STRESS DISTRIBUTION AND CRACK OPENING IN THE PREFRACTURE ZONE (NEUBER –NOVOZHILOV APPROACH)

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ABSRACT

The Neuber-Novozhilov approach is proposed for description of the stress distribution and crack opening in the prefracture zone in the vicinity of rupture cracks when solutions of the classical elasticity theory has a singular component. The stress distribution in the classical Leonov-Panasyuk-Dugdale model is chosen for a null approximation. Modification of this model is proposed when the scheme of a fiber bundle is used for the prefracture zone, the fiber bundle occupying a rectangle in the front of the tip of the initial crack. The stress distribution in the prefracture zone and in the front of it is obtained in the first approximation in the case when the standard $\sigma-\varepsilon$ diagram of material has jump discontinuity. The function that describes the stress distribution may also have jump discontinuity. The function that describes a crack opening is a smooth function. For critical lengths of prefracture zones and the critical coefficient of material crack toughness, relations between structural and strength characteristics of the material are obtained.

1 INTRODUCTION

Comprehensive formulation of a problem on the stress distribution and displacements in the prefracture zone for elastic-plastic materials and partially fractured ones refers to nonlinear fracture mechanics. We use classical representations of linear fracture mechanics for prefracture zones when both the initial and imaginary rupture cracks are modeled by bilateral cuts. Discussion on appropriateness of simplifications being proposed will be done in paragraph 3 where special attention is given to description of nonlinear force bonds in the prefracture zone and to peculiar interpretation of the Leonov-Panasyuk-Dugdale model [1, 2].

Conception of prefracture zones at the tips of rupture cracks is found to be a highly constructive approach. Let tensile stresses σ_∞ acting normally to the crack plane be specified at infinity. A real inner straight crack-cut of length $2l_0$ is modeled by some imaginary crack-cut of length $2l = 2l_0 + 2\Delta$ when linear equations of the elasticity theory are used (Δ is the length of prefracture zones, each being located on the continuation of the inner crack). The field of normal stresses $\sigma_y(x,0)$ on the imaginary crack continuation can be represented as a sum of two terms (the origin of Cartesian coordinate system Oxy coincides with the right tip of an imaginary crack)

$$\sigma_y(x,0) \cong K_I/(2\pi x)^{1/2} + O(1), \quad K_I = K_{I\infty} + K_{I\Delta}, \quad K_{I\infty} > 0, K_{I\Delta} < 0 \tag{1}$$

where $K_I = K_I(l,\Delta)$ is the total stress intensity factor (SIF), $K_{I\infty}$ is the SIF caused by stresses σ_∞, $K_{I\Delta}$ is the SIF caused by stresses $\sigma_{m\Delta}$ that act in accordance with the classical Leonov-Panasyuk-Dugdale model. Both first and second terms in relation (1) are singular and smooth parts of the solution, respectively.

When prefracture zones are described, two classes of solutions are possible:

$$K_I = 0, \tag{2}$$

$$K_I > 0. \tag{3}$$

The third class of solutions corresponding to unequally $K_I < 0$ is not under consideration because crack-cut flanks overlap in the model being studied at such a limitation. Construction of the first class of solutions (2) may be related to Khristianovich's hypothesis [3] on the absence of singularity at the tip of an imaginary crack. When the prefracture zone is studied, solutions without singularity [4] receive primary emphasis. One of recent publications on the theme of solutions having no singularity [5] contains the wide bibliography.

The Neuber-Novozhilov approach [6, 7] allows one to extend the class of solutions for solid with a structure, see relations (1) and (3), works [8 - 10] and their references. In accordance with N.N. Novozhilov's terminology, the strength criteria under consideration are referred to as sufficient criteria. Infinite stresses at the imaginary crack tip, see (1) and (3), which are not assumed by the continual strength criterion, are not contradictory to discrete criteria [6, 7] if the singular component of solution has integrable singularity. We will consider the stress distribution in the Leonov-Panasyuk-Dugdale model as some null approximation when inequality (3) is valid.

