# LOCAL COMPLIANCE EXPERIMENTS AND CRACK CLOSURE MODELS

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ABSTRACT

The "plasticity-induced crack closure" phenomenon is the leading mechanism which controls the main effects on fatigue crack growth (e.g. stress ratio and load interaction effects) in metallic materials. Experimental tests are usually carried out to quantify the physical phenomenon, but some aspects concerning the elaboration of acquired signals are not yet clear. From the analytical point of view, the so-called Strip Yield model has proven to be the most versatile and powerful tool for estimating crack closure levels, but its application to steels is not yet straightforward. The present work tries to add some new ideas on the elaboration of local compliance experimental data in crack closure analysis simulating experimental P-$\varepsilon_{offset}$ loops by means of an optimised Strip Yield model implementation enriched by a novel and recently presented module based on the Westergaard's elastic complex potentials. Analytical results gave the possibility to investigate some of the different parameters which influence local compliance measurements.

## 1 INTRODUCTION

Plasticity-induced crack closure (CC), originally introduced by Elber (Elber [1]), is the leading mechanism to explain the influence of different factors acting on fatigue crack growth in metallic materials, such as stress ratio (R) effects and load interaction phenomena typically occurring under variable amplitude loading.

Experimental evaluation of CC mainly consists in the determination of the opening load ("$P_{op}$" or "$S_{op}$" respectively in terms of load or stress) of crack surfaces during a single load cycle. Two kind of methods are proposed in literature to achieve this goal: "global methods" (ASTM E647 [2]), based on the analysis of the global compliance of the specimen, and "local methods" (Toyosada *et al.* [3]), based on local compliance measurements near the crack tip. Particularly (Fig. 1), the latter consists firstly in the acquisition of a P-$\varepsilon$ loop that eventually is transformed in a P-$\varepsilon_{offset}$ loop by means of a regression line drawn onto the upper linear part of the unloading branch, where the crack is supposed to be completely open. Then, $P_{op}$ value corresponds to the point on the loading branch that presents the same slope of the points just under the maximum load on the unloading branch. The typical problems that arise during the elaboration and analysis of acquired signals are: i) smoothing the (always present) noise; ii) the definition of the portion of the unloading branch on which define the regression line useful to transform the P-$\varepsilon$ loop in the P-$\varepsilon_{offset}$ loop. The first problem has already been faced in a previous work (Skorupa *et al.* [4]), while some considerations on the second topic are presented in this paper.

Among many existing analytical models to account for CC effects, the so called Strip Yield (SY) model, first developed by Newman (Newman [5]), is the most flexible and powerful tool since it allows to compute crack opening levels under arbitrary load histories (Skorupa [6]). The key point in SY models is the evaluation of the amount of constraint on yielding at the crack tip, quantified by the so called "constraint factor" ($\alpha$). Estimation of the constraint factor is usually done on the basis of crack growth tests carried out at different R-ratios during which CC levels are measured: the correct $\alpha$ value is the one matching the SY model predictions with the experimental data on CC and crack growth. Recent results by the Author (Beretta *et al.* [7]) show that, in order to correctly evaluate CC in structural steels, local compliance measurements should be performed. Furthermore, a novel module (Beretta and Carboni [8]) based on Westergaard's elastic complex potentials (Westergaard [9]) has been added to the SY model in order to determine displacements of any given point in the region of crack tip. An optimised SY model implementation (Beretta *et al.* [10]) together with this new module will be so adopted in the present paper in order to simulate the previously determined experimental P-$\varepsilon_{offset}$ loops and to investigate some of the different parameters which influence local compliance measurements.

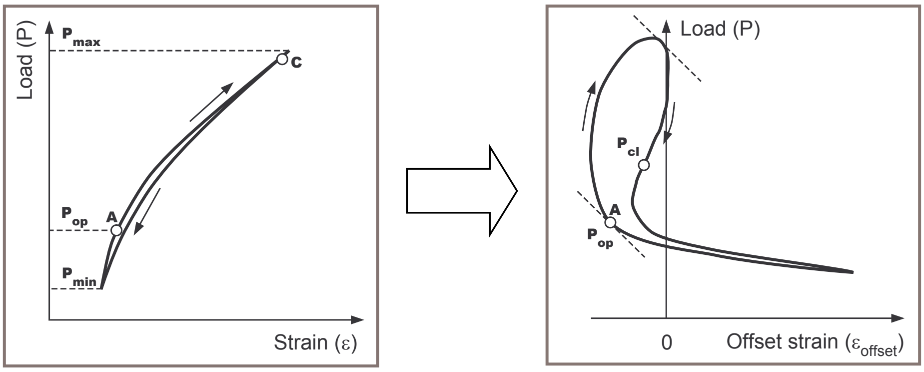

Figure 1: Elaboration of local compliance P-ε loops in P-$\varepsilon_{offset}$ loops.

