APPLICATION OF DISCRETE STOCHASTIC FRACTURE NETWORKS FOR MODELLING OF GROUNDWATER FLOW

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ABSTRACT

The paper gives overview of present state achieved by our research group in the field of the modelling of the fluid flow in fractured rock environment.

First, we introduce known possible approaches for solving the problem. Our system is based on one of these approaches, so called *the discrete stochastic network* approach. It means that particular fractures are represented as 2D entities (polygons in our case) placed randomly (according to given distribution and frequency) in 3D space. The resulting mesh has the same statistical characteristic (density of fractures, their orientation, permeability etc.) as real fractured environment in the rock massif.

The second part of the paper describes our simulation system. We start with a description of a process of generation of a mesh, i.e. transforming the geological date about fractures into the form of the grid for numerical methods (FEM/FVM). Then we show the numerical model of the groundwater flow. This model solves problem of the linear Darcy flow on the system of mutually interconnected fractures. The mixed-hybrid FEM is used for approximation of the PDE's.

In the third part of the paper we introduce a real-world hydrogeological problem calculated by our system — simulation of the injection test and communication between drillholes PTP3 and PTP4a through the system of fractures in locality Krušné Hory Mountains in the Czech Republic. We conclude our paper with brief summary of experiences gained by development and application of our system. We also mention its possible improvements and extensions.

1 INTRODUCTION

Numerical modelling of processes in the fractured rock environment becomes an important tool for solving many hydrogeological, geochemical and ecological problems. The most important practical application is expected in choosing suitable localities for the permanent repositories of the dangerous (especially radioactive) waste. This kind of modelling is relatively new branch of research. There exist lot of software for modelling the processes in fractured rock, but no of them is considered to be generally applicable for all possible types of problems.

The start of the construction of the radioactive waste repository in the Czech Republic is planned in the time horizon of approximately twenty years but some preparation works have already started. In scope of this works, a team of researchers from the Czech Geological Survey, Technical University of Liberec and Institute of Computer Science of the Academy of Sciences of the Czech Republic has been established. The goal of the work of this team is to develop a software system for the simulation of the processes in fractured rock, with special respect to geological conditions typical for the crystalinic massifs in the Czech Republic. There were two grant projects (GAČR 205/00/0480 and MŽP VaV 630/3/00) focused on this area. As a result of solution of these projects, there exists first functional and applicable version of the simulation system.

2 STOCHASTIC DISCRETE FRACTURE NETWORK APPROACH

Underground granitoid massifs are proposed as nuclear waste repositories. However, they are always disrupted by a system of geological faults, fractures. The percolation of groundwater through such massifs is called *fracture flow*.

According to Bear (1993) there are three main approaches to modelling of fracture flow:

1. *Equivalent porous medium models* are used for large-scale problems if there is no need for knowing the flow field in detail.
2. *Double porosity models* work with two connected continua – representing fractures and porous blocks.
3. *Stochastic discrete fracture network models* are trying to create as exact representation of the fractured environment as possible by simulating particular fractures. Due to computational costs these models can be used only for solution of the problems on relatively small domains (up to tens of meter).

Our simulation system is based on the third approach. The main idea of that approach is to approximate the original three-dimensional fractures by planar elliptic or polygonal disks whose frequency, size, assigned aperture, and orientation are *statistically* derived from field measurements and consider hydraulical and geochemical processes on such a network. Therefore the procedure of calculation of a flow field by such approach has two important steps:

1. Generate a stochastic discrete fracture network.
2. Find an approximation of the flow field on such network.

We will describe both of these topics in following two sections.

3 GENERATION OF THE FRACTURE NETWORK

We generate the fracture network on the basis of statistical data obtained from field measurements. We enable the definition of hydraulically important fractures, zones with an increased density of fractures, or insertion of deterministic fractures. Planar circle disks approximate the original three-dimensional fractures and each disk is subsequently discredited into a triangular mesh respecting the intersections with his neighbours. In order to simplify the geometrical situation in fracture planes, the computed intersections are moved and stretched slightly. In this way, one obtains a mesh of a higher quality; however, the three-dimensional geometrical correspondence vanishes and has to be replaced by an element edges correspondence. Finally, we assign an aperture to each element. Based on it, the hydraulic permeability of the element is set, considering also fracture wall roughness and filling. The classical parallel plate model is thus avoided and the channelling effect is simulated. We can see an example of a simple triangular mesh in Figure 1.

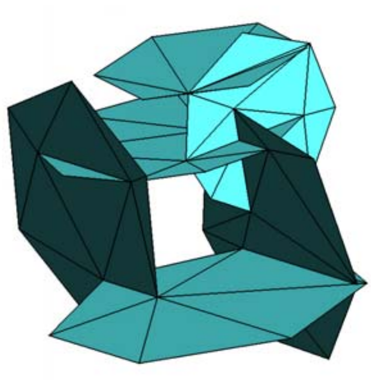

Figure 1. Fracture network made of six polygons, discredited into a triangular mesh

4 APPROXIMATION OF THE FLOW FIELD

System of fractures S can be denoted as:

$$S \equiv \left\{ \bigcup_{l \in L} \overline{\alpha_l} \setminus \partial S \right\}, \tag{1}$$

with α_l an opened 2-D polygon placed in the 3-D Euclidean space. We call $\overline{\alpha_l}$ the closure of α_l, a *fracture. L* is the index set of fractures, ∂S is the set of those boundaries of α_l which do not create the connection with other fractures. We suppose that all $\overline{\alpha_l}$ are connected into one *fracture network*; the connection is possible only through an edge, not a point.

