COMPARISON BETWEEN DYNAMIC CRACK EQUATION AND SOME OF THE PUBLISHED EXPERIMENTS

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ABSTRACT

Recent progress on dynamic fracture of brittle materials has been remarkable and the validity of the continuum theory of dynamic brittle fracture has been confirmed experimentally (Sharon and Fineberg, Nature, Vol. 397 (1999) 333). We have then examined and discussed whether some of the published experiments are consistent with the dynamic crack equation.

1 INTRODUCTION

Much work has been done on dynamic crack propagation in brittle materials and numerous problems have been discussed. In 1980's, for example, the uniqueness of the stress intensity factor - crack velocity relationship has been discussed [1]. The idea behind this subject seems to be the following. When the stress intensity factor, K_I, is equal to the fracture toughness of the material, i.e., the condition, $K_I = K_{IC}$, is satisfied, the static crack starts to propagate, which is unique independent of such experimental conditions asCT, SEN and other tests. In the experiments of dynamic crack propagation, one of the important parameters measured is the stress intensity factor or the dynamic fracture toughness, of which measuring techniques were also reviewed [2]. The dynamic crack equation, which corresponds to the condition, $K_I = K_{IC}$, in static case, is given as

$$(1 - v/c_R)G(l) = G\ , \tag{1}$$

where G is the fracture energy which is the energy needed to create a crack of unit length. The quantity, l, is the instantaneous crack length, and v is the crack velocity and c_R, the Rayleigh wave speed. $G(l)$ is the amount of energy per unit area present at the tip of a static crack of length l and contains all of the effects of the applied stresses and specimen geometry. We emphasize here that eqn (1) is associated with the singular stress field at the tip of the dynamically propagating crack. Eqn (1) is the function of the velocity, and the typical parameters measured in the experiment of dynamic crack propagation are "the fracture toughness", G and the crack velocity, v. Thus it seems to be natural to expect the relationship, $G - v$ could be obtained in any of the experiments associated with dynamic crack propagation although whether this relationship is unique independent of the experimental conditions or not is a different problem, and should be examined carefully. Making use of the dynamic crack equation (1), theoretical discussion on this subject has been given [3], in which the series of the extensive work on dynamic crack propagation have also been discussed [4]. The subject discussed above is still of interest [5] and the detailed experiment has been performed [6].

Recent progress on dynamic fracture has been remarkable and validity of the continuum theory of dynamic brittle fracture has been confirmed experimentally [7]. We now know that the continuum theory, which means eqn (1), derived by Eshelby, Kostrov and Freund [8], can be used to analyze the experiment of the dynamic crack propagation even with microbranching instability [9]. In this paper we have examined whether the experiment [6] is consistent with eqn (1) or not. Since the dynamic stress intensity factor, K_d, the crack velocity, v, and the crack acceleration, a, are measured as functions of crack length, l, we can directly obtain the relationship between the crack velocity, v, and the crack acceleration, a. We can also obtain this relationship assuming eqn (1) can be used to analyze the data, which are shown in the following section. Comparing the relationship between a and v obtained by two different methods we determine the consistency between the experiment [6] and eqn (1).

2 COMPARISON BETWEEN THE DYNAMIC CRACK EQN (1) AND THE EXPERIMENT [6]

Materials used in the experiment [6] are polymers, PMMA and Epoxy. We first analyze the data for PMMA, which is given in Fig. 6 of [6], from which we find the relationship between v and a, which is shown in Fig. 1. We have calculated "the normalized fracture toughness", $\tilde{G}_D$, from the following formulae,

$$\tilde{G}_D = G(v, a) / G_0, \tag{2}$$

where $G(v, a) = K_d^2 / E$ for the case of plane stress. The quantity, E, is the Young's modulus. The energy release rate at the initiation of the crack propagation is obtained as, $G_0 = 542.7$ J/m². The crack velocity, v_0 at the condition $a = 0$, is found to be $v_0 = 294.3$ m/s, which can be seen from Fig. 1. The Rayleigh wave velocity for PMMA is found to be $c_R = 1309$ m/s. Making use of these quantities, we define the normalized crack velocity, $\tilde{v}_1$, and the normalized crack acceleration, $\tilde{a}$, as

$$\tilde{v}_1 = (v - v_0) / c_R, \tag{3}$$

$$\tilde{a} = aDl / c_R^2, \tag{4}$$

where Dl $(= 6.6 \times 10^{-4}$ m) is the mesh length which we used to extract various quantities from the data. We then make the table for the normalized quantities, $\tilde{G}_D$, $\tilde{v}_1$ and $\tilde{a}$ from Fig. 6 of [6]. Making use of this table we show the graphs of $\tilde{G}_D$ versus $\tilde{v}_1$, and $\tilde{G}_D$ versus $\tilde{a}$, which are

