# LOCAL CRITERIA OF BRITTLE AND DUCTILE FRACTURE AND APPLICATION OF LOCAL APPROACH TO FRACTURE MECHANICS PROBLEMS

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#### ABSTRACT

New local criteria of brittle and ductile fracture are considered. Models based on the proposed local criteria for prediction of critical fracture mechanics parameters are presented. These models are as follows: (i) a probabilistic model for prediction of the temperature dependence of brittle fracture toughness, named now the Prometey model; (ii) a model for prediction of  $J_R$ -curves; (iii) a model for prediction of brittle fracture toughness after ductile crack growth. A new engineering method named the Unified Curve concept is proposed for the K<sub>IC</sub>(T) curve prediction for reactor pressure vessel (RPV) steels with various degrees of embrittlement, up to extremely high levels.

## 1 INTRODUCTION

Application of local approach for fracture toughness predictions is known to be very important for irradiated RPV materials for which full-sized fracture toughness specimens cannot be tested. This explains the reason as to why the local approach models are now intensively developed. In the present paper, local fracture criteria and model for prediction of critical fracture mechanics parameters which were proposed in works of Margolin with co-authors [1-6] are briefly considered.

### **2** THE LOCAL CRITERION FOR BRITTLE FRACTURE

The formulation of the local cleavage fracture criterion in a probabilistic manner includes the following steps (Margolin [1]).

1. The polycrystalline material is viewed as an aggregate of cubic unit cells. The mechanical properties for each unit cell are taken as the average properties obtained by standard specimen testing. The size of the unit cell  $\rho_{uc}$  is never less than the average grain size. The stress and strain fields in the unit cell are assumed to be homogeneous.

2. The local criterion for cleavage fracture of a unit cell is taken as

$$\sigma_1 + m_{\mathrm{T}\varepsilon} \sigma_{\mathrm{eff}} \ge \sigma_{\mathrm{d}} \tag{1a}$$

$$\sigma_1 \ge S_C(\boldsymbol{x}) \tag{1b}$$

where the critical brittle fracture stress,  $S_C(\alpha)$ , is calculated by

$$S_{C}(\alpha) = \left[C_{1}^{*} + C_{2}^{*} \exp(-A_{d}\alpha)\right]^{-1/2}$$
(2)

Here,  $\sigma_1$  is the maximum principal stress, the effective stress is  $\sigma_{eff} = \sigma_{eq} - \sigma_Y$ ,  $\sigma_{eq}$  is the equivalent stress,  $\sigma_Y$  is the yield stress,  $\mathfrak{a} = \int d\varepsilon_{eq}^p$  is Odqvist's parameter,  $d\varepsilon_{eq}^p$  is the equivalent plastic strain increment,  $C_1^*, C_2^*$ ,  $A_d$  are material constants,  $\sigma_d$  is the strength of carbides or "carbide-matrix"

interfaces or other particles on which cleavage microcracks are nucleated,  $m_{T\epsilon}$  is a parameter that depends on temperature T and plastic strain and may be written as  $m_{T\epsilon}=m_T(T)m_{\epsilon}(\boldsymbol{x})$  with  $m_{\epsilon}(\boldsymbol{x}) = S_0/S_C(\boldsymbol{x})$  and  $m_T(T) = m_0\sigma_{Y_S}(T)$ , where  $S_0=S_C(\boldsymbol{x}=0)$ ,  $m_0$  is a constant which may be experimentally determined and  $\sigma_{Y_S}$  is the temperature-dependent component of the yield stress.

Condition (1a) is the nucleation condition for cleavage microcracks. Condition (1b) is the propagation condition for cleavage microcracks.

3. To formulate criteria (1) in a probabilistic way, it is assumed that the parameter  $\sigma_d$  is stochastic and the remainder of the parameters controlling brittle fracture are deterministic. Such an assumption is based on analysis of the stochastic nature of various critical parameters controlling cleavage fracture of RPV steels.

4. To describe the distribution function for the parameter  $\sigma_d$ , the Weibull law is used

$$p(\sigma_d) = 1 - \exp\left[-\left(\frac{\sigma_d - \sigma_{d0}}{\widetilde{\sigma}_d}\right)^{\eta}\right]$$
(3)

where  $p(\sigma_d)$  is the probability of finding in each unit cell a carbide with minimum strength less than  $\sigma_d$ ;  $\tilde{\sigma}_d$ ,  $\sigma_{d0}$  and  $\eta$  are Weibull parameters.

5. The weakest link model is used to describe the brittle fracture of the polycrystalline material.

6. It is considered that brittle fracture may happen only in unit cells for which the conditions  $\sigma_{eq} \ge \sigma_Y$  and  $\sigma_1 \ge S_C(a)$  are satisfied.

