# FREE VIBRATION CHARACTERISTICS OF DELAMINATED COMPOSITE ROTATING CANTILEVER SHALLOW SHELLS

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## ABSTRACT

Delamination in fibre-reinforced laminated composites resulting from interlaminar debonding of constituting laminae causes strength degradation and thereby promotes instability. The delaminated structure exhibits new vibration characteristics depending on size and location of the delamination. Turbomachinery blades with low aspect ratio could be idealized as rotating cantilever shallow shells. The initial stress system in a rotating shell due to centrifugal body forces affects natural frequencies appreciably. An extensive knowledge of these natural frequencies is of fundamental importance in order to prevent vibration induced fatigue failures and to ensure operational safety. The present study is concerned about the effects of delamination on free vibration characteristics of graphite-epoxy composite rotating cantilever shallow shells. Lagrange’s equation of motion is used to derive the dynamic equilibrium equation and moderate rotational speeds are considered wherein the Coriolis effect is negligible. An eight noded isoparametric plate bending element is employed in the finite element formulation. Effects of transverse shear deformation and rotary inertia are included. To satisfy the compatibility of deformation and equilibrium of resultant forces and moments at the delamination crack front where the separated plates meet, a multipoint constraint algorithm is incorporated which leads to unsymmetric stiffness matrices. Parametric studies are performed in respect of relative length and location of delamination, fibre orientation and rotational speed for cylindrical, spherical and hyperbolic paraboloidal shells. Numerical solutions obtained for bending stiff, quasi-isotropic and torsion stiff laminates are the first known non-dimensional fundamental frequencies for the type of analyses carried out here.

## 1 INTRODUCTION

Turbomachinery blades with low aspect ratio could be idealized as rotating cantilever shells. In a weight sensitive application such as turbomachinery engine, composite materials are advantageous because of their light weight, high stiffness and strength. One of the major causes of failure in laminated composites is the debonding of constituting laminae (delamination). Delamination of a critical size can result in strength degradation and can lead to instability. The delaminated structure exhibits new vibration frequencies depending on the size and location of the delamination. The presence of invisible delamination can be be detected with the help of prior knowledge of natural frequencies for a composite containing delamination. Moreover, the initial stress system in a rotating shell due to centrifugal body forces affects natural frequencies appreciably. As a precursor to the application of composites in the critical parts of aero-engines an extensive knowledge of these natural frequencies is essential in order to prevent vibration induced fatigue failures and to ensure long life structurally sound blades.

The works on free vibration of rotating composite plates were first carried out by Wang et al. [1] and Shaw et al. [2]. Pagano and Soni [3] derived two approximate analytical models for determining the stress-strain field within each layer of a rotating composite turbine blade. Bhumbla et al. [4] studied free vibration behaviour of shear deformable, composite rotating blades including geometric non-linearity. Seshu et al. [5] presented the details of fabrication and experimentation of glass-epoxy composite blades for determining natural frequencies and steady state strain measurement along with the effect of rotational speed on it. McGee and Chu [6] carried out three-dimensional continuum vibration analysis for rotating, laminated composite blades using the Ritz method. Bhumbla and Kosmatka [7] developed nonlinear finite element technique to study the nonlinear static deflection and vibration behaviour of spinning pretwisted composite plates. Kee and Kim [8] carried out vibration analyses of twisted rotating cylindrical shell type composite blades. To the best of the authors' knowledge there is no literature available which deals with delaminated composite rotating shells. The present work is aimed at investigating the effects of delamination on free vibration characteristics of graphite-epoxy composite rotating cantilever shallow shells.

## 2 THEORETICAL FORMULATION

The dynamic equilibrium equation for moderate rotational speeds is derived employing Lagrange's equation of motion and the equation in global form is expressed as

$$[M]\{\ddot{\delta}\}+([K]+[K_{\sigma}])\{\delta\}=\{F(\Omega^{2})\} \qquad (1)$$

where [M], [K] and $[K_\sigma]$ are global mass, elastic stiffness and geometric stiffness matrices, respectively . $\{F(\Omega^2)\}$ is the nodal equivalent centrifugal forces and $\{\delta\}$ is the global displacement vector. $[K_\sigma]$ depends on initial stress distribution and is obtained by the iterative procedure upon solving

$$([K]+[K_{\sigma}])\{\delta\}=\{F(\Omega^{2})\} \qquad (2)$$

The natural frequencies are determined from the standard eigenvalue problem which is represented below and is solved by QR iteration algorithm.

$$[A]\{\delta\}=\lambda\{\delta\} \text{ where } [A]=([K]+[K\sigma])^{-1}[M] \text{ and } \lambda=1/\omega_{n}^{2} \qquad (3)$$

The nodal displacements of the plate elements 1, 2 and 3 at a delamination crack front (Fig. 1) are expressed as (Gim [9])

$$u_{j}=\overline{u_{j}}-(z-\overline{z_{j}})\phi_{xj}, \quad v_{j}=\overline{v_{j}}-(z-\overline{z_{j}})\phi_{yj}, \textit{ and } \quad w_{j}=\overline{w_{j}} \qquad (4)$$

where $\overline{z_j}$ (j=2 or 3 ) is the z- coordinate of mid-plane of element j.

