FRACTURE ROUGHNESS FOR THE TWO-DIMENSIONAL CENTRAL FORCE MODEL

Jan Ø. H. Bakke[1] Thomas Ramstad[1] Alex Hansen[1]
[1]Department of Physics, NTNU, Trondheim Norway

ABSTRACT

We study the fracture roughness for a two-dimensional central force model by numerical simulations to check the conjecture that the fracture roughness is due to the fracture process being a stress-weighted percolation process. The simulations are done on a triangular lattice and the fracture process is quasi-static. The simulations are done in mode I and the elastic equilibrium equations are solved with the iterative conjugated gradient method. The material has non-uniform material properties through disorder in the breaking thresholds for the bonds on the lattice. We find that for small disorders in the breaking thresholds the fracture is localised and the fracture roughness is found to be $\zeta = 0.70 \pm 0.05$. We also measure the percolation correlation length exponent for large disorders and find $\nu = 1.43 \pm 0.10$, which suggests that the value of ζ is in the upper part of the interval. The result for ν also excludes generic rigidity percolation as the universality class of the stress weighted percolation process in this model. In the infinite disorder limit the gradient imposed on the damage distribution by the fracture process makes this model different from ordinary rigid percolation.

1 INTRODUCTION

The studies of the fractal properties of fracture surfaces started in the mid-eighties when Mandelbrot et al studied fracture surfaces in steel. Mandelbrot[1] In the early- and mid-nineties the evidence of an universal self affine scaling was established. Parallel to the experimental studies researchers started to study these scaling relations both numerically and theoretically.

Self affinity can be described as a anisotropic scaling of a quantity. If we scale the system in one direction with one scaling factor we must scale with another scaling factor in another direction. For fracture surfaces this self affine scaling manifests itself through the probability of the surface passing at height y at position x when it was at height 0 when $x = 0$.

$$\lambda^{\zeta}\pi(\lambda x, \lambda^{\zeta} y) = \pi(x, y), \tag{1}$$

where ζ is the self affine scaling exponent or fracture roughness. $\zeta = 0.5$ implies a non-correlated surfaces, $\zeta > 0.5$ implies persistence and $\zeta < 0.5$ implies anti-persistence.

The universality of the self affine scaling exponent implies that it is material independent. This universality suggests that the the fracture roughness is due to the fracture process and not the material properties of fractured material. To determine which physical process that governs the shaping of the fracture surface and to be able to model this process is an important question in the studies of the morphology of fracture surfaces.

For three-dimensional materials a fracture roughness $\zeta = 0.8$ has been measured for large length scales, μm to mm. Daugier[2] For smaller length scales, $< \mu$m, a fracture roughness $\zeta = 0.5$ has been observed. Daugier[2]. For two-dimensional materials $\zeta = 0.7$ has been measured for large length scales. Engøy[3].

We can measure this fracture roughness by considering the scaling of the

average fracture width. The average fracture width is defined as

$$w = (< y >^2 - < y >^2)^{1/2} \propto L^{\zeta}, \tag{2}$$

where y is the direction perpendicular to the fracture and the averages are taken over the length of the samples parallel to the fracture
A conjecture made by Hansen and Schmittbuhl connects the fracture roughness to the critical exponent for the correlation length of a stress weighted percolation process, ν. Schmittbuhl[4]

$$\zeta = \frac{2\nu}{1+2\nu} \tag{3}$$

Using the equation above we can calculate an expected value for ζ if ν is known and vice versa.

We can measure ν when the disorder is so large that the correlation length is larger then the lattice size. Finite size scaling then gives the following expression for the fluctuations in the density of broken bonds

$$\sigma(p_{eff}) = (< p_{eff}^2 > - < (p_{eff} >^2)^{1/2} \propto L^{-1/\nu} \tag{4}$$

We model the elastic forces in our simulations with the central force model. The expression for the force on a bond connecting the nodes i and j in the lattice is shown in eqn (5)

$$f_{ij} = \sigma_{ij}\{(\mathbf{r}_j - \mathbf{r}_i) \cdot \mathbf{n}_{ij}\} \cdot \mathbf{n}_{ij}. \tag{5}$$

Where σ_{ij} is the spring constant for the bond, r_k is the displacement of the kth node, and $\mathbf{n}_{ij}$ is the unit vector parallel with the bond.

The central force model only transmits forces parallel to the axis of a bond and does not transmit angular forces. Since this model is vectorial a first guess for the percolation process is rigidity percolation. Using the value for ν found by Moukarzel and Duxbury Duxbury[5], $\nu = 1.16$ in eqn (3) gives us $\zeta = 0.70$.

2 NUMERICAL SIMULATIONS

The numerical simulations were done on a triangular lattice with rigid crossbars at the upper and lower boundary and dimension $L \times L$. The network was periodic perpendicular to the crossbars. The spring constants were all set to unity and then a random noise was added to the spring constants to ensure generic rigidity. To model the heterogeneity of the material each bond in the model was also assigned a different threshold value drawn from an uncorrelated distribution, $t_i = r_i^D$, where D is a disorder parameter describing the disorder in the system. The limit $D = 0$ equals no disorder, and the limit $D = \infty$ equals infinite disorder.

The fracture process was considered quasi-static which implies that we let the time scale of force relaxation after a bond has been broken be much smaller than the time between succeeding bond failures. The algorithm for the fracture process is as follows:

- Solve the equilibrium equations for the lattice
- Find the largest ration f_i/t_i

- Remove the bond with the largest ratio
- Repeat until elasticity module is zero

The equilibrium equations are solved iteratively by the conjugated gradient method. When the sample is broken we identify the lower fracture surface and calculates the average width of this surface.