We are looking for the fracture flow velocity **u** (2-D vector in each α_l), which is the solution of the problem

$$\mathbf{u} = -\mathbf{K}(\nabla p + \nabla z) \qquad \text{in } S, \tag{2}$$

$$\nabla \cdot \mathbf{u} = q \qquad \text{in } S, \tag{3}$$

$$p = p_D \quad \text{in } \Lambda_{\mathrm{D}}, \quad \mathbf{u} \cdot \mathbf{n} = u_N \quad \text{in } \Lambda_{\mathrm{N}}, \tag{4}$$

where all variables are expressed in local coordinates of appropriate α_l, and also the differentiation is always expressed in towards these local coordinates. The equation (2) is Darcy's law, equation (3) is the mass balance equation and equation (4) is the expression of appropriate boundary conditions. The variable p denotes the modified fluid pressure $p = \mathrm{p}/(g\rho)$, g is the gravitational acceleration constant, ρ is density of the fluid, q represents stationary sources/sinks density and z is the elevation, positive upward taken vertical 3-D coordinate expressed in local coordinates of appropriate α_l. The second rank tensor **K** of hydraulic conductivity is the function of original 3-D fracture properties mentioned above (aperture...). We require it to be symmetric and uniformly positive definite on each α_l.

We pose also the requirement

$$\Lambda_D \cap \Lambda_N = \emptyset, \overline{\Lambda_D} \cup \overline{\Lambda_N} = \partial S, \Lambda_D \neq \emptyset .$$

The detailed description of the process of the approximation of the problem described above is out of the scope of this paper. For details see for example Maryška, Severýn, Vohralík 2002. There are three main ides of the formulation of the continuous problem.

i.) Divide the whole domain *S* to subdomain *e* – called *elements* – of a simple geometrical shape.

ii.) Require a weak fulfilment of the equations (2), (3), (4) on each *e*.

iii.) Express conservation of the mass on each boundary between two elements e_i and e_j of the mesh.

Mathematical expression of these ideas leads to a system of integral identities

$$
\begin{aligned}
&\sum_{e\in\mathcal{E}_h}\{(\mathbf{A}\mathbf{u}^e,\mathbf{v}^e)_{0,e}-(p^e,\nabla\cdot\mathbf{v}^e)_{0,e}+\langle\lambda^e,\mathbf{v}^e\cdot\mathbf{n}^e\rangle_{pe\cap\Lambda_{h,D}}\} = \\
&=\sum_{e\in\mathcal{E}_h}\{-\langle p_D^e,\mathbf{v}^e\cdot\mathbf{n}^e\rangle_{pe\cap\Lambda_D}+(z^e,\nabla\cdot\mathbf{v}^e)_{0,e}-\langle z^e,\mathbf{v}^e\cdot\mathbf{n}^e\rangle_{0,\partial e}\}\,, \\
&\sum_{e\in\mathcal{E}_h}-(\nabla\cdot\mathbf{u}^e,\phi^e)_{0,e}=\sum_{e\in\mathcal{E}_h}-(Q^e,\phi^e)_{0,e} \\
&\sum_{e\in\mathcal{E}_h}\{\langle\mathbf{n}^e\cdot\mathbf{u}^e,\mu^e\rangle_{0,\Gamma_h}-\langle\sigma\lambda^e,\mu^e\rangle_{0,\partial e\cap\partial\Omega}\}=\sum_{e\in\mathcal{E}_h}\langle\mathbf{u}_N^e-\sigma\lambda_N^e,\mu^e\rangle_{0,\partial e\cap\partial\Omega} \\
&(\mathbf{v},\phi,\mu)\in\mathbf{H}(div,\mathcal{E}_h)\times L_2(\Omega)\times H^{\frac{1}{2}}(\Gamma_h),
\end{aligned}
\tag{5}
$$

solved in appropriate functional spaces. We use a mixed-hybrid finite element method with lowest order Raviart-Thomas basis to discretize the system (5). This leads to a system of linear algebraic equations

$$
\begin{array}{lllllll}
\mathbb{A}\mathbf{u} & + & \mathbb{B}\mathbf{p} & + & \mathbb{C}\boldsymbol{\lambda} & = & \mathbf{q}_1 \\
\mathbb{B}^T\mathbf{u} & & & & & = & \mathbf{q}_2 \\
\mathbb{C}^T\mathbf{u} & + & & & \mathbb{F}\boldsymbol{\lambda} & = & \mathbf{q}_3.
\end{array}
\tag{6}
$$

for unknowns **u** (velocity of flow expressed as an inter-element flux), **p** (modified pressure on the elements) and **λ** (modified pressure on the sides of elements). This system is symmetric and indefinite. We use a specialized iterative solver for solution of the system (6), based on the Schur complement method (see Maryška, Rozložník, Tůma 2000).

