining the precise form of an appropriate composition scheme for asymmetric notches, which realises the potential accuracy, flexibility and efficiency achieved for symmetrically notched components. A composition scheme closely related to that presented here, which fulfils these criteria, has been successfully developed. As yet unpublished results demonstrate the composition schemes robust nature and accuracy when validated against FE solutions.

4 FUTURE DEVELOPMENTS

Ongoing development of the composition theme has identified the possibility of combining geometric influences of two or more complex forms to build SIF solutions for more intricate geometric profiles in a manner that is both conceptually and mathematically simple. Realisation of this observation allows the rapid determination of an almost limitless number of SIF solutions from a small library of constituent geometry solutions.

Cracks at external notches in real three-dimensional engineering components invariably develop as two-dimensional surface flaws. An extension to the existing methodology envisages application of the composition scheme to such crack systems. Reference [8] describes a general procedure to compose weight functions to determine SIF solutions for surface cracks in complex three-dimensional geometries. This is achieved, in essence, via the composition of two-dimensional geometry solutions with three-dimensional 'base' geometry solutions. At present the analysis is restricted to the deepest point of the crack utilising weight functions of the form employed for the two-dimensional analyses. Preliminary solutions, validated against full three-dimensional FE solutions, indicate that the fundamental concept is applicable and may be used to generate new solutions for notched plates and pipes. Future work aims to implement the composition of more representative weight functions for surface cracks.

5 SUMMARY AND CONCLUSIONS

Modern computers allow advanced fracture mechanics analysis using Finite Element and Boundary Element methods. There are few situations today that can not be numerically analysed within a reasonable amount of time. These however should be seen in a research context and are beyond the capabilities of the non specialist engineer. Their use by non specialists can in fact be dangerous. FEA and BE methods can however be used to generate reference solutions that if presented in an appropriate manner can be used by maintenance engineers to safely and reliably generate complex solutions for rapid defect assessment of components in service.

The paper demonstrates the power of a composition approach where a SIF solution can be generated for an infinite number of plate thicknesses from a combination of known solutions and one newly generated specific reference solution.

Finally, an introduction of current work at UCL is given into the approach to asymmetric and three-dimensional geometries.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support of the UK EPSRC and MoD in carrying out this work.

REFERENCES

[1] Brennan, F. P., and Teh, L.S., Determination of Crack Tip Stress Intensity Factors in Complex Geometries by the Composition of Constituent Weight Function Solutions, *Fatigue Fract Engng Mater Struct* **27**, 1–7, 2004.

[2] Teh, L. S., Library of Geometric influences for Stress Intensity Factor Weight Functions, Ph.D. Thesis, University of London, 2002.

[3] Ngiam, S.S. and Brennan, F. P. Experimental Investigation of Crack Shape Evolution in Cold Rolled Components, *International Conference on Fatigue Crack Paths, Parma*, 2003.

[4] Niu, X. and Glinka, G., The Weld Profile Effect on Stress Intensity Factors in Weldments. *International Journal of Fracture* **35**, 3-20, 1987.

[5] Niu, X., Glinka, G., Stress Intensity Factors for Semi-Elliptical Surface Cracks in Welded Joints. *International Journal of Fracture* **40**, 255-270, 1989.

[6] Mattheck, C., Munz, D. and Stamm, H., Stress Intensity Factor for Semi-elliptical Surface Cracks Loaded by Stress Gradients. *Engineering Fracture Mechanics* **18**, No. 3, 633-641, 1983.

[7] Impellizzeri, L. F. and Rich, D. L., Spectrum Fatigue Crack Growth in Lugs. *ASTM STP 595,* 320-336, 1976.

[8] Brennan, F. P. and Dover, W. D., Stress Intensity Factors for Threaded Connections. *Engineering Fracture Mechanics* **50,** No. 4, 545-567, 1995.

[9] Brennan, F.P., Dover, W.D., Karé, R.F. and Hellier, A. K., Parametric equations for T-butt weld toe stress intensity factors, *International Journal of Fatigue*, **21**, 1051-1062, 1999.

[10] Brennan, F. P. Evaluation of stress intensity factors by multiple reference state weight function approach. *Theoretical and Applied Fracture Mechanics* **20**, 249-256, 1994.