## 2 EXPERIMENTAL PROCEDURE

Local compliance measurements have been carried out, during propagation tests, on M(T) specimens (width 80 mm, thickness 9 mm and notch length 12 mm) made in a S275J2G3 (PN-EN 10025) mild structural steel characterised by $\sigma_{y,monotonic}$=320 MPa, $\sigma_{y,cyclic}$=240 MPa and $\sigma_u$=475 MPa (Carboni [11]). Tests were carried out at constant amplitude and at stress ratio equal to –1, 0.1, 0.3, 0.5 and 0.7. Fig. 2 shows the typical gluing of uni-axial strain gages across the expected crack path. More details associated with CC measurements and experimental $P_{op}$ estimation are provided elsewhere (Beretta *et al.* [7]).

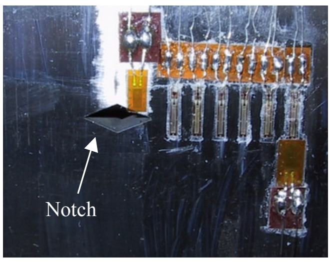

Figure 2: Local strain gages glued along the expected crack path.

## 3 ANALYTICAL MODEL

SY models have been defined and are used to simulate and predict crack propagation in structural components keeping the influence of CC. Particularly, the dimension of the plastic zone is determined applying the Dugdale's theory (Dugdale [12]). Then the plastic zone is discretised in elements that break during propagation and that, remaining plastically deformed, originate the plastic wake. The entity of the stress internal to a given element can be achieved by equilibrium and compatibility equations of the whole system and is a function of the remote applied load, of the geometry and of the material. The opening load is then computed resolving the SIF equilibrium of the system at the crack tip.

The drawback of this approach, a said before, is that 2D SY model results are strongly influenced by the constraint factor $\alpha$. Particularly, the yield stress for the elements is supposed to be $\alpha \cdot \sigma_y$, where $\alpha$ can vary between 1 (pure plane stress) and 3 (pure plain strain). There is not a definite rule for finding the correct $\alpha$ value: it should be chosen as the one which predict $P_{op}$ estimates close to experimental results. From this point of view, it's important to add that different constraint concepts have been proposed in literature: the original Newman's model considers only one constraint (here called $\alpha_t$, relative to tensile yielding in the plastic zone), while three different constraint factors ($\alpha_t$, $\alpha_c$ for compressive yielding in the plastic zone and $\alpha_w$ for compressive yielding in the plastic wake) have been considered in the present SY model. Particularly, the simple constraint concept adopted here chooses $\alpha_c=\alpha_t$ and $\alpha_w \approx 1/\alpha_t$, where $\alpha_t$ can be defined from expressions available in literature (Guo *et al.* [13]). This constraint concept has proven to be effective for steady state conditions, but not for variable amplitude loads: in this case other formulations seem to produce better results (NASA [14], Skorupa *et al.* [15]).

Westergaard's elastic complex potentials Z are particular solutions for the Airy's stress function. Since the SY model can provide information only for points constituting crack surfaces and the plastic zone, the complex potentials permit to derive all the stress components and the entity of the displacements for any given (x,y) point in the crack region. Different Z solutions are available in literature depending on crack geometry.

The analytical tool, adopted in the present paper, discretises the load cycle in steps and for each of them it calculates, by the SY model, the entity of contact stresses ($\sigma_a$ and $\sigma_\rho$) between crack surfaces (Fig. 3).

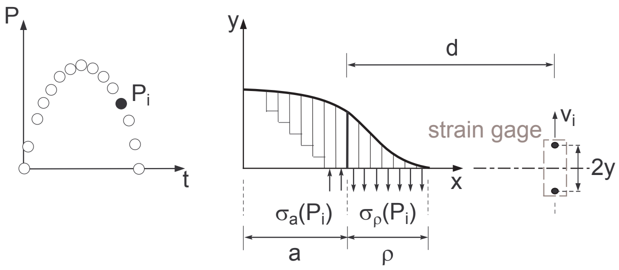


Figure 3: Simulation of P-ε loops ahead of the crack tip.