## **3** THE LOCAL CRITERION FOR DUCTILE FRACTURE

The proposed local criterion (Margolin [2]) of ductile fracture caused by the evolution of voids is based on the idea of plastic collapse for a unit cell that is a regular structural mezovolume of polycrystalline material. The void evolution is described by equations for void nucleation and growth that occur simultaneously. The criterion does not require the introduction of any empirical parameters, such as critical void size, critical size of ligament between voids and critical void volume fraction, which are used in most models. The first consideration of this criterion is the same as formulated in Section 2. The other considerations are as follows.

1. As a local criterion for ductile fracture the criterion of plastic collapse of a unit cell with size of  $\rho_{uc}$  is used

$$\frac{\mathrm{dF}_{\mathrm{eq}}}{\mathrm{d}\boldsymbol{\varpi}} = 0 \tag{4}$$

where  $F_{eq}=\sigma_{eq}(1-S_{\Sigma})$ ,  $S_{\Sigma}$  is the relative area of voids, i.e. area of voids per unit area of the unit cell section. In other words,  $F_{eq}$  is the stress in a conglomerate of matrix and voids, and  $\sigma_{eq}$  is the stress in the matrix of a material. The value of the relative area of voids,  $S_{\Sigma}$ , is calculated by equations for void nucleation and growth according to the procedure presented in Margolin [2].

As known the ductile fracture criterion is usually used in the form

$$\mathfrak{E} = \varepsilon_{\mathrm{f}}$$

(5)

where  $\varepsilon_f$  is the critical strain. According to eqn (4) the value of  $\varepsilon_f$  is calculated as

$$\varepsilon_{f} = \boldsymbol{\mathfrak{E}}_{\left|\frac{\mathrm{dF}_{\mathrm{eq}}}{\mathrm{d}\boldsymbol{\mathfrak{R}}}\right| = 0} \tag{6}$$

2. Equation for void nucleation in structural materials is taken in the form

$$\rho_{\rm s} = \rho_{\rm f} [1 - \exp(-A_{\rho}(\boldsymbol{\varpi} - \boldsymbol{\varpi}_0))] \tag{7}$$

where  $\rho_s$  is the void concentration, i.e. the number of voids per unit of the undeformed area,  $\rho_f$  is the maximum number of void nucleation sites per unit of the undeformed area,  $\boldsymbol{\varpi}_0$  is the value of  $\boldsymbol{\varpi}$  at which void nucleation begins, i.e. at  $\boldsymbol{\varpi} \leq \boldsymbol{\varpi}_0$  voids are not nucleated, and  $A_{\rho}$  is the temperature-independent constant of a material. For reactor pressure vessel steels, void nucleation begins at small plastic strain therefore it may be taken in eqn (10) that  $\boldsymbol{\varpi}_0=0$ .

3. Growth of a spherical single void caused by plastic deformation for a triaxial stress state is described by equation of Huang [7]

$$\frac{\mathrm{dR}}{\mathrm{R}} = 0.427 \left(\frac{\sigma_{\mathrm{m}}}{\sigma_{\mathrm{eq}}}\right)^{\mathrm{k}} \cdot \exp\left(\frac{3}{2} \frac{\sigma_{\mathrm{m}}}{\sigma_{\mathrm{eq}}}\right) \cdot \mathrm{d}\boldsymbol{\boldsymbol{\varpi}}$$
(8)

with k=0.25 at  $\sigma_m/\sigma_{eq} \le 1$  and k=0 at  $\sigma_m/\sigma_{eq} \ge 1$ , R is the void radius,  $\sigma_m/\sigma_{eq}$  is the stress triaxiality,  $\sigma_m$  is the hydrostatic stress.

To predict the ductile fracture it is necessary to determine the parameters  $A_{\rho}$  and  $\rho_{f}$  in eqn (7) and parameters describing plastic deformation to enable the stress and strain fields to be calculated. These parameters may be determined on the basis of test results from smooth and notched cylindrical specimens.

#### 4 A PROBABILISTIC MODEL FOR THE K<sub>IC</sub>(T) CURVE PREDICTION

A probabilistic model for prediction of the  $K_{IC}(T)$  curve (Margolin [1]) (now known as the Prometey model) is based on the local criterion of brittle fracture described above. The stress and strain fields near the crack tip are calculated by FEM or on the crack extension line with an approximate analytical solution.

