MODELING DAMAGE LINKAGE IN HETEROGENEOUS ALLOYS AND METAL MATRIX COMPOSITES

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ABSTRACT
Predicting the ductility of two-phase alloys and metal matrix composites has been an outstanding problem in materials science for many years. One of the most challenging aspects of this problem involves the final coalescence of damage leading to component failure. This is a largely stochastic process and not readily amenable to analytical modeling approaches. In recent years we have developed a series of models for the development of damage – in the form of particle cracking – and its influence on tensile deformation. The approach we have used is based on self-consistent methods using an incremental effective medium approach. These models have tended to correlate well with measured tensile curves but overpredict the onset of tensile instability and thus material ductility. More recently we have added a microcrack coalescence step to the models. This is based on a localized version of the Considere criterion in which coalescence is assumed to occur once the local ligament stress falls below the global work hardening rate of the alloy or composite. The material is assumed to contain a random distribution of particles which crack according to Weibull statistics. Thus both the stochastic nature of particle damage and of coalescence and incorporated into the averaging process. The model gives rather good predictions of ductility.

INTRODUCTION
The fracture behaviour of ductile alloys and of composites which utilize ductile matrices involves a process of damage accumulation. This process is thought to consist of three separate (although overlapping) stages – namely void nucleation, growth and coalescence. This process has been extensively modeled over several decades. These models address the progressive loss of the mechanical resistance through either the nucleation and growth of voids [1,2] or the cracking of the second phase particles [3,4]. The models are however generally deficient in their treatment of damage coalescence. There are several reasons for this. Coalescence is first of all a stochastic process. This makes it difficult to observe experimentally and more difficult to treat theoretically as well. The classical model for damage coalescence is that by Brown and Embury [5]. This is based on a simple geometric concept; namely that voids coalesce once the void length along the tensile axis is equal to the lateral spacing between adjacent voids. It assumes that void linkage occurs through shear localization. This model is however, not universally applicable. One key example, from the perspective of metal matrix composites, involves systems in which damage occurs primarily through particle cracking. In these systems (see Fig. 1 for example) the particles crack; however, these do not expand significantly in the tensile direction. Nor do they propagate into the matrix until the final stage of the fracture process. Such voids never meet the Brown and Embury condition for coalescence.
Figure 1: Damaged (cracked) particles near the fracture surface of an Al-20%Si alloy. The particles are essentially pure silicon [12].

Over the past several years we have developed a suite of models which successfully describe the strength and tensile behaviour of particulate metal matrix composites [3, 5-8]. These models utilize a self-consistent methodology based on a tangent modulus construction that lends itself to incremental analysis. The first phase of this effort focused on the effect of particle clustering on strength and work hardening [6]. These models show how particle clustering enhances the work hardening behaviour as compared to composites with the same particulate volume fraction in a uniform distribution. This concept has been tested experimentally through the development of model bimodal materials based on Al-Cu alloys which can be thermomechanically processed to generate CuAl\textsubscript{2} particles [7]. Figure 2 shows the stress-strain curves for two composites based on the Al-17wt\%Cu alloy (resulting is about 20vol\% CuAl\textsubscript{2}). These samples have been tested in compression so as to minimize the effect of damage. In one material the Cu content is uniform, while the other contains a mixture of 10 and 24wt\% regions. The clustered material is clearly stronger to an extent that is well predicted by the model. In this case the behaviour is modeled in two stages. In the first we develop a model for a uniform composite containing a given particle volume fraction. For the non-uniform composites a second stage is added in which each volume fraction region is treated as a continuum “phase” with constitutive behaviour as determined in the first stage.