These stresses are then introduced in the complex potentials expressions, together with the remote applied stress:

$$Z(P_i) = Z_{P_i} + Z_{\sigma_a(P_i)} + Z_{\sigma_\rho(P_i)}. \tag{1}$$

Eventually, the instantaneous vertical displacement $v_i$ (i.e. in the direction of adopted uni-axial strain gages) for any given point (x, y) of the region of the crack tip can be derived (McKellar *et al.* [16]):

$$v_i = \frac{2 \cdot \mathrm{Im}\left[Z^*(P_i)\right] - y(1+\nu) \cdot \mathrm{Re}\left[Z(P_i)\right]}{E} \tag{2}$$

where Z* is the integration of Z on the complex variable z=x+iy, E and ν are the Young and Poisson moduli, "Im" and "Re" are the imaginary and real part of a complex number and y is the vertical co-ordinate of the chosen point of analysis. Finally, repeating the procedure for all the $P_i$ values, the correspondent strains are calculated so that a complete theoretical P-ε loop is determined (Beretta *et al.* [7]).

*3.1 Application of the analytical model to constraint factors calibration*

An important application of the analytical tool is the possibility to calibrate the constraint factors by comparison with experimental P-$\varepsilon_{offset}$ loops. Fig. 4 shows, as an example, the comparison for a=13.7 mm, R=0.1 and $S_{max}$=64 MPa, in a point positioned 0.4 mm away from the crack tip (x=14.1 mm and strain gage base 2 mm). The analytical results coming from both the original Newman's constraint concept ($\alpha_t$=2.5 and $\alpha_c$=$\alpha_w$=1) and the symmetrical constraint concept here adopted ($\alpha_t$=$\alpha_c$=2.05 and $\alpha_w$=0.45) are reported.

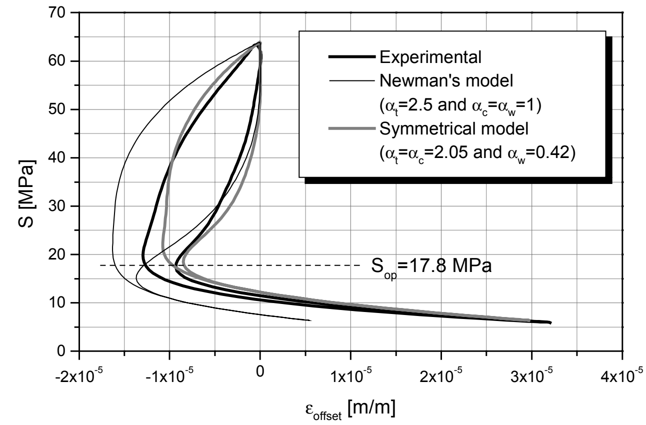


Figure 4: An example of simulation of P-$\varepsilon_{offset}$ loops (a=13.7 mm, R=0.1, $S_{max}$=64 MPa, x=14.1 mm and strain gage base 2 mm).

As it can be seen, even if the $P_{op}$ values produced by the analytical concepts correspond to the experimental one, the loop shape seems much better in the case of the symmetrical concept. This is due to the fact that there are infinite α triplets that produce the right opening load value, but just a little amount of them can contemporaneously give a good match of the loop shape. Moreover, it is intuitive that a good match of loop shapes automatically satisfies the $P_{op}$ requests, with the advantage that, in this way, constraint factor calibration is based on a physical foundation.

## 4 ANALYSIS OF LOCAL COMPLIANCE MEASUREMENTS

The analytical tool permitted firstly to investigate the influence of the distance between crack tip and strain gage. Fig. 5 shows the variation of analytical loop shapes for the cases of R=0.1 and R=0.5. As it can be seen and as expected considering "local" compliance acquisitions, the sensitivity of the method (i.e. the hysteresis on which relies $P_{op}$ estimation) gets lower and lower increasing the distance of the strain gage. This fact suggests that a criterion for applicability should be further investigated. Particularly, the lower bound can be fixed in terms of plastic zone dimension, inside which large deformation prevents reliable measurements with strain gages. The maximum distance corresponds to a significant width of the hysteresis loops: as a first instance from Fig. 5 data, it appears that a "wide" hysteresis loop is achieved till a distance equal to $10 \cdot r_p$, where $r_p$ is the dimension of the process zone computed by the SY model. A more systematic study of this parameter is yet being carried out.