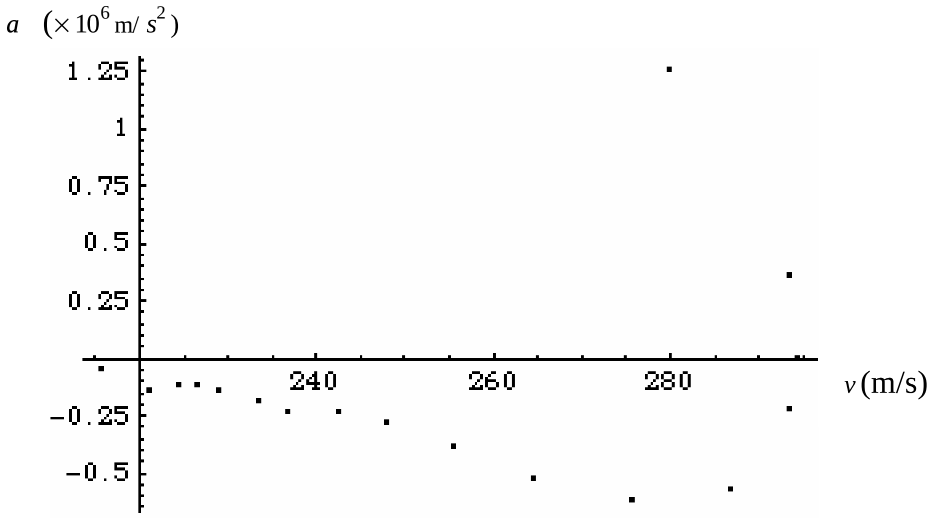

Fig. 1 The crack acceleration versus the velocity of the crack for PMMA

shown in Fig. 2 and Fig. 3, respectively. Since the quantity, $\tilde{G}_D$ is the functions of $\tilde{v}_1$ and $\tilde{a}$ we expand $\tilde{G}_D$ in the following,

$$\tilde{G}_D = 3.412 + a_1 \tilde{v}_1 + a_2 \tilde{v}_1^{\,2} + \tilde{a}\,(b_0 + b_1 \tilde{v}_1) + b_2 \tilde{a}^{\,2}, \qquad (5)$$

where the number 3.412 in right hand side of eqn (5) is the value of $\tilde{G}_D$ at the condition $a = 0$. We have five unknown constants, a_1, a_2, b_0, b_1 and b_2, which are obtained by substituting the values of $\tilde{G}_D$, $\tilde{v}_1$, and $\tilde{a}$ from the table described above. Since the graph of $\tilde{G}_D$ versus $\tilde{v}_1$

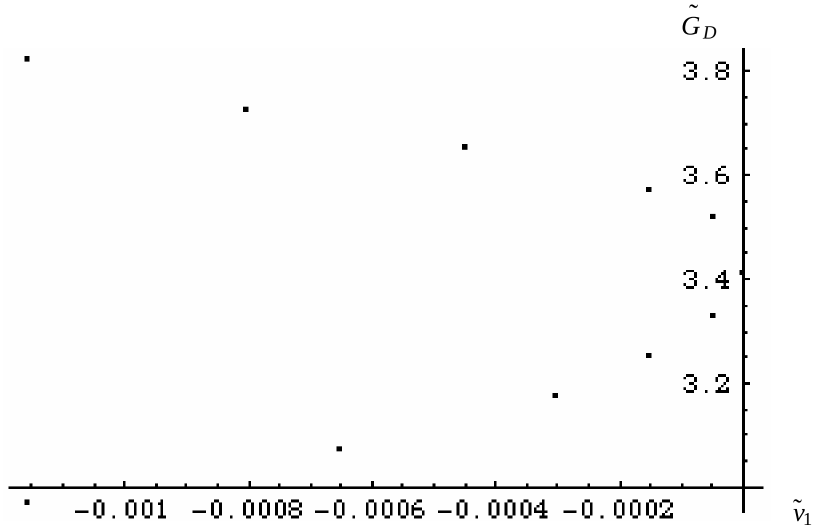

Fig. 2 The normalized fracture toughness versus the normalized velocity of the crack for PMMA