2 SUFFCIENT STRENGTH CRITERION

The sufficient discrete-integral criterion of quasi-brittle strength has the form ($\Delta > 0, \delta_{m\Delta} > 0$), see. [8 - 10],

$$\frac{1}{kr_0}\int_0^{nr_0}\sigma_y(x,0)dx \le \sigma_{m0}\,,\ 0 \le x \le nr_0\,; \tag{4}$$

$$2v(x) \le \delta_{m\Delta}\,,\ -\Delta \le x \le 0\,. \tag{5}$$

Here σ_y is normal stresses (1), which have the singular component (3) with integrable singularity; r_0 is the specific linear size of the initial material structure; n,k are integers ($n \ge k$); nr_0 is the averaging interval; $(n-k)/n$ is the coefficient of material damage within the averaging interval; σ_{m0} is the "theoretical" strength of granular material (in the general case $\sigma_{m0} \ne \sigma_{m\Delta}$); $2v = 2v(x)$ is the imaginary crack opening; $2v^*(-\Delta) = \delta_{m\Delta}$ is the critical imaginary crack opening at which a prefracture zone structure nearest to the center of the crack is broken .

The sufficient discrete-integral criterion (4) and (5) allows limiting passage to the necessary discrete-integral criterion (4) when the prefracture zone length vanishes, i.e. $\Delta \to 0$. Stresses σ_∞^0 are critical stresses obtained by the necessary criterion (4) and these correspond to brittle material fracture. At $\Delta \to 0$, there is no imaginary crack opening: lengths of the imaginary and initial cracks coincide, i.e., at $\Delta = 0$, we have $2v(0) = 0$. Consider the sufficient criterion (4) and (5): for critical parameters $\sigma_\infty^*, 2v^*, K_I^*, \Delta^*$, relations (4) and (5) change into equalities (critical parameters are marked by stars). It is obvious that $\sigma_\infty^* > \sigma_\infty^0$ and $0 \le 2v(x) \le \delta_{m\Delta}$ ($-\Delta^* \le x \le 0$) when the standard $\sigma - \varepsilon$ material diagram has a nonlinear part besides the linear part of deformation. It is reasonable to measure the prefracture zone length Δ in the terms of material structure length r_0. We refine the description of the $\sigma - \varepsilon$ material diagram for construction of critical fracture parameters.

3 PHYSICAL-MECHANICAL DESCRINTION OF PREFRACTURE ZONE

Emphasize that, generally speaking, in relation (4), $\sigma_{m0} \ne \sigma_{m\Delta}$ for reinforced or multicomponent materials. For example, in works [8, 10], the idea on a fiber bundle for describing the standard $\sigma - \varepsilon$ diagram is used. When the Neuber-Novozhilov approach [6, 7] is implemented in the classical Leonov-Panasyuk-Dugdale model [1, 2], the limitation $\sigma_{m0} = \sigma_{m\Delta}$ is chosen. This limitation corresponds to behavior of elastic-plastic materials with the pronounced yield area.

At the top right of Fig. 1a, the initial stress distribution (null approximation) is shown as in the prefracture zone in accordance with the Leonov-Panasyuk-Dugdale model for rupture cracks, so in the immediately ahead of this zone with allowance for averaging according to the criterion (4) at $\sigma_\infty > 0$ ($\sigma_\infty^0 < \sigma_\infty \le \sigma_\infty^*$): averaged stresses $\sigma_{m\Delta}$ in the studied model are the horizontal straight line 1, see relation (6); stresses σ_{m0} acting within the averaging interval are shown by the horizontal straight line 2 with accordance the criterion (4); the curve 3 has two parts: the singular (dashed curve) and smooth (solid curve) parts and the curve corresponds to solution of linear fracture mechanics. It should be emphasized that before averaging, the function describing the stress distribution on the crack continuation has the discontinuty at $x = 0$, when solutions of the classical elasticity theory has a singular component. Fig. 1b demonstrates a force loading scheme for the right tip of an imaginary crack. The scheme in Fig. 1b does not take into consideration distortion of crack-cut flanks. In Fig. 1c, the scheme of opening $2v(x)$ of an imaginary crack flanks at $\sigma_\infty > 0$ ($\sigma_\infty^0 < \sigma_\infty \le \sigma_\infty^*$) is given when the unequally (3) is valid.