The brittle fracture probability of a cracked specimen,  $P_{\rm f}$ , is presented in the form used in Beremin [8]

$$P_{f} = 1 - \exp\left[-\left(\frac{\sigma_{w}}{\widetilde{\sigma}_{d}}\right)^{\eta}\right]$$
(9)

where the Weibull stress  $\sigma_w$  is

$$\sigma_{w} = \left[\sum_{i=1}^{k} \left(\max(S_{nuc}^{i}) - \sigma_{d0}\right)^{\eta}\right]^{1/\eta} \text{ and } S_{nuc}^{i} \equiv \begin{cases} \sigma_{nuc}^{i}, \text{ if } \sigma_{1}^{i} \ge S_{C} \left( \mathbf{a}_{i} \right) & \text{and } \sigma_{nuc}^{i} > \sigma_{d0} \\ \sigma_{d0}, & \text{if } \sigma_{1}^{i} < S_{C} \left( \mathbf{a}_{i} \right) & \text{or } \sigma_{nuc}^{i} \le \sigma_{d0} \end{cases}$$
(10)

Here  $\sigma_{nuc} \equiv \sigma_1 + m_{Te} \sigma_{eff}$ ; k is the number of unit cells in the plastic zone, i is the number of a unit cell. For each unit cell, the parameter max( $S_{nuc}^i$ ) is the maximum value of  $S_{nuc}^i$  from the beginning of deformation up to the current moment. The above equations provide the calculation of the dependence  $P_f(K_I)$  as the parameter  $\sigma_w$  is a function of  $K_I$ .

To predict the  $K_{IC}(T)$  curve on the basis of the Prometey model, it is necessary to know the parameters  $S_C(a)$ ,  $m_T(T)$ ,  $\tilde{\sigma}_d$ ,  $\sigma_{d0}$  and  $\eta$  and also parameters describing plastic deformation to enable the stress and strain fields to be calculated. The parameters  $\tilde{\sigma}_d$  and  $\eta$  may be determined from test results of small-sized fracture toughness specimens at one temperature. The rest of parameters are determined from uniaxial tension tests of standard cylindrical specimens.

The Prometey model was verified by application to RPV steels for WWER in various states (Margolin [1, 4]). The calculated  $K_{IC}(T)$  curves were compared with test results and the predicted curves from the Master Curve approach. It was shown (Margolin [4]) that the  $K_{IC}(T)$  curves for the initial state calculated with the Master Curve approach and the Prometey model show good agreement. At the same time, for highly embrittled RPV steel, the  $K_{IC}(T)$  curve predicted with the Master Curve approach is not an adequate fit to the experimental data, whereas the agreement of the test results and the  $K_{IC}(T)$  curve calculated with the Prometey model is good. As an illustrative example, the  $K_{IC}(T)$  curves for RPV steel predicted with the Prometey model are given hereafter.

## 5 A MODEL FOR PREDICTION OF J<sub>R</sub>-CURVES

Following a model for prediction of  $J_R$ -curves in Margolin [3], the ductile crack growth is

simulated as a successive fracture of unit cells with size  $\rho_{uc}$  located on the crack extension line. Fracture of the unit cell nearest to the growing crack tip happens when the plastic strain of this unit cell reaches the critical value  $\mathfrak{a}=\varepsilon_f$  (eqn (5)). It was shown that the loading histories of the unit cells for stationary crack and for growing crack are different. Therefore, according to the ductile fracture model the critical strain values at the start  $\varepsilon_f^{start}$  and the growth  $\varepsilon_t^{gr}$  of the crack are also different. At the same time the loading history of the unit cells on the growing crack extension line located outside the zone of the blunting stationary crack tip ( $\Delta a > CTOD_C$ ) is practically the same. Therefore the value  $\varepsilon_f^{gr}$  may be taken the same for all unit cells on the crack extension line located outside the zone of the blunting stationary crack tip ( $\Delta a > CTOD_C$ ).

Thus, the crack start and growth conditions may be formulated as

$$\mathbf{\mathfrak{x}}(\mathbf{r})_{|\mathbf{r}=\mathbf{r}_{\mathrm{f}}} = \varepsilon_{\mathrm{f}}^{\mathrm{start}} \tag{11}$$

$$\mathfrak{X}(\mathbf{r})_{|\mathbf{r}=\mathbf{r}_{\mathrm{f}}} = \varepsilon_{\mathrm{f}}^{\mathrm{gr}} \tag{12}$$

where  $r_f$  may be determined from

$$r_{\rm f} = l / \sqrt{\rho_{\rm s}(\epsilon_{\rm f}^{\rm start})} \tag{13}$$

where  $1/\sqrt{\rho_s}$ , the average distance between the voids at the  $\mathfrak{a}=\epsilon_f^{\text{start}}$  is calculated using eqn (7). The size of unit cell  $\rho_{uc}$  is determined from

$$\frac{1}{\rho_{uc}} \int_{0}^{\rho_{uc}} \alpha(r) dr = \varepsilon_{f}^{start}$$
(14)

where  $\mathfrak{A}(r)$  is taken at the load corresponding to eqn (11).

