STRESSES AND FAILURE IN JOINTS

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ABSTRACT
A short overview about the stress distribution of joints under mechanical and thermal loading and the fracture criterion is presented. The stresses near the free edge of an interface or near an internal corner are singular in most cases. Parameters describing the stress field are the stress exponent, the stress intensity factor and the angular function. All these quantities depend in a different way on the material properties of the joint materials and the contact angles. The stress distribution is especially complex in a three material joint with an interlayer between two materials. There exists no straightforward way to define a fracture criterion. Different approaches presented in the literature are discussed.

1 INTRODUCTION
Joints of different materials have many applications in structural engineering or microelectronics. Different methods of joining exist, such as soldering or welding. Due to the different elastic and plastic properties and thermal expansions of the joined materials, stresses develop under mechanical loading and after a change in temperature. The reliability assessment of joints requires an accurate calculation of these stresses under the applied loading condition and an assessment of the stresses. If an appropriate procedure has been established for the assessment of the stresses, geometric design of a joint and selection of suitable materials are possible. The main problem in this area consists in the fact that the stresses are especially high at the free edge of the interface (open wedge) or at an internal corner. Fig. 1 shows part of a microelectronic component, where the critical positions are marked.

Fig. 1 Critical locations of a component

2 STRESSES AT AN OPEN WEDGE OR AN INTERNAL CORNER
The geometry of a bimaterial wedge and an internal corner is characterized by the two contact angles $\theta_1$ and $\theta_2$ and by the angle $\theta_i$, respectively (Fig.2). Calculations were performed for isotropic and anisotropic materials under elastic and elastic-plastic material behavior with perfect bonding or friction sliding contact. Here, isotropic, elastic behavior and perfect bonding at the interface shall be assumed. A two-dimensional situation is considered. In this case the stresses can be calculated by applying an Airy stress function and the appropriate boundary conditions for perfect bonding at the interface. For an open wedge, stress-free conditions are assumed at the open faces. Then, the stress tensor is
\[
\sigma_y = \frac{K_L}{(r/L)^\omega} f_\theta(\theta) + \sigma_0 g_\theta(\theta)
\] (1)

The distance \( r \) from the corner point is related to a characteristic size parameter \( L \) of the component. Then, the stress intensity factor \( K_L \) has the dimension of a stress. \( \omega \) is the stress exponent. In most cases, \( \omega > 0 \) and, therefore, a stress singularity exists. \( f_\theta \) and \( g_\theta \) are functions of the angle \( \theta \). The quantities \( K_L, \sigma_0, \omega, f_\theta \) and \( g_\theta \) depend on the angles \( \theta_1 \) and \( \theta_2 \). The elastic constants, \( K_L \) and \( \sigma_0 \), additionally depend on the applied loading. All these quantities, with the exception of \( K_L \), are independent of the overall geometry of the component and can be calculated analytically. \( K_L \) depends on the overall geometry and has to be calculated by applying numerical methods. The applied load is characterized by an external stress \( \sigma_\text{ext} \) for mechanical loading and by \( \Delta T(\alpha'_1 - \alpha'_2) \) for a change in temperature \( \Delta T \). \( \alpha'_i = \alpha_i(1+\nu_i) \) for plane strain, where \( \alpha_i \) denotes the thermal expansion coefficients. It will be shown that the second term in eq.(1), the regular term which is independent of the distance \( r \), is quite important to thermal loading. From eq.(1), it can be seen that the stress distribution depends on the quantities \( K_L, \sigma_0, \omega \) and on the two angular functions \( f_\theta \) and \( g_\theta \). All these quantities are dependent on the elastic constants in different ways. Whereas the stresses in a homogeneous material with a crack can be characterized by the stress intensity factor, this is not possible for the singular stress field of a joint. For some combinations of wedge angles and elastic constants the stress exponent is complex: \( \omega = p + qi \). Then, eq.(1) has to be replaced by

\[
\sigma_y = \frac{K_L}{(r/L)^\omega} \left[ \cos[q \log(r/L)] f_\theta(\theta) + \sin[q \log(r/L)] f_\theta(\theta) \right] + \sigma_0 g_\theta(\theta)
\] (2)

As an example the stress exponent of an open wedge and an internal corner is plotted versus the ratio \( E_2/E_1 \) for fixed Poisson’s ratios in Fig.3. It can be seen that there are ranges with real and complex stress exponents. Furthermore ranges with two or three positive real stress exponents exist. Then, the singular term in eqs. (1) and (2) has to be replaced by the sum of two or three terms, with different \( K_L, \omega, f_\theta \). In Fig.4 \( K_L, \omega, \sigma_0 \) are plotted versus \( E_2/E_1 \) within the range of real