5 AN EXAMPLE OF A REAL PROBLEM

There were drilled two drillholes, called PTP3 and PTP4A, as a part of solution of the grant project VaV 630/3/00. The drillholes are located in an extremely compact granitoid massif, near village Potůčky in Krušné Hory Mountains in the Czech Republic. Depth of the drillholes is 350 (PTP3) and 300 (PTP4A) meter; their horizontal distance is 10 meter at the surface and approx. 15meter at the bottom of the drillhole PTP4A. There were performed almost all available field measurements for achieving the data about fractured environment. These measurements contained core scanning, acoustic television, electrical conductivity measurements, mineralogical analysis, chemical analysis, electron microscopy of the fracture surfaces and several others. The main result of these measurements is a description of fractures in the massif. Figure 2 shows an example of the results of the measurements.

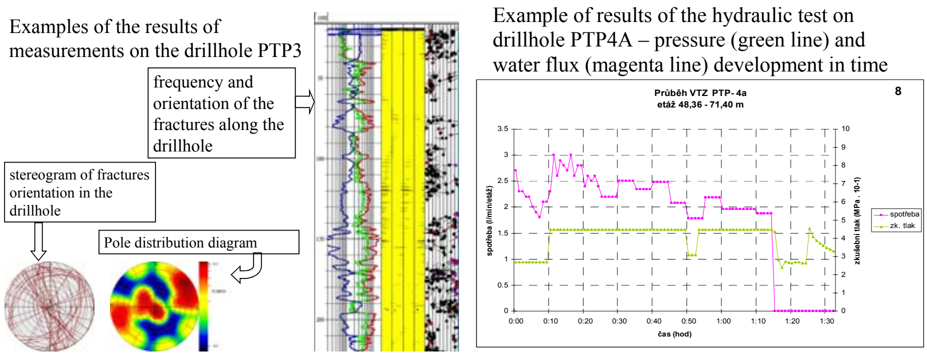

Figure 2. Example of the results of the field measurements

The pumping and injection tests were also performed as well as tests of hydraulic communication between various parts of the drillholes. A tracer test was also done.

Results of the tests gave us suitable set of data for numerical modelling of the close neighbourhood of the drillholes. At the beginning we tried to create mesh consisting of the same number of fractures as it was observed in the drillhole. This attempt was not very successful because of huge amount of generated fractures. When the computational domain had dimensions 20x30x20 meter and when we included fractures bigger than 20cm in diameter then we had got fractured mesh with several tens of thousands of fractures. Calculation on such a large mesh is not possible due to computational costs caused by the excessive number of fractures and ill-conditioned state matrix of the linear equation system.

Therefore if was necessary to reduce the number of fractures. We have eliminated all fractures smaller than 2 meter in diameter. There was proved that the importance of such fractures for the flow field is not very high. After this elimination there left only approx. 50 fractures in the domain. Calculation in this case was successful. Then we calibrated the model by the results of injection tests. After calibration, we achieved the difference between model and the reality better than 15%. Examples of the results are shown in figure 3.

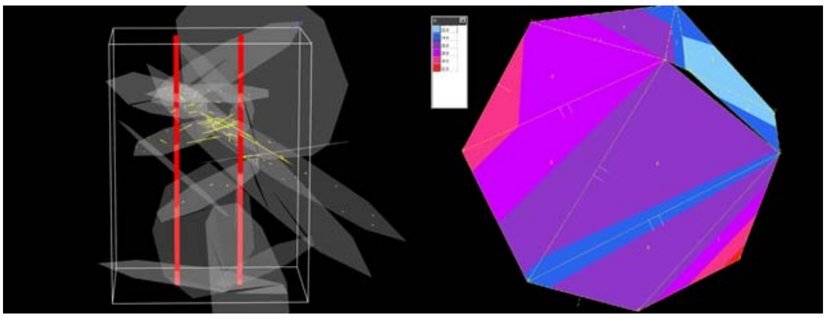

Figure 3. Results of the calculation of a real problem.

On the left of the figure 3 the computational domain with drillholes is shown. Fractures are drawn as semi-transparent. Calculated vectors of velocity of flow are also shown. The right part shows pressure field and inter-element flux on one particular fracture of the mesh.

6 CONCLUSIONS

We have presented the most important information about our system for simulation flow in the fractured rock environment. The system is based on the discrete stochastic fracture network approach and uses mixed-hybrid FEM for approximation of the partial differential equations. It can be used for solution of hydrogeological problems of scale of approximately tens of meter. The implementation of the system proved, that the most important and most critical part of the system is the module for the mesh generation. The algorithms for discretizing shapes with complex topology, as the fracture systems are, are very complicated and lot of special situation have to be solved. The low regularity of the triangular elements is the reason for limitation of size for the problems.

In near future we want to improve this limitation. We want to combine our approach with equivalent porous medium approach. We plan to use the discrete stochastic fracture networks only for small number of hydraulically important fractures and the rest of the domain will be filled with porous blocks representing the less significant fractures.

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