CONSTRAINT-DEPENDENT FRACTURE TOUGHNESS OF GLASS AND PMMA

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ABSTRACT
Data on Mode I fracture collected in an extensive program of uniaxial and biaxial tests of glass and PMMA specimens are compared in an effort to quantify uniformly the constraint effects in brittle and quasi-brittle materials. We seek to refine our understanding of the interplay between of the constraint-related parameters of crack growth resistance tests and the results of single-point characterization of fracture toughness. The long-term aim is to develop a sufficiently general engineering approach allowing one to predict the critical values of loads and displacements and the amounts of the subcritical crack extension in plates and shells made from brittle and ductile metallic or nonmetallic materials. It is demonstrate that in general terms, the changes of the plane strain fracture toughness characteristics with constraint may occur in a roughly similar manner for brittle, quasi-brittle, and ductile materials.

1 INTRODUCTION

Since Griffith's pioneering work [1], the problem of a sufficiently large crack growing along a single plane in transparent glassy materials is of great scientific interest. Being the best model materials, they allow studying the fundamentals of the near-crack-front fracture apart from the complexities that are intrinsic to cracking in heterogeneous and disordered materials like concrete. This paper deals with experimental data on Mode I crack growth in silicate- and polymethylmethacrylate (PMMA)-based glasses. The data collected on a wide range of specimen geometries are analysed in terms of the Conventional Methodology (CM) of the linear elastic fracture mechanics. A basic inadequacy of the CM to describe uniformly the Mode I fracture in tension and compression necessitates the use of a new semi-analytical approach called the Unified Methodology (UM) of fracture investigation by Naumenko et al. [2-6]. The central issue of the UM is a search for a common way of quantifying the stable propagation of a through-the-thickness crack for the cases of uniaxial and biaxial loading in tension and/or compression (Fig. 1).

The CM approach relies on the modelling of a crack by a mathematical cut (Fig. 1b) and single-point representation of plane strain fracture toughness by the critical value K_{Ic} of the stress intensity factor K_I. This value is assumed to be a material property when a specimen meets the size requirements set forth in the ASTM E399. To explain the constraint effect in brittle fracture, the notion of "apparent fracture toughness K_c" has been incorporated in updated analyses. According to the recent studies by Chao and Zhang [7], Chao et al. [8], and Liu and Chao [9], the K_c values obtained on PMMA specimens vary significantly with the crack-tip constraint. The latter for strain-controlled fracture was represented by the elastic T stress term in Williams' asymptotic series solution. For a through crack, T is the constant stress acting parallel to the crack line and its magnitude under tensile loading is proportional to the applied stress σ.

In the UM, attention is focused on the change of the entire crack border geometry during the Steady State Tearing (SST) in the typical structural element, that is, in the unconstrained rectangular plate shown in Fig.1. The mechanical behaviour and fracture criteria are expressed in terms of the remote stresses σ and q, displacements of the extreme points m and n on the inner boundary of this element and M and N on its outer boundary. The main distinction between the CM and UM is in using the conceptually different model descriptions of brittle fracture.

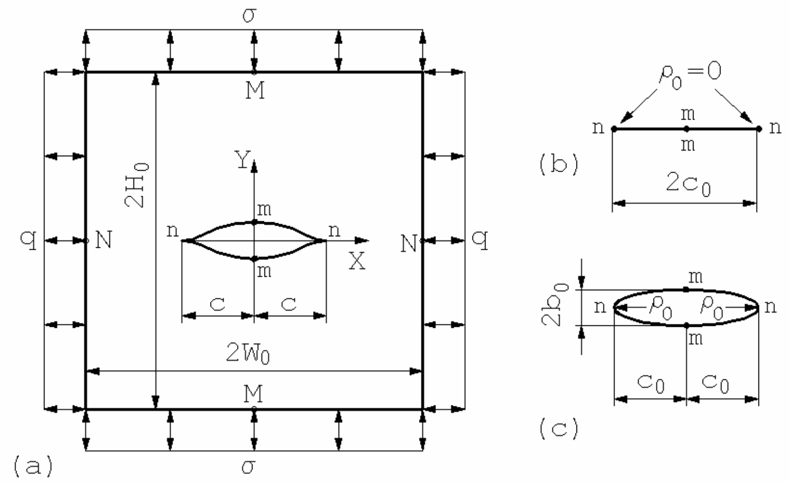

Figure 1: Centre-cracked plate under the action of uniform boundary stresses (a) and geometries of ideal cracks representing an actual crack in a stress-free plate, which are basic for the CM (b) and UM (c) analyses.

2 THE UM MODEL DESCRIPTION OF BRITTLE FRACTURE

Unnotched specimens made from glass and PMMA both fail under uniaxial tensile loading in a brittle manner. The mirror-like fracture surfaces are externally identical and the fracture toughness characteristics of these materials are comparable. However, in transition to uniaxial compression, when only the sign of the applied load is changed, the behaviour of the specimens becomes diametrically opposite. Glass retains all the features of brittle fracture by Mode I crack growth along the loading line, whereas the pronounced barreling of the PMMA specimens indicates that the ductility of this material is incomparably higher. Such a type of the brittle-ductile transition was reported in numerous investigations into the mechanical behaviour of amorphous polymers under hydrostatic pressure. An example of the stress-strain response and fracture of PMMA under multiaxial loading in tension and lateral pressure is described in [10].

