# ON A BIMODAL REPRESENTATION OF SMALL FATIGUE CRACK SCATTER

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#### ABSTRACT

Fatigue cracks growing from micro-notches are used to obtain relationships between the standard deviation of crack growth rates, the crack length and the number of grains intersected by a crack front. The growth of micro-cracks on smooth surfaces is also considered. It is suggested that the crack population may be represented by bi-modal distributions consisting of propagating cracks and non-propagating cracks. An experimental procedure for separating the two distributions is proposed.

#### **1 INTRODUCTION**

The basic features and the operative mechanisms of small fatigue crack growth are well known, and have been thoroughly summarized by Schijve [1]. One of the features, significant data scatter, has been the subject of a number of investigations [2,3,4,5]. In these studies the changes in the statistical crack size distributions of with increasing load cycles have been determined.

One objective of the present paper is to examine the role of the number of grains intersected by a small crack front in scatter. An analytical procedure that provides a relationship between the standard deviation of crack growth rate and the number of grains intersected by a crack front is presented.

A second objective is to review methods that have been used for describing the statistical distributions of small crack lengths as functions of the number of loading cycles. In these representations the early growth stage includes both cracks that ultimately are described as propagating and those that are described as non-propagating. The consequences of the form of presentation are discussed.

#### 2 AN ANALYSIS OF SMALL CORNER CRACKS

The data analyzed were obtained from fatigue experiments on small corner cracks in 6061-T651 [6]. The average transverse grain size was 200 microns, and initial notches were 150 microns deep. A cubic regression fit was made for the crack length, a, versus loading cycle, N, for each test specimen. The crack growth rate versus loading cycle was obtained by differentiating the cubic regression equations. A plot of the results is presented in Figure 1.



Figure 1: Crack Growth Rate Results

A plot of the standard deviation of the crack growth rates versus the crack length is presented in Figure 2.



Figure 2: S.D of Crack Growth Rates and Number of Grains Intersected Vs. Crack Length

The corner crack front was assumed to be a quarter arc of a circle. Metallographic examinations of fractured surfaces confirmed this to be a valid assumption. The number of grains intersected by

the crack front is then given by the equation  $n = 0.5\pi$  (a/d), where a is the crack depth and d is the grain diameter. A second plot shows the number of grains, n, intersected by the crack fronts versus the crack length.

The trend of the data indicates that, initially, the rate of decrease in the standard deviation increases with increasing crack length. Ultimately, however, the rates of decrease begin to decrease with increasing crack length. This behavior is reasonable because it would be expected that the standard deviation should tend to approach an asymptotic value as the 'long' crack regime is approached.

The features of the standard deviation in Figure 2 indicate that it may be possible to represent the observed behavior by an exponential function of the form

$$S=C \exp(D\phi(\alpha)), \tag{1}$$

in which s is the standard deviation, a is the crack length, and C and D are constants. The constants can be determined by the application of a nonlinear regression analysis [7]. Natural logarithms are applied to eqn (1). Then,

$$\ln S = \ln C + D\phi(a). \tag{2}$$

Let  $y = \ln S$ ,  $A = \ln C$ , and  $\phi(a) = x$ . Then

$$Y = A + Dx.$$
(3)

The values of A and B have been determined by the use of a least squares analysis, and the resulting equation for S becomes

$$S = 0.81 \exp[-2.29 \times 10^{-6} (a - 800)].$$
(4)

Values of eqn (4) are plotted in Figure 2, and they indicate that the equation provides a good fit.

The linear relation between a and n indicates that S can also be represented as a function of n. The resulting substitution has been used to obtain the plot in Figure 3.



Figure 3: Relation Between Growth S.D. and Number of Intersected Grains

This version may provide a basis for anticipating differences in the evolution of scatter for different crack surface shapes. Thus, for the same crack depth, the crack front of a semi-circular crack intersects twice as many grains as a corner crack. The scatter for the semi-circular may, therefore, be expected to diminish more rapidly than that of a corner crack.

# **3 MULTI-SITE CRACKING**

Much of the statistical analyses of scatter have been concerned with the initiation of micro, multisite cracking on smooth surfaces. In experiments conducted to obtain scatter data, the lengths of cracks within clusters of cracks have been measured as soon as they can be detected. Because the cracks are of the order of microns, replication techniques have often been used. Crack length measurements have been made by periodically interrupting cyclic loading to obtain crack length distributions.

As the number of loading cycles increases, the growths of some cracks are arrested, and cease to grow. These have been described as non-propagating cracks. Cracks that continue to grow, sometimes through coalescence, are described as propagating cracks. These dominant or primary cracks can, if loading continues, lead to a failure by fracture. Swain [2] has proposed a criterion for evaluating crack interaction effects for tests in which multiple cracks evolve as cyclic loading increases.

As long as a dominant crack is within the region of influence of non-propagating cracks, it can be expected to be subject to the shielding effects of the networks of non-propagating cracks. These networks, along with grain boundaries, form the neighborhood from which a propagating crack emerges.

If separate small crack distributions are identified as propagating cracks and as non-propagating cracks, it follows that the total population has a bi-modal distribution.

During the initial stages of loading, an identification of the two separate distributions is difficult. Thus, the procedure of using early crack length measurement interruptions is not an effective method for separating the two distributions.

A tentative test procedure for accomplishing this separation is being developed for use with the current testing being performed on aluminum alloy 7075-T7351. Instead of starting measurements early in fatigue tests, loading will be continued until a dominant or primary crack of the order of ten times the grain size is clearly developed. Crack measurements on additional specimens for the same number of cycles will then be made to determine the distribution of the crack sizes. Subsequent tests will then to be conducted at successively decreasing values of loading cycle. The lengths of the dominant, propagating cracks will then be measured for each of the subsequent loading cycle values. The objective of the above testing procedure is to obtain crack distribution data that can be used as a basis for extrapolating back to the regime in which the crack sizes are of the order of the grain size.

## **4 FUTURE WORK**

The test procedure described above will be used to conduct multiple specimen tests to provide data for statistical analyses. To obtain statistical bounds on fatigue lives, the possibility of applying the recent proposals of Halliday, et al [8] on the use of Newman's effective stress intensity factor [9] will be investigated.

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