Later models were developed to incorporate the effect of damage by means of particle cracking. The models predict the loss of material stiffness and strength resulting from particle fracture [3]. As expected the addition of damage to such a model lowers the strength of the material considerably. What is less obvious though is the effect of particle clustering on damage. In general, since particle cracking is stress-controlled and load shedding results in higher stresses within high particle volume fraction regions, damage occurs preferentially in these regions. This tends to compensate for the enhanced strengthening associated with cluster in the absence of damage. In fact our results suggest that the stress-strain response of a clustered composite which undergoes damage by particle cracking will not be significantly different from a uniform composite of the same overall composition. This is again borne out by experiment in the Al-Cu model material – tested now in tension in order to allow for particle damage [7]. Here we find that the uniform and clustered materials now have very similar stress-strain behaviour. Moreover the
models give rather good predictions of the work hardening behaviour of the composite up to strain very close to fracture. All initial material parameters follow those established by Maire et al.[8].

Despite the success of these models they tend to over predict the experimentally observed ductility – by at least a factor of two. Thus in our most recent work we have attempted to address this deficiency by incorporating a realistic description of the micro-crack linkage process at the local scale such that the accumulation of linked cracks precipitates the ultimate failure of the material. A statistical approach is used which accounts for the heterogeneous distribution of cracked particles. This is incorporated into a self-consistent model for deformation or damage. The new model leads to excellent predictions of both the stress-strain response and ductility under tensile testing.

A SELF-CONSISTENT MODEL INCORPORATING DAMAGE LINKAGE

We use an incremental self-consistent formulation based on a tangent modulus construction. At each increment of strain, the average value of particle stress within each phase is calculated. This value is then used to determine the fraction of particles that have fractured based on Weibull statistics. The damaged material is distinguished from the undamaged material by treating it as an additional phase with properties defined through the results of finite element calculations by Brockenbrough and Zok [4].

The increment in damage is assumed to occur instantaneously at constant total plastic strain. Since the elastic modulus of the damaged region is much lower than that of the same region without damage, the composite modulus and as a result the composite stress is reduced due to damage. The stress reduction in each phase and in the composite is calculated following the formulation of Wilkinson et al.[3] through a process of elastically unloading at constant total plastic strain.
Observation near the fracture surface of a material that damages primarily through particle cracking reveals that some damage coalescence occurs, through a form of micro-crack linkage between adjacent cracked particles. In multi-phase materials both the work hardening rate of the matrix and the local stress state influence the local fracture of the matrix in the ligament between particles. When the reinforcing particle size is well within the continuum range, the work hardening of the composite is governed by both dislocation accumulation and by the apparent work hardening (i.e. the plastic constraint) resulting from the stress partitioning between the ductile matrix and the rigid reinforcement. When the particle spacing is below a critical distance in which high stress triaxiality can develop, the apparent work hardening of the matrix locally is further enhanced through elevated plastic constraint. However, once the reinforcement fractures, the work hardening capability of the matrix between particles resulting from the stress partitioning and plastic constraint is significantly diminished.

A model for the linkage of damage between adjacent cracked particles can be developed which incorporates Thomason’s [9] concept of a localization mechanism, involving the unstable failure of the inter-particle matrix, either through localized shear or load limit failure. In this case, internal necking or localized instability may be rationalized by considering the plastic constraint that the reinforcing particles impose on the matrix. Once the particles fracture this plastic constraint is lost and the stress acting on the matrix between the two cracked particles can exceed the fracture stress of the matrix. This can be approximated by setting the far field work hardening rate, $\theta$, equal to the stress acting between two penny shaped cracks separated a distance, $\lambda$, apart:

$$\theta = \sigma_a \left[ 1 + \alpha \frac{a}{\lambda} \right].$$

The value of $a$ is equal to the average particle diameter as it is assumed, and experimentally observed [8], that particles in these composites fracture across their entire diameter. The term $\alpha$ is a stress concentration factor of the order 2 and $\sigma_a$ is the far field applied stress of the participating phase. We assume that the distribution of particle spacing may be treated using the Poisson distribution. From this we can find the fraction of cracked particles at each strain and thus the distribution function for the spacing of cracked particles [10]. This is illustrated in Figure 3. As strain is increased, the stress in the particle increases. Due to Weibull statistics this results in an increase in the volume fraction of cracked particles. The Poisson distribution progressively narrows while the mean cracked particle spacing decreases.