![Fig.3 Stress exponents, left: open wedge (\( \theta_1 = 165^\circ, \theta_2 = 55^\circ, \nu_1 = \nu_2 = .30 \)), right: Internal corner (\( \theta_1 = 60^\circ, \nu_1 = 0.5, \nu_2 = 0.1 \))](image-url)
stress exponents for the example of the open wedge. When \( \omega \) passes zero, the stress intensity factor and the regular stress term \( \sigma_o \) approach infinity with opposite sign.

It is important to realize that not only the singular term, but also the regular term and even a non-singular term may contribute significantly to the stresses also very close to the singular point. This is shown in Fig. 5 for a joint under thermal loading \((\theta_1 = 115^\circ, \theta_2 = 45^\circ, E_1 / E_2 = 50, \omega_1 = 0.088, \omega_2 = -0.059, K_1 / \Delta T = -104104MPa / ^\circ C, K_2 = -210MPa / ^\circ C)\). The contribution of the singular term, non-singular term and regular term are plotted versus the distance from the singular point.

![Fig. 4 K_L, \( \sigma_0, \omega \) for open wedge of Fig.3](image1)

![Fig. 5 Stress components \( \sigma_1 / \Delta T \)](image2)

Now, a joint with contact angles of 90° and an interlayer of thickness \( H \) shall be considered (Fig. 6). The stresses in the three materials depend on the material properties of all three materials. However some general rules apply. In Fig. 6 the stress along the free surface of material 1 is shown. It can be distinguished between a near field, a far field, and a transition region. Both, the near field and the far field can be described by eq. (1). For the near field, \( \omega, \sigma_{ij}, f_\theta \) and \( g_{\theta} \) depend on the elastic parameters of the adjacent materials (in material 1 on those of materials 1 and 2) only.

![Fig. 6 Stresses in a joint with interlayer under thermal loading](image3)
Only the stress intensity factor \( K_{LNF} \) also depends on the thermal expansion coefficients of all three materials and on the elastic parameters of the third material. The stress distribution of the far field depends on the material properties of material 1 and 3 only and is independent of the properties of the interlayer. Especially if \( K_{LNF} \) and \( K_{LFF} \) have different signs, the curves of the stress components versus the distance have a maximum or a minimum. Fig. 6 on the right site shows the stress distribution of two other material combinations. In example B, the thermal expansion coefficients of materials 1 and 3 are identical. Hence the stresses are zero in the far field.

3 FAILURE OF JOINTS

The fracture of joints under mechanical or thermal loading is caused by the high - mostly singular - stresses at the corner point. In linear-elastic fracture mechanics a relation similar to eq.(1) exists for cracks in homogeneous materials. For all crack configurations, the stress exponent is 0.5 and the angular function \( f_\theta \) is the same. Under a homogeneous change of temperature, no stresses occur and the regular term (the T-stress) is not important in many cases. Hence the stress state is characterized by the stress intensity factor alone, and fracture occurs at a critical value for a given material, the fracture toughness \( K_{IC} \). In a joint all quantities \( f^{ij}, \sigma^{ij}, \omega, f_y, g_y \) depend on the material properties and contact angles in different ways. Therefore, a critical value of \( K_L \) does not exist and a more complicated fracture criterion has to be found. First, let us look on the strength under mechanical loading. Eq.(1) is reduced to

\[
\sigma_y = \frac{K_L}{(r/L)^{\omega}} f_y(\theta) = \frac{M}{r^{\omega}} f_y(\theta)
\]

Failue occurs at

\[
M = M_c = K_L L^{\omega}
\]

(7)

\( K_L \) is proportional to the external stress \( \sigma^* \) applied:

\[
K_L = \sigma^* Y(\alpha,\beta,\theta_1,\theta_2)
\]

(8)

Therefore, the critical external stress \( \sigma^*_c \) is

\[
\sigma^*_c = \frac{M_c}{Y L^{\omega}}
\]

(9)

Fig. 7 Strength versus thickness of interlayer [1]

Fig. 8 Strength versus thickness of an aluminium-epoxy joint [2]

For identical material combinations and contact angles, the strength is proportional to \( L^{\omega} \), provided that the overall geometry is constant. If the notch depth \( a \) is varied, then \( Y \) is a function of the relative notch depth. Eq.(9) is confirmed by results given in literature.