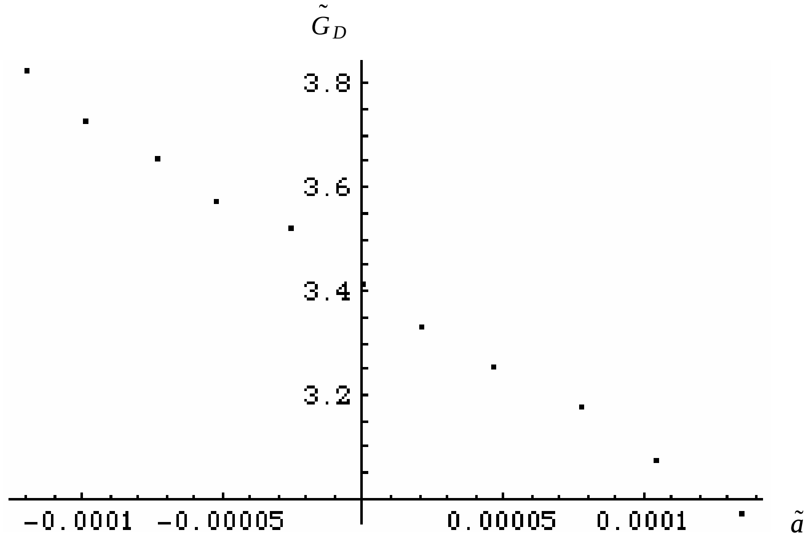

Fig. 3 The normalized fracture toughness versus the normalized crack acceleration for PMMA

in Fig. 2 is not a single valued function, we have obtained the unknown constants for two cases, $\tilde{G}_D \geq 3.412$ and $\tilde{G}_D < 3.412$. We now divide eqn (1) by the quantity, G_0, and obtain the normalized equation of the crack as follows

$$(1-\tilde{v}_1 - v_0/c_R)G(l)/G_0 = \tilde{G}_D. \tag{6}$$

Substituting the quantity, $\tilde{G}_D$ in eqn (5) into eqn(6), we have the following equation to obtain the relationship between $\tilde{v}_1$, and $\tilde{a}$,

$$b_2\tilde{a}^2 + \tilde{a}(b_0 + b_1\tilde{v}_1) + \tilde{v}_1(a_1 + G(l)_{a=0}/G_0 + a_2\tilde{v}_1) = 0, \tag{7}$$

where $G(l)_{a=0}/G_0 = 4.402$, is substituted into eqn (7), which is obtained by the following equation ,

$$(1-v_0/c_R)G(l)_{a=0}/G_0 = 3.412. \tag{8}$$

Solving eqn (7) by the formula for the quadratic equation, we find

$$\tilde{a} = \{-(b_0 + b_1\tilde{v}_1) - \sqrt{(b_0 + b_1\tilde{v}_1)^2 - 4b_2\tilde{v}_1(a_1 + G(l)_{a=0}/G_0 + a_2\tilde{v}_1)}\}/(2b_2). \tag{9}$$

Substituting the values of the constants,, a_1, a_2, b_0, b_1 and b_2, obtained from eqn (5) we have obtained the graph of $\tilde{a}$ versus $\tilde{v}_1$, which corresponds to Fig. 1. We find, however, the acceleration, $\tilde{a}$ does not take negative values for the case, $\tilde{G}_D \geq 3.412$. We then find the experimental result for PMMA [6] is not consistent with the dynamic crack equation (1).

Taking the similar procedure we have examined the data for Epoxy from Fig. 7 in [6] and found the following, $v_0 = 351$ m/s, $Dl = 5.26\times 10^{-4}$ m, $c_R = 1097$ m/s, $G_0 = 111.2$ J/m^2, and