The transverse displacements and rotations at a common node must have values as given below

$$w_{1}=w_{2}=w_{3}=w, \quad \phi_{x1}=\phi_{x2}=\phi_{x3}=\phi_{x}, \textit{ and } \phi_{y1}=\phi_{y2}=\phi_{y3}=\phi_{y} \qquad (5)$$

The in-plane displacements of all three elements at crack tip are equal and they are related as

$$\begin{aligned} \overline{u_{2}}&=\overline{u_{1}}-\overline{z_{2}}\phi_{x} \qquad & \overline{v_{2}}&=\overline{v_{1}}-\overline{z_{2}}\phi_{y} \\ \overline{u_{3}}&=\overline{u_{1}}-\overline{z_{3}}\phi_{x} \qquad & \overline{v_{3}}&=\overline{v_{1}}-\overline{z_{3}}\phi_{y} \end{aligned} \qquad (6)$$

Equations (5) and (6) relating the nodal displacements and rotations of elements 1, 2 and 3 at the delamination crack tip are the multipoint constraint equations used in the finite element formulation to satisfy the compatibility of displacements and rotations.

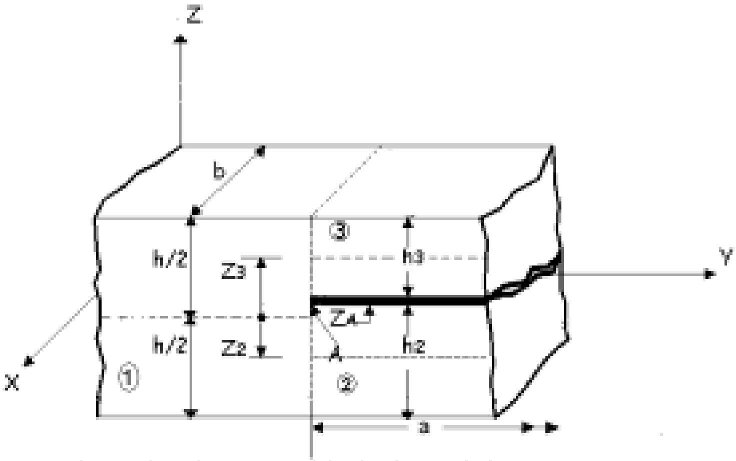

Fig. 1. Plate elements at a delamination crack tip

## 3 RESULTS AND DISCUSSION

The results obtained from the computer code developed on the basis of present finite element modelling are validated with those in the literature as shown in Tables 1 and 2. The predictive capability of the code is successfully demonstrated through the excellent agreement. Parametric studies are performed in respect of relative length and location of delamination, fibre orientation and rotational speed for cylindrical ($R_y/R_x=0$), spherical ($R_y/R_x=1$) & hyperbolic paraboloidal ($R_y/R_x=-1$) shells wherein $R_x$ and $R_y$ represent the radii of curvature in the x and y directions, respectively. Non-dimensional fundamental frequencies are obtained for bending stiff ($[O_2/\pm 30]_S$), quasi-isotropic ($[0/\pm 45/90]_S$) and torsion stiff ($[\pm 45/\mp 45]_S$) laminates (Crawley [10]). The following material properties for graphite-epoxy composite [11] are adopted for computation:

$E_1$ = 138.0 GPa, $E_2$ = 8.96 GPa, $G_{12}$ = 7.1 GPa, $G_{13}$ = 7.1 GPa, $G_{23}$ = 2.84 GPa, $\nu_{12}$= 0.30 ;