The first example is a joint of stainless steel with an epoxy interlayer [1]. The relevant size parameter is the thickness of the interlayer \( H \). From the elastic constants of stainless steel and epoxy, a stress exponent of \( \omega = 0.30 \) is calculated. The failure strength is plotted versus \( H \) in the log-log plot of Fig. 7. The data are in agreement with the predicted slope. The second example is a
joint of epoxy and aluminum [2]. The specimen is loaded at the edges and, thus, a 3D loading situation exists. In this case, the angular function $f$ depends on two angles. In Fig. 8 the strength is plotted versus the size $W$ on a log-log scale. Again, the slope is in agreement with the theoretical value of - 0.351.

So far, we have considered joints, where the materials and contact angles were identical and the size was varied. The more general problem is to predict the failure load of a component with arbitrary contact angles from the measured strength of a simple test specimen. No straight forward method can be applied. Therefore, different fracture criteria have been proposed:

- Critical stress at a distance $r_0$ from the wedge tip along the interface, where the normal stress or the shear stress can be considered
- Critical average stress at a distance $r_0$ along the interface. For normal stress as the relevant stress component, this leads to

$$\overline{\sigma}_c = \frac{K_i L_0^\omega}{(1 - \omega)r_0^\omega} + \sigma_0 = \frac{M}{(1 - \omega)r_0^\omega} + \sigma_0 \quad (10)$$

Under mechanical loading ($\sigma_0 = 0$) the critical external stress is

$$\sigma_c^* = \frac{\overline{\sigma}_c (1 - \omega)r_0^\omega}{L^\omega Y} \quad (11)$$

For both criteria the critical distance $r_0$ has to be determined. As an example, results of Hattori et al. [4] are presented. As shown in Fig. 9, an epoxy and an Fe-Ni alloy were bonded and different geometries were selected. The bonding temperature was 100°. During cooling, delamination was observed. The critical delamination temperature was determined. From stress analysis, the critical stress intensity factor $M_c$ was calculated. In Fig.9 $M_c$ is plotted versus the stress exponent. Fig.10 shows the shear stress along the interface versus $r$. Selecting a critical distance of $r_0 = 0.64$ mm leads to a common average shear stress of about 14.4 MPa.

![Fig. 9 Fracture toughness $M_c$ versus stress Exponent for an epoxy/Fe-Ni-alloy [3]](image)

![Fig. 10 Normal stress at interface for the different joints of Fig. 9](image)

In joints with one component being a ceramic, the failure very often starts in the ceramic near the free edge of the joint. Failure in ceramics is initiated at inherent flaws. A large scatter in the strength is observed due to the scatter in the flaw size. This scatter can be described by a Weibull distribution. By applying multiaxial Weibull statistics [4] it is possible to predict the failure probability of a component from the failure probability of test specimens. The probability of failure below an applied stress $\sigma$ is
\[ F = 1 - \exp \left( -\left( \frac{\sigma}{\sigma_0} \right)^m \right) \]  

(12)

The Weibull exponent \( m \) should be a material parameter independent of the geometry of the component and the stress distribution in the component. The second parameter \( \sigma_0 \) depends on the geometry of the component and the multiaxial stress distribution in the component. In Fig 16 the Weibull distribution of the strength of different geometries of a joint of alumina with Fe-Ni alloy is shown together with the strength distribution obtained from bending tests of alumina.

Fig. 11 Weibull-plot of different geometries of an Al\(_2\)O\(_3\)/Fe-Ni-alloy

The parameter \( m \) (slope of the straight lines) depend on the joint geometry and are smaller than \( m \) of the alumina specimens. This difference is due to the violation of a basic assumption of the Weibull theory: A constant stress along the flaw, which causes failure. This requirement is not fulfilled for cracks near the singular stress field. To obtain an improved Weibull-distribution the relation

\[ K = \sigma \sqrt{aY} \]  

(13)

with \( \sigma \) and \( Y \) being constant has to be replaced by

\[ K = \int_0^a \sigma(x)h(x,a)dx \]  

(14)

where \( \sigma(x) \) is the normal stress and \( \tau(x) \) the shear stress along the crack, and \( h_1 \) and \( h_{II} \) are the mode I and mode II weight functions, that depend on the elastic properties of the materials and the distance from the interface [6].

REFERENCES