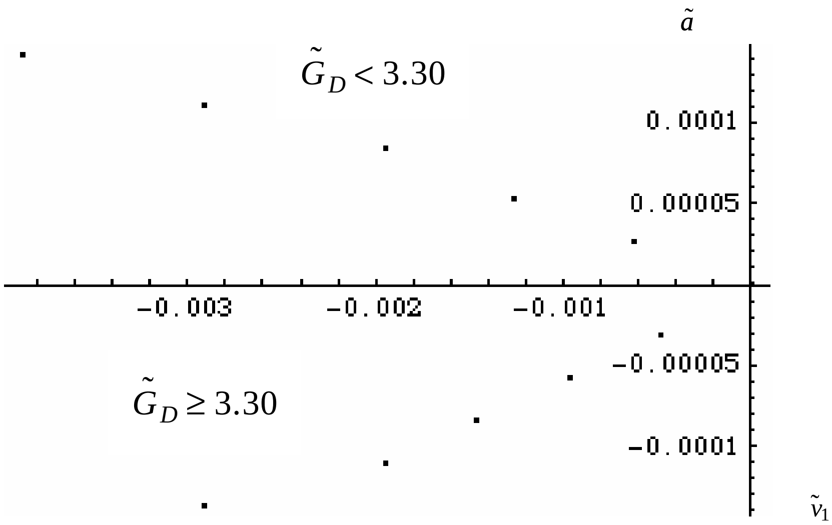

Fig. 4 The graph of $\tilde{a}$ versus $\tilde{v}_1$,for the case of Epoxy in the experiment [6]

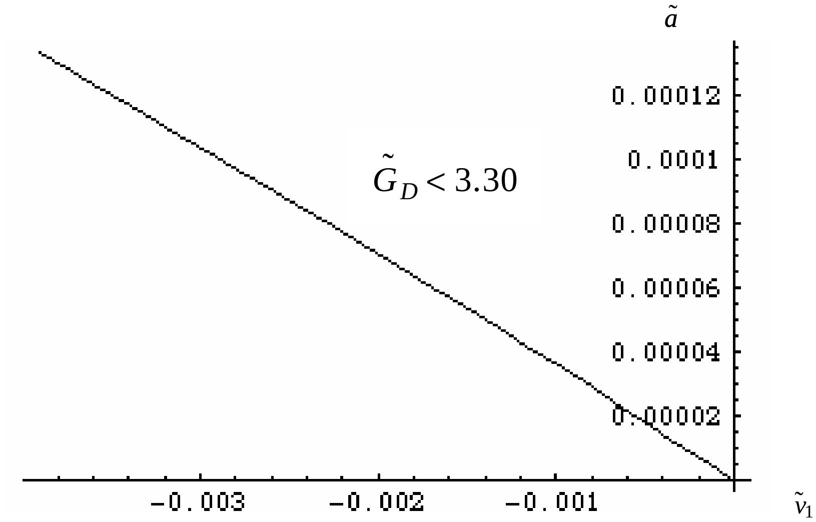

Fig. 5 The graph of $\tilde{a}$ versus $\tilde{v}_1$ for the case of Epoxy [6] , which is obtained from eqn (9).

$G(l)_{a=0}/G_0 = 4.853$. Fig. 4 is the graph of $\tilde{a}$ versus $\tilde{v}_1$, directly obtained from the data. We note that the normalized crack velocity, $\tilde{v}_1$, always takes the negative value since we expand the quantity $\tilde{G}_D$ at $v = v_0$, $a = 0$. Fig. 5 is the graph of $\tilde{a}$ versus $\tilde{v}_1$, which is obtained by eqn (9). We find no real solution for $\tilde{a}$ for the case, $\tilde{G}_D \geq 3.30$, which corresponds to the region of $\tilde{a} < 0$ in Fig. 4. We again conclude that the experiment for Epoxy[6] is not consistent with eqn (1).

3 DISCUSSION

It is rare to find such experiment [6], which one could find the detailed behavior of the experimental parameters. In this experiment, however, it is logically irrelevant to measure both of the crack acceleration and the stress intensity factor or the fracture toughness, K_d, because the parameters associated with the crack acceleration can only be obtained from the higher orders stress fields [10], while the stress intensity factor is the parameter associated with the singular stress field at the tip of the crack. Thus the data itself is relevant in the sense of the approximation. Beside we should point out that the boundary condition of the experiment [6] and eqn (1) is different, i.e., the crack in an infinitely large elastic plate is assumed in deriving eqn (1) [8, 11]. Thus the sound waves reflected from the nearest boundary could interact a few times with the moving crack during the experiment. The effect of the sound wave on the dynamic fracture toughness, K_d, is clearly demonstrated in the experiment of the crack arrest [2]. However, we do not know the detailed effect of the sound wave on the moving crack. Strictly speaking one must derive the dynamic crack equation which corresponds to eqn (1), taking account of the boundary condition of this experiment and then compare it with the experimental result, which is very hard in practice. Thus comparison between eqn (1) and the experiment is only approximation at best, and we simply find that such comparison does not have any meaning. In the conference we will also discuss the consistency of the experiments [4] and [7] with the dynamic crack equation (1), which is the main objective of this paper.

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