The term "brittle" is used in the UM to denote decohesion that occurs without crack-tip blunting and localized necking, i.e., without plastic deformation. We treat the SST in silicate-based glasses as brittle fracture when the active damage zone adjacent to the Fracture Process Zone (FPZ) is negligibly small in size. In the case of quasi-brittle fracture, plasticity (crazing in PMMA) develops concomitantly within the active damage zone, but it is not treated as an essential part of the material decohesion process. The following practical classification of the glassy materials has been incorporated in the UM fracture analysis [2]. In brittle materials, the Mode I fracture process can be initiated from inherent structural defects under both uniform uniaxial tension and uniform uniaxial compression. In quasi-brittle materials, the same process can be initiated only by tensile loading and not by compressive one. Thus, in contrast to the usual practice of utilizing PMMA as a brittle material [7-9, 11, 12], we consider that PMMA is a quasi-brittle material. In tests under monotonically increasing load, PMMA exhibits acceleration of slow (visco-elastic) crack growth if K_I attains a certain level (about 0.6-0.9 MPa $m^{1/2}$) designated K_{Is} [13-15].

It is instructive to outline the UM approach in terms of the energy exchange during brittle fracture of a typical structural element (Fig. 1) at a prescribed value of the stress biaxiality ratio $k = q / \sigma$. The following hypothesis was taken as a starting point: the physical essence of the

Mode I cracking micromechanisms does not depend on the sign and value of the stress biaxiality ratio k. This suggests that the model descriptions of the SST in the tension-dominant and compression-dominant crack geometries should be identical. The simplest and still physically relevant description is to treat the SST crack growth as the process of omnidirectional extension of an ideal crack in the form of an elliptic hole (Fig. 1c). The latter in the stress-free plate has a fixed radius of its tips $\rho_0 = b_0^2 / c_0$, where b_0 and c_0 are the minor and major semi-axes of the hole. For an isolated SST crack of size $c_s \le 0.1 \cdot W_0$, where $W_0 \le H_0$, the potential energy release rate $J_s(k)$ and the value $K_{Is}(k)$ of the effective stress intensity factor $K_I(k)$ are given by [2, 3]

$$J_s(k) = \frac{\pi \cdot \sigma_s^2 \cdot c_s}{E'} \cdot \left[1 + 0.5 \cdot \frac{\rho_0}{c_s} \cdot k^2 + 0.375 \cdot \left(\frac{\rho_0}{c_s} \right)^{1/2} \cdot (1-k)^2 \right] = \frac{\pi \cdot \sigma_s^2 \cdot l_s(k)}{E'} = \frac{K_{Is}^2(k)}{E'}, \quad (1)$$

where $E' = E$ for the plane stress state and $E' = E / (1 - \nu^2)$ for the plane strain, E is the elastic modulus, ν is the Poisson ratio, and $l(k)$ is the half-length of an effective mathematical cut ($\rho_0 = 0$).

During brittle fracture all energy dissipation is associated with the creation of new free surfaces in the FPZ. The behaviour of an idealized cohesive zone was accepted [2, 3] in the form

$$J_s(k) = \Gamma_s(k) = \int_0^{\delta_s(k)} p(\delta) d\delta = p_s(k) \cdot \delta_s(k) \ . \quad (2)$$

Here $\Gamma_s(k)$ is the cohesive fracture energy appropriate for the traction-separation law $p(\delta)$, δ is the relative displacement of the FPZ faces, and $p_s(k)$ is the constant level of the cohesive stress during the SST stage. The parameters $\Gamma_s(k)$, $p_s(k)$, and $\delta_s(k)$ have a definite physical meaning when, at least, two of them can be determined experimentally. The input data must be collected from the so-called basic and additional tests of the UM. The basic tests of centre-cracked specimens provide constraint-dependent values of $\Gamma_s(k)$ and the additional tests determine the ultimate tensile stress σ_{ult} of the material. The tensile tests of glass whiskers containing inherent structural defects of minimally possible size were adopted as the most practical route for estimating the value of σ_{ult}. It is assumed that the cohesive strength $p_s(k) = p_s = \sigma_{ult}$ for a damage-free material. Specifically, based on the analysis of literature on the strength of silicate-based glasses, we took the constraint-independent characterstic p_s to be equal to 13.74 GPa [2, 3].