3 RESULTS

The results for the fracture roughness are for uniaxial tension and a threshold disorder of 0.7. We used 10000 realisations for the 10×10 lattices and 90 realisations for the 180×180 lattices.

With a finite disorder we found that the average fracture width scaled well with a fracture roughness equal around 0.7, see figure 1. The localisation of the fracture is seen in figure 2.

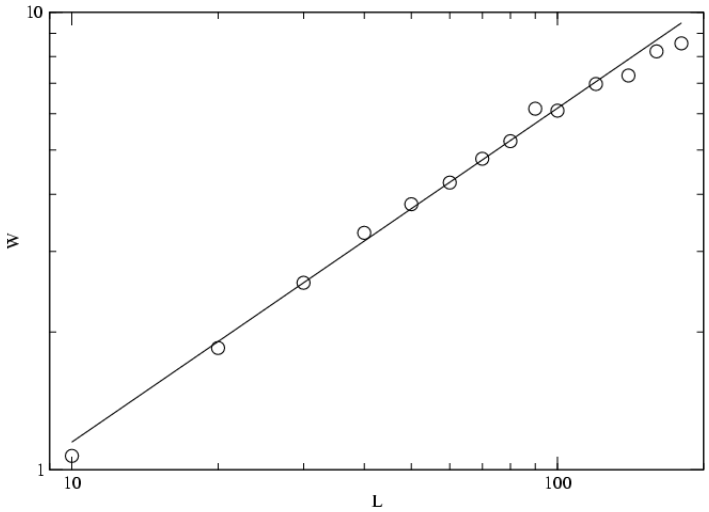

Figure 1: Scaling of the average fracture width for lattices up to 180×180. the solid line is for $\zeta = 0.74$.

A measurement of ν showed good agreement with the values around $\nu = 1.4$ for a disorder of $D = 20$, see figure 3. At this disorder there is no localisation of the fracture. These measurement had 30000 realisations for the 10×10 lattices and 400 for the 70×70 lattices.

4 CONCLUSION

For a finite disorder the fracture is localised and we have measured $\zeta = 0.70 \pm 0.05$. This result is close to earlier experimental values for ζ, and also close to the predicted value by eqn (3). Our result for ν suggests however that the correlation length exponent associated with this fracture process is close to 1.4 which gives a fracture roughness $\zeta = 0.74$. Neither $\zeta = 0.70$ or $\zeta = 0.74$ is

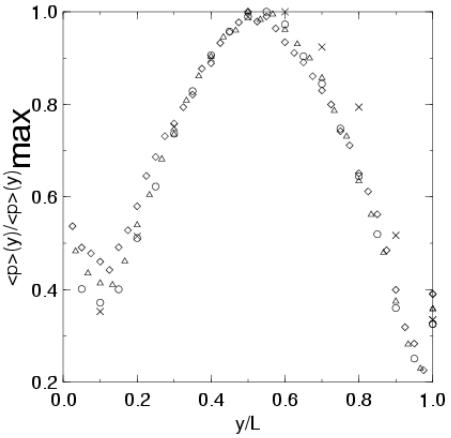

Figure 2: Normalised damage profile perpendicular to the fracture for $D = 0.7$ and lattice sizes $L = 10(\times), 20(\circ), 30(\triangle)$ and $40(\diamond)$. The profiles are averages where each profile is shifted such that they are centred around the mean value. The localisation is clearly seen.

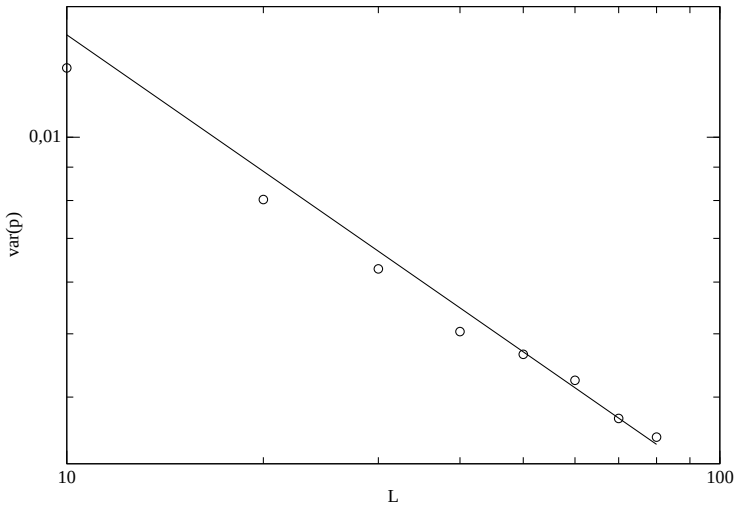

Figure 3: Measurement of ν for the disorder $D = 20$ The fluctuations in the density of broken bonds scale as $L^{-1/\nu}$. The solid line is for $\nu = 1.43$.

excluded by our data for ζ so more high quality measurements are needed to be able to separate between the two proposed values for ζ.

REFERENCES

[1] Mandelbrot B. , Passoja D. and Paully A., Fractal characters of fracture surfaces of metals, *Nature*, **308**,721, 1984

[2] Daugier P., Nghiem B. , Bouchaud E. and Creuzet F., Pinning and depinning of crack fronts in heterogeneous materials, *Phys. Rev. Lett.* **78**,1062, 1997

[3] Engøy T. and Måløy K., Roughness of two-dimensional cracks in wood, *Phys. Rev. Lett.* **73**, 834, 1994

[4] Schmittbuhl J. and Hansen A., Origin of the universal roughness exponent of brittle fracture surfaces: Stress-weighted percolation in the damage zone, *Phys. Rev. Lett.* **90**, 045505, 2003

[5] Moukarzel C. and Duxbury P., Comparison if rigidity and connectivity percolation in two dimensions, *Phys. Rev. E.* **59**, 2614, 1999