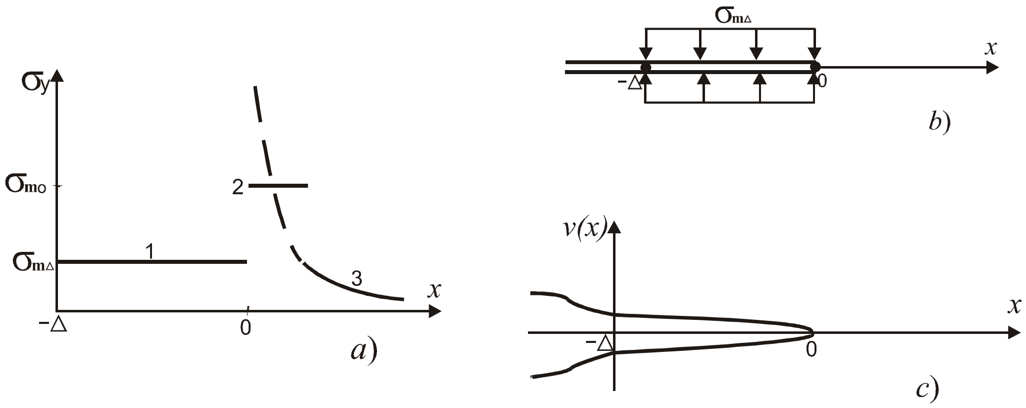

Fig. 1. The stress distribution in the vicinity of the imaginary crack tip (a).
The scheme of loading of an imaginary crack (b). Opening of imaginary crack flanks (c).

It is supposed to use approximation of such standard $\sigma - \varepsilon$ diagrams that the averaging is used on the nonlinear segment

$$(\sigma_{m\Delta})_j = \sigma_{m\Delta} = \frac{1}{\varepsilon_{m\Delta} - \varepsilon_{m0}} \int_{\varepsilon_{m0}}^{\varepsilon_{m\Delta}} \sigma(\varepsilon) d\varepsilon, \quad j = 0. \tag{6}$$

Here ε_{m0} is the extreme relative lengthening of initial material when the “theoretical” strength of the initial material is reached, $\varepsilon_{m\Delta}$ is the extreme relative lengthening of the prefracture zone material, index $j = 0$ refers to the null approximation in the chosen model.

Simple tension is realized ahead of a rupture crack tip on the Ox axis, so material behavior in the prefracture zone is supposed to describe as behavior of a bundle of s fibers. The number of tensile fibers in the bundle may be estimated as follows: integer Δ / r_0, i.e. $s = [\Delta / r_0]$. Nonlinear force bonds that are modeled by behavior of the fiber bundle at $\sigma_\infty^0 < \sigma_\infty \le \sigma_\infty^*$ occupy the prefracture zone. The criterion of critical crack opening displacement (CCOD criterion) is realized in the Leonov-Panasyuk-Dugdale model. Formally, the CCOD criterion is already written as relation (5) and it is the component of the sufficient strength criterion (4), (5).