With each increment of load and damage, the Poisson distribution shifts to the left and narrows while the fraction of linked particles sweeps to the right. To set a lower bound, it is assumed that once linkage between two particles occurs, the particles and the matrix between them can no longer sustain loading and thus this region possess zero stiffness. Using this approach a phase is essentially treated as being composed of three regions – the undamaged region, a damaged region possessing damage in the form of cracked particles and linked cracked particles, as illustrated in Figure 5.
The experimentally measured mechanical response of the AA2618/SiC system can now be compared to the model predictions. In Fig. 4 we plot the work hardening rate predicted by various version of the model compared with experimental data for the material with 18 vol% SiC particles. All of the models fit the data rather well up to a strain of about 2%. However, the model without damage coalescence predicts a gradual loss of work hardening and continued deformation without any onset of tensile instability until quite high strain. Experimentally however we observe an increase in the slope of the hardening curve. This suggests that a new damage process such as coalescence must be occurring. The coalescence model predicts the failure strain rather well. Moreover, it also captures in the increased negative slope.

Incorporations of the micro-crack linkage model greatly improves upon the prediction of flow response provided by the original SCEMA damage model. In particular, when comparing the models with the experimental material used in the original study of Maire et al.[8] the agreement with the new model is exceptionally good, particularly for the composite of the highest volume fraction.

This description suggests that the linkage event between cracked particles occurs when the stress acting on the remaining ligament exceeds the work hardening capacity of the matrix. It is important to note that this provides a lower bound to composite ductility since it is assumed that the work hardening rate of the matrix material between the two cracked particles is equal to that of the unreinforced matrix. Moreover, the model assumes that damage linkage continues to proceed in a spatially random fashion up to the point of fracture. In point of fact one would expect that as the number of damage linkage events increases a local instability will develop somewhere in the

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**Figure 3**: Conceptual illustration of the progressive linkage of cracks in the damaged region of the concept as described by the micro-crack linkage model. Each “phase” obeys a somewhat different constitutive behaviour which recognizes the effect of particle cracking and damage linkage on the strength of the composite.

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![Diagram](image-url)
material leading to the development of a rapidly propagating crack. Our conclusion is that while this indeed must happen it does so only once the life of the material is almost completely exhausted.

The model suggests that the spatial distribution of particles plays a significant role in the damage process. Thus the version that assumes a uniform particle distribution, utilizing the same micro-crack linkage model, yields results that fail to capture the experimentally observed sensitivity to volume fraction and significantly over-predict the strain at which failure occurs for the composite (by a factor of three or so at high volume fraction). Assuming a random distribution of particles however leads to excellent predictions of ductility in the high volume fraction composite. It also captures the additional loss in work hardening experimentally observed in the 18% volume fraction composite at strains near failure.

Comparing the modeling predictions to the experimental data for composites with 10 and 18 vol% particles suggests that some additional features may be important at lower volume fractions. This is because the model predictions tend to underestimate the ductility at the lower volume fraction (even though the error remains much smaller than with the previous models). This may reflect to effects related to the local distribution of damaged particles and tortuosity of the fracture path as a function of particle volume fraction. Particle clustering scales with volume fraction and as the volume fraction increases, the fraction of particles located within particle rich regions also rises. Our model assumes that linkage occurs when the separation of cracked particles is below a threshold value, independent of the relative orientation between the particles. In reality however, particle linkage will be favoured for particle pairs that are separated by a low energy shear plane at say 45°. At low volume fractions such planes may be harder to find, resulting in a delay in the final linkage and a small but significant increase in ductility. This buffers the sensitivity of micro-crack linkage to the orientation of the cracked particles relative to the normal stress axis. Once the volume fraction of particles exceeds a threshold value, the sensitivity to the local orientation of particles diminishes and the micro-crack linkage process provides a stronger effect on the global loss of work hardening.

REFERENCES