As seen from eqns (11)-(14), the proposed approach allows one to calculate the process zone parameters  $\rho_{uc}$  and  $r_{f}$ . In most models, similar parameters are known to be taken as the adjusted ones. So, application of the proposed model is simpler than the Tvergaard-Needleman-Gurson model (Tvergaard [9]) as requires calibration of two parameters  $A_{\rho}$  and  $\rho_{f}$  only.

As an illustrative example, in Fig. 1, the  $J_R$ -curves predicted with the above model for RPV steel in different conditions are given and compared with test results.



Figure 1: The predicted and the experimental  $J_R$ -curves for RPV steel in the initial (a) and embrittled (b) conditions: SZW – stretched zone width,  $\Delta a$  – ductile tearing.

# 6 A MODEL FOR PREDICTION OF CLEAVAGE FRACTURE TOUGHNESS AFTER DUCTILE CRACK GROWTH

Modelling of cleavage fracture after ductile crack growth according to Margolin [5] is performed on the basis of an approach which unites the Prometey model (Section 4) and the ductile fracture model (Section 3). Practical realization of this approach consists in using a two stage calculation procedure. In the first stage, the ductile crack growth is simulated by FEM in accordance with the procedure presented in Section 5. Using this the stress and strain fields, for a specimen with ductile growing crack, the ductile tearing  $\Delta a$  and the J-integral are calculated. In the second stage, the brittle fracture probability of a cracked specimen is calculated according to the procedure in Section 4.

Predictions of the temperature dependence of cleavage fracture toughness  $K_{JC}^{cl}(T)$  have been performed as applied to a 2Cr-Ni-Mo-V reactor pressure vessel steel in the initial and embrittled states (Fig. 2). On the basis of the performed calculations, the following conclusions may be drawn. The character of the dependence  $K_{JC}^{cl}(T)$  varies for  $K_{JC}^{cl} \ge K_{JC}^{duct}$  ( $K_{JC}^{duct}$  - the upper shelf level), i.e. when the ductile tearing occurs. For material in the initial state, cleavage fracture toughness calculated with regard for ductile crack growth,  $\left[K_{JC}^{cl}(T)\right]^{DCG}$ , exceeds cleavage fracture toughness calculated without regard for ductile crack growth,  $\left[K_{JC}^{cl}(T)\right]^{0}$ . For material in highly embrittled state, fracture toughness  $\left[K_{JC}^{cl}(T)\right]^{DCG}$  is less than fracture toughness  $\left[K_{JC}^{cl}(T)\right]^{0}$ .

# 7 THE UNIFIED CURVE CONCEPT

As it was shown in Margolin [4] an applicability of the Master Curve approach may be restricted for highly embrittled steels as this approach uses the lateral temperature shift to describe the  $K_{IC}(T)$  curves. The Prometey model does not include any assumptions concerning the shape of the  $K_{IC}(T)$  curve and the temperature lateral shift condition and provides a prediction of the  $K_{IC}(T)$ curve allowing for the possibility of both a shift and a variation in shape. At the same time, the Master Curve approach is more suitable for engineering application as simple method as compared with the Prometey model. Therefore a task arose to elaborate an engineering method that allows the prediction of the  $K_{IC}(T)$  curve for RPV steels with various degrees of embrittlement, including extremely high levels. Such an engineering method named the Unified Curve concept was proposed on the basis of the generalized results obtained by the Prometey model (Margolin [6]).

The main considerations of the Unified Curve concept may be summarized as follows.

1. The temperature dependence of fracture toughness at  $P_i=0.5$  for specimen with thickness B=25 mm from RPV steel for any degree of embrittlement may be described by

$$K_{JC(med)} = K_{JC}^{shelf} + \Omega \left[ 1 + \tanh\left(\frac{T - 130}{105}\right) \right], MPa\sqrt{m}$$
(15)

where  $K_{JC}^{shelf} = 26 \text{ MPa}/\text{m}$ ;  $\Omega$  is constant for a given state of a material, T the temperature in °C.

2. It is assumed that for the embrittled materials the one parameter,  $\Omega$ , varies, the rest of the numerical parameters in eqn (15) are fixed.

3. Equations describing the thickness effect on  $K_{JC}$  and the scatter in  $K_{JC}$  results are the same as for the Master Curve concept (Wallin [10]).

The parameter  $\Omega$  is determined on the basis of fracture toughness test results at one or several temperatures. Requirements for the number and size of specimens are the same as for determination of the parameter T<sub>0</sub> in the Master Curve. The Unified Curve concept was verified by using more than 30 sets of experimental data for ferritic steels (Margolin [6]).



Figure 2: The  $\left[K_{JC}^{cl}(T)\right]^0$  curves (solid lines) and  $\left[K_{JC}^{cl}(T)\right]^{DCG}$  curves (dotted lines) for RPV steel in the initial (a) and embrittled (b) conditions.

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