The parameter of extreme relative lengthening of prefracture zone material $\varepsilon_{m\Delta}$ is chosen from real $\sigma-\varepsilon$ diagrams or their approximations, then the critical parameter of imaginary crack $\delta_{m\Delta}$ is calculated by the relation

$$\delta_{m\Delta}=(\varepsilon_{m\Delta}-\varepsilon_{m0})a\,,\quad a=5(K_I^0)^2/(4\pi\sigma_{m0}^2)\,,\quad K_I^0=\sigma_\infty\sqrt{\pi l_0} \tag{7}$$

Above we used the hypothesis to the effect that the prefracture zone is a rectangle with sides a and Δ, see [11]. From [12, 13] follows that the hypothesis being used reflects behavior of plastic material in the vicinity of the crack tip only qualitatively. Thus, three parameters for the sufficient deformation-strength criterion (4) and (5) are obtained in the general case: two strength parameters $\sigma_{m0}>0$ and $\sigma_{m\Delta}>0$ (the following variations are possible $\sigma_{m0}=\sigma_{m\Delta}$, $\sigma_{m0}\neq\sigma_{m\Delta}$), and one deformation parameter $\delta_{m\Delta}>0$. These parameters characterize material behavior.

4 CRITICAL FRACTURE PARAMETERS FOR BOTH INNER AND EGE CRACKS

We now turn to definition of critical fracture parameters, at first, for null approximation $j=0$ (in this section, the index j is omitted in all relations). The simplest asymptotic representations are used for stresses $\sigma_y(x,y)$ in relation (4) when the smooth part of solution is omitted in the approximate equality (1) and SIFs $K_I^*=K_I(l^*,\Delta^*)$, $K_{I\infty}=K_{I\infty}(l^*)$, and $K_{I\Delta}=K_{I\Delta}(l^*,\Delta^*)$ for an inner rupture crack are determined as, see. [14],

$$K_I^*=K_{I\infty}+K_{I\Delta}=\sigma_\infty^*\sqrt{\pi l^*}-\sigma_{m\Delta}\sqrt{\pi l^*}\left[1-\frac{2}{\pi}\arcsin\left(1-\frac{\Delta^*}{l^*}\right)\right]. \tag{8}$$

Here K_I^*, $l^*=l_0+\Delta^*$, Δ^* are total critical SIF and both the half-length of an imaginary crack and prefracture zone length. For crack opening $2v(x)$ in relation (5), we use the simplest relation when secondary terms of the order $O(-x)$ are omitted in the asymptotic relation for crack opening in the vicinity of its tip, see [14]. Then critical opening $2v^*(-\Delta)$ of the imaginary crack takes the form

$$2v(x)\cong\frac{\eta+1}{G}K_I\sqrt{\frac{-x}{2\pi}}+O(-x)\,. \tag{9}$$

Here $\eta=3-4\mu$ or $\eta=(3-\mu)/(1+\mu)$ are parameters for plane deformation or plane stress state, respectively; μ is the Poisson ratio; G is the shear modulus.

After obvious transformations of equalities (4) and (5) using relations (8) and (9), we obtain the system of two nonlinear equations for critical parameters in one of the forms:
the first form of the system for arbitrary crack is

$$\frac{K_I^*}{\sqrt{\pi}}\frac{1}{k}\sqrt{\frac{2n}{r_0}}=\sigma_{m0}\,,\quad \frac{\eta+1}{G}K_I^*\sqrt{\frac{\Delta^*}{2\pi}}=(\varepsilon_{m\Delta}-\varepsilon_{m0})a\,; \tag{10}$$

the second form of the system for inner crack is

$$\frac{\sigma_\infty^*\sqrt{\pi l^*}-\sigma_{m\Delta}\sqrt{\pi l^*}\left[1-(2/\pi)\arcsin\left(1-(\Delta^*/l^*)\right)\right]}{\sqrt{\pi}}\frac{1}{k}\sqrt{\frac{2n}{r_0}}=\sigma_{m0}\,, \tag{11}$$

$$\frac{\eta+1}{G}\left\{\sigma_{\infty}^{*}\sqrt{\pi l^{*}}-\sigma_{m\Delta}\sqrt{\pi l^{*}}\left[1-\frac{2}{\pi}\arcsin\left(1-\frac{\Delta^{*}}{l^{*}}\right)\right]\right\}\sqrt{\frac{\Delta^{*}}{2\pi}}=(\varepsilon_{m\Delta}-\varepsilon_{m0})a\ .$$