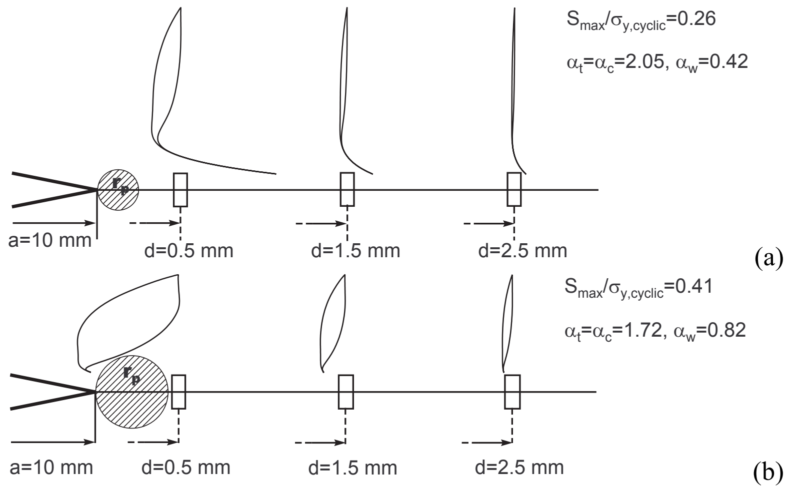


Figure 5: Influence of the distance between crack tip and strain gage: a) case of R=0.1; b) case of R=0.5. (dashed area corresponds to the "process zone" computed by the SY model)

A second aspect to consider is that to univocally calibrate the constraint factors from local measurements, it's necessary to know (estimate) *a priori* the entity of the linear part ($\Delta P_{linear}$) of the experimental unloading branch. As an example, Fig 6a shows the effect, in terms of the portion (10%, 20% and 30%) of the unloading branch on which the regression line used for the offset transformation is defined, on the experimental loop already shown in Fig. 4. Even if the $P_{op}$ value is the same for all the considered cases, the loop shape is significantly different: this means that constraint factors results should be different, too.

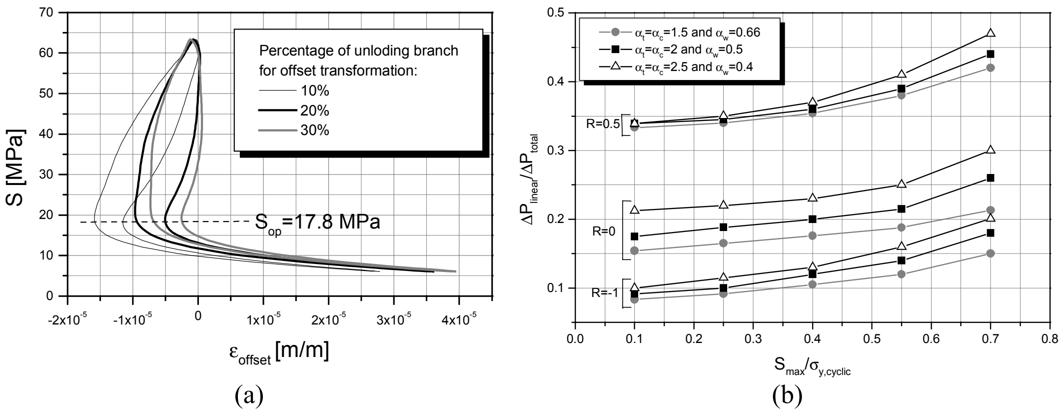


Figure 6: Effect of different definitions of regression line on P-$\varepsilon_{offset}$ loops transformation: a) case of a=13.7 mm, R=0.1, $S_{max}$=64 MPa, x=14.1 mm and strain gage base 2 mm; b) results obtained in terms of $\Delta P_{linear}$ as a function R, $\alpha$ and $S_{max}$.

In order to define a rule for offset transformation, the analytical procedure has been applied to determine $\Delta P_{linear}$ as a function of $\alpha$ and maximum applied stress for different R values. Particularly, since for the considered material it was observed (Carboni [11]) that typical calibrated values of $\alpha_t$ are between 1.5 and 2.5, values of this parameter equal to 1.5, 2 and 2.5 were considered together the symmetrical constraint concept. Results are shown in Fig. 6b: it has been found that in order to apply an offset transformation always valid in respect to different R and $S_{max}$ conditions, it's necessary to set a $\Delta P_{linear}$ included between 8% and 15% of the $\Delta P_{total}$.

## 5 CONCLUDING REMARKS

Elaboration of experimental local compliance measurements of crack closure has been addressed. Particularly, a previously proposed analytical tool, formed by an optimised Strip Yield model together with a module implementing the Westergaard's elastic complex potentials and able to simulate experimental P-$\varepsilon_{offset}$ loops, has permitted to investigate some of the different parameters which influence local compliance measurements.

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