3 TEST RESULTS AND DISCUSSION

Different kinds of test specimens containing a single through-the-thickness or part-through-the-thickness crack with a well-defined initial front line are shown in Fig. 2. In glass specimens, the value $(dc / dt)_s$ of crack growth rate upon attaining the characteristic state "*s*" depends on the constraint level and varies roughly in the range from $1 \cdot 10^{-5}$ to $1 \cdot 10^{-3}$ m/s [3, 14,16]. Similar data for PMMA presented in Table 1 are of limited occurrence [13, 14]. The K_{Is} characterisitics of PMMA are to be in line with the K_c and K_{Ic} values reported in [7-9]. The latter, if determined in accordance with the ASTM E399, are more closely associated with the initiation of a stable crack growth than with the instability event "*c*". It must be emphasized that the transition from stable to unstable fracture in tensile loading is marked by an easily distinguished line on the exposed fracture surface of glass and PMMA. At the same time, there is no evidence of attaining the crack instability event "*c*" under uniform compressive loading.

Experimental results in Table 1 taken together with the earlier data on crack growth in glass and PMMA partly presented in [2, 3, 13-16] demonstrate that (i) fracture toughness value K_{Is} determined for bend-dominant crack geometries are systematically lower then those obtained for

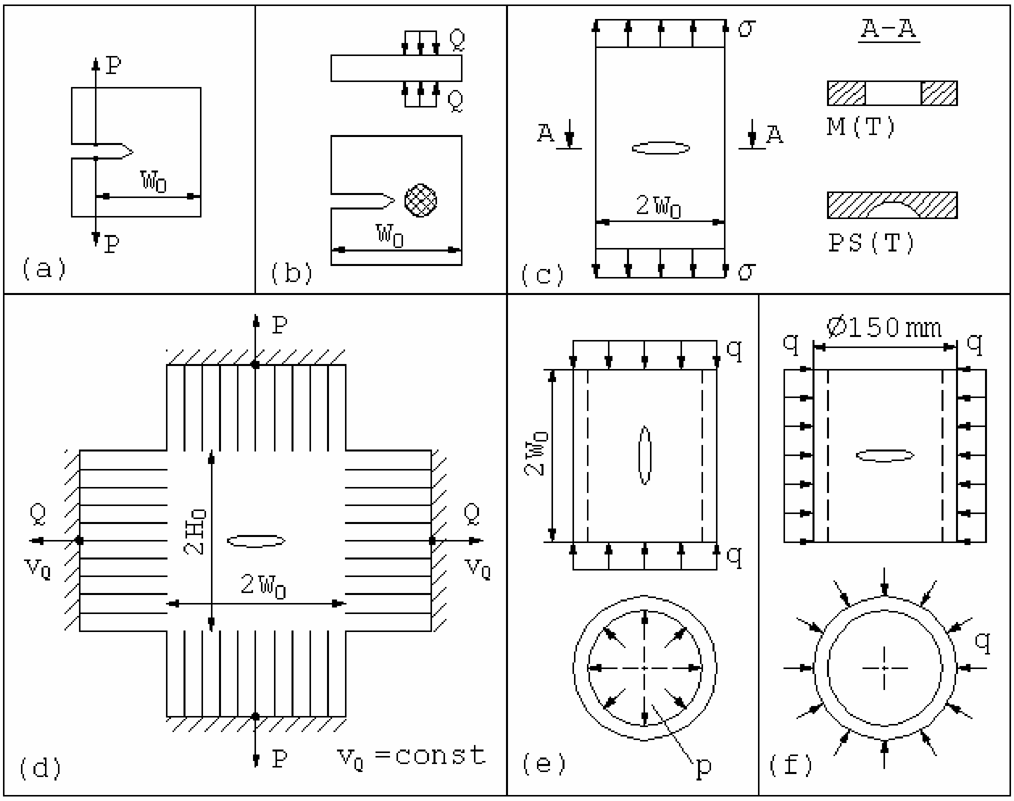

Figure 2: Schemes of the specimen geometries and loading designated as *a* – C(T), *b* – C(LC), *c* - M(T) and PS(T), *d* – MM(T-TC), *e* – ML(IP-C), *f* – MT(EP).

tension- and compression-dominant crack geometries; (ii) under conditions of the superimposed compressive stress *q* acting parallel to a growing crack, the fracture toughness of both materials enhances, and (iii) the tensile stress *q*, conversely, reduces the K_{Is} values for glass and PMMA. These findings agree with the direction of constraint-dependent variations in the fracture toughness of brittle and ductile metallic materials. The case in point are the well-established dependencies of the type J_{Ic} - (T / σ_Y) or J_{Ic} - Q, where Q is the constraint parameter of N.P. O'Dowd and C.F. Shih. In the light of this consensus with respect to the constraint effect in metallic and nonmetallic materials, the surprising thing is the literature data on the constraint-dependent fracture toughness of PMMA. Thus, the transition from high-constrained to low-constrained crack geometries was reported to increase [11], decrease [7-9] or leave unchanged [12] the K_{Ic} or K_c values. This disagreement suggests that more attention should be directed towards the reliable evaluation of the constraint-dependent fracture toughness. Some difficulties in interpreting the test data may be obviated by using the multi-point characterization of the resistance to brittle fracture [13-16].

As an illustration, the constraint-dependent parameter $\delta_s(k)$ of the FPZ in glass can be determined by reference to Eq. (2). For the values of (T / σ