Three parameters $\sigma_{m0}>0$, $\sigma_{m\Delta}>0$, and $\delta_{m\Delta}=(\varepsilon_{m\Delta}-\varepsilon_{m0})a>0$, enter the systems (10) and (11) with allowance for material damage if $k<n$. Therefore, the sufficient criterion (4) and (5) is three-parametric deformation-strength criterion.

Systems (10) and (11) describe fracture on the continuation of long cracks, i.e. $l_0/r_0>>1$, since for normal stresses $\sigma_y(x,y)$, smooth components of solution of the $O(1)$ order in relation (1) are omitted, and for opening $2v(x)$ of imaginary cracks, components of the order of $O(-x)$ are omitted. The system (11) can be simplified for long cracks at quasi-brittle fracture when $\Delta^*/l^*<<1$. Approximate relations are obtained, which are equivalent to the system (11). Two other parameters K_I^*, $l^*=l_0+\Delta^*$ are obtained by the obvious way from critical parameters σ_∞^*, Δ^* after appropriate calculations. Critical stresses at quasi-brittle $\sigma_\infty^*(l^*)$ and brittle $\sigma_\infty^0(l_0)$ material fracture at the same crack length can differ by several times. The cross-section of the prefracture zone is refined when these critical stresses σ_∞^*, σ_∞^0 essentially differ. In quasi-brittle approximation $l^*\approx l_0$ for the critical dimensionless parameter of tensile stresses $\sigma_\infty^*/\sigma_{m0}$, correction is made for material elasticity as compared with $\sigma_\infty^0/\sigma_{m0}$, this correction being mainly dependent on deformability $\varepsilon_{m\Delta}-\varepsilon_{m0}$ of plastic material. For quasi-brittle fracture, dimensionless critical parameters of tensile stresses $\sigma_\infty^*/\sigma_{m0}$ and lengths of prefracture zones Δ^*/r_0 (in null approximation $j=0$) are obtained. Two strength parameters σ_{m0}, $\sigma_{m\Delta}$ and one deformation parameter $\delta_{m\Delta}=(\varepsilon_{m\Delta}-\varepsilon_{m0})a>0$ characterizing material behavior are components of these relations.

The same relations are also obtained for edge cracks.

The null stress distribution in the prefracture zone has already been constructed in accordance with the Leonov-Panasyuk-Dugdale model in (6) and this distribution was used in determining critical parameters. The stress distribution in the prefracture zone at $-(\Delta^*)_0\le x\le 0$ for an imaginary crack was refined using critical fracture parameters of null approximation. The part of the $\sigma=\sigma(\varepsilon)$ material diagram at $\varepsilon_{m0}\le\varepsilon\le\varepsilon_{m\Delta}$ is realized for the prefracture zone. Consider the relative lengthening $\varepsilon=\varepsilon(-x)$ as a function of the x coordinate. Function $\sigma=\sigma(\varepsilon)$ is considered as a complex function, i.e., $\sigma=\sigma[\varepsilon(-x)]$. Finally we have the closed form relation of the stress distribution in the prefracture zone of the right crack tip for the first and highest approximations. At every step $j=1,2,3,...$ of successive approximations, nonlinear scale transformation takes place that distorts the real $\sigma-\varepsilon$ diagram.

6 DISCUSSION

The proposed sufficient strength criterion (4) and (5) allows the following description: i) development of the prefracture zone Δ when its length is changed ($0\le\Delta\le\Delta^*$), ii) the stress distribution $\sigma=\sigma[\varepsilon(-x)]$ when the zone length is changed, and iii) imaginary crack opening $2v(x)$ at successive extra loading and initiation of the real crack tip.

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