Non-dimensional fundamental frequencies of composite ($[O_2/\pm 30]_S$) cantilever cylindrical shell with relative lengths of delamination of 0.5 & 0.25 are shown in Tables 3 & 4 for various non-dimensional rotational speeds defined as $\Omega = \Omega / \omega_0$ ( $\Omega$ =rotational speed, $\omega_0$ =fundamental frequency of non-rotating shell). The influence of the relative delamination position along the shell thickness on the fundamental frequency is also presented. In case of higher value of relative length of delamination (0.5) frequency values are observed to decrease for both stationary condition and low non-dimensional speed of rotation (0.2) as the location of delamination moves towards the neutral plane and the minimum value is obtained at the mid-plane. But such trend is absent for higher speeds of rotation (0.6, 1.0). In case of lower value of delamination length (0.25) the frequency parameter has a maximum fall when the delamination is located at mid-plane. This is observed for both stationary condition and all rotating speeds unlike the previous case. It is also noted that for symmetric laminate ($[O_2/\pm 30]_S$) the frequencies across thickness are mirror images with respect to the value at mid-plane and excepting at d=0.125 for relative delamination length of 0.5 frequency increases with the increase of non-dimensional speed of rotation.

Non-dimensional fundamental frequencies of cantilever cylindrical and spherical shells are furnished in Tables 5 & 6 considering bending stiff, quasi-isotropic and torsion stiff laminates. The

Table 1. Comparison of non-dimensional fundamental frequencies for four layer [θ, -θ, -θ, θ] cantilevered shells. a/b =1, b/h=100, $b/R_y = 0.5$, $\varpi = \omega_n b^2 \sqrt{\rho / E_1 h^2}$ ; \$ Present FEM, # Qatu and Leissa [11]

| θ (Deg) | Cylindrical shell | Spherical shell | Hyperbolic paraboloidal shell |
|---|---|---|---|
| 15 | $2.0762^{\$}$ $2.0812^{\#}$ | $1.6535^{\$}$ $1.6639^{\#}$ | $1.2732^{\$}$ $1.2730^{\#}$ |
| 30 | 2.0606 2.0655 | 1.5937 1.6055 | 1.2086 1.2086 |
| 45 | 1.8622 1.8622 | 1.3396 1.3491 | 1.1515 1.1502 |
| 60 | 1.5475 1.5521 | 1.1561 1.1607 | 1.0403 1.0384 |

Table 2. Comparison of fundamental frequencies (Hz) for composite cantilever beam for different relative locations of delamination along its span.

| Relative Position | 0.25 | 0.50 | 0.75 |
|---|---|---|---|
| Present FEM | 63.45 | 69.40 | 71.57 |
| Krawczuk et al. [12] | 63.25 | 69.50 | 72.00 |

Table 3. Non-dimensional fundamental frequencies of graphite-epoxy composite ($[0_2/\pm30]_S$) rotating cylindrical shell along its thickness (d) with relative length of delamination of 0.5 from free end. a/b=1, b/h=100, $b/R_y$=0.5

| $\overline{\Omega}$ | d=0.125 | d=0.25 | d=0.375 | d=0.5 | d=0.625 | d=0.75 | d=0.875 |
|---|---|---|---|---|---|---|---|
| 0.0 | 1.9887 | 1.9339 | 1.7743 | 1.5940 | 1.7743 | 1.9339 | 1.9887 |
| 0.2 | 2.0088 | 1.9555 | 1.8411 | 1.6795 | 1.8411 | 1.9555 | 2.0088 |
| 0.6 | 1.9335 | 2.2575 | 2.0211 | 1.8878 | 2.0211 | 2.2575 | 1.9335 |
| 1.0 | 1.8341 | 2.7853 | 2.5121 | 2.6598 | 2.5121 | 2.7853 | 1.8341 |

relative positions of delamination along length are 0.25, 0.5 and 0.75 measured from fixed end. In each case 25% delamination is considered in the neutral plane of the shell. Although delamination length is same, frequency values are higher when the crack front is located at a greater distance from the fixed end. In general, cylindrical shell provides higher values of non-dimensional frequencies compared to spherical one. It is also found that for cylindrical shell of quasi-isotropic laminate variation of frequencies with respect to relative positions of delamination is negligible. For all the laminates cylindrical shell leads to increase in frequency with higher rotational speed.

Non-dimensional fundamental frequencies of cylindrical, spherical and hyperbolic paraboloidal shells of bending stiff laminate with multiple delaminations are presented in Table 7. The crack front is located at mid-span and relative length of each delamination is 0.25. The first case refers to three delaminations located at relative positions across thickness of d=0.125, 0.5 and 0.75 (i.e. 0//0/30/-30//-30/30//0/0, // indicates location of delamination), while the second case considers two delaminations at d=0.375 and 0.875 (i.e. 0/0/30//-30/-30/30/0//0). Excepting for spherical shell at non-dimensional speed of one, frequencies are higher for less number of delaminations as expected.

Table 4. Non-dimensional fundamental frequencies of graphite-epoxy composite ($[0_2/\pm30]_S$) rotating cylindrical shell along its thickness (d) with relative length of delamination of 0.25 from free end. a/b=1, b/h=100, $b/R_y$=0.5

| $\bar{\Omega}$ | d=0.125 | d=0.25 | d=0.375 | d=0.5 | d=0.625 | d=0.75 | d=0.875 |
|---|---|---|---|---|---|---|---|
| 0.0 | 2.0185 | 1.9980 | 1.9548 | 1.9207 | 1.9548 | 1.9980 | 2.0185 |
| 0.2 | 2.0662 | 2.0444 | 2.0020 | 1.9641 | 2.0020 | 2.0444 | 2.0662 |
| 0.6 | 2.3805 | 2.3508 | 2.3018 | 2.2538 | 2.3018 | 2.3508 | 2.3805 |
| 1.0 | 2.8629 | 2.8224 | 2.7731 | 2.7216 | 2.7731 | 2.8224 | 2.8629 |

Table 5. Non-dimensional fundamental frequencies of graphite-epoxy composite rotating cylindrical shell at different locations along its span with 25% mid-plane delamination. a/b=1, b/h=100, $b/R_y$=0.5

| Relative location | Bending stiff | | Quasi-isotropic | | Torsion stiff | |
|---|---|---|---|---|---|---|
| | $\bar{\Omega}$=0.5, | 1.0 | $\bar{\Omega}$=0.5, | 1.0 | $\bar{\Omega}$=0.5, | 1.0 |
| 0.25 | 2.1659 | 2.5991 | 2.0658 | 2.6490 | 2.1148 | 2.7469 |
| 0.50 | 2.1768 | 2.7302 | 2.0664 | 2.6467 | 2.1361 | 2.7759 |
| 0.75 | 2.1611 | 2.7216 | 2.0793 | 2.6557 | 2.2089 | 2.8650 |

Table 6. Non-dimensional fundamental frequencies of graphite-epoxy composite rotating spherical shell at different locations along its span with 25% mid-plane delamination. a/b=1, b/h=100, $b/R_y$=0.5

| Relative location | Bending stiff | | Quasi-isotropic | | Torsion stiff | |
|---|---|---|---|---|---|---|
| | $\bar{\Omega}$=0.5, | 1.0 | $\bar{\Omega}$=0.5, | 1.0 | $\bar{\Omega}$=0.5, | 1.0 |
| 0.25 | 1.5914 | 1.7923 | 1.6226 | 1.2921 | 1.2203 | 1.1127 |
| 0.50 | 1.6418 | 6.7105 | 1.7210 | 2.3493 | 1.3512 | 1.9860 |
| 0.75 | 1.6936 | 2.3691 | 1.8033 | 5.4811 | 1.4010 | 2.8656 |

Table 7. Non-dimensional fundamental frequencies of graphite-epoxy composite ($[0_2/\pm30]_S$) rotating shells with 25% delamination located at several positions across the thickness. a/b=1, b/h=100, $b/R_y$=0.5

| $\bar{\Omega}$ | Cylindrical shell | | Spherical shell | | Hyperbolic Paraboloidal shell | |
|---|---|---|---|---|---|---|
| | 3-delam. | 2-delam. | 3-delam. | 2-delam. | 3-delam. | 2-delam. |
| 0.0 | 1.8994 | 1.9332 | 1.4255 | 1.4793 | 1.1244 | 1.2559 |
| 0.5 | 2.1495 | 2.1921 | 1.5440 | 1.6432 | 1.3042 | 1.4558 |
| 1.0 | 2.6874 | 2.7475 | 6.9108 | 1.9039 | 1.6921 | 1.8663 |

Geometric stiffness matrix at non-dimensional speed of unity for spherical shell has substantial influence compared to elastic stiffness. In all the cases frequency parameter increases with speed. It is also to be noted that maximum values of frequency are obtained for cylindrical shell while the minimum for hyperbolic paraboloidal shell for all the cases, excepting at non-dimensional speed of unity wherein spherical shell provides the maximum one.

## 4 CONCLUSIONS

Natural frequencies are the functions of size of delamination, its position along length and location across thickness and rotational speed. Cylindrical shallow shell is favourable compared to spherical and hyperbolic paraboloidal shells as indicated in frequency values. The fall of fundamental frequency has a maximum value when the delamination is located at the neutral plane of a laminated composite cylindrical shell (e.g. $[O_2/\pm30]_S$). In general, the value of fundamental frequency increases with the increase of speed of rotation. Numerical solutions obtained for bending stiff, quasi-isotropic and torsion stiff laminates are the first known non-dimensional fundamental frequencies for the type of analyses carried out here and may serve as the reference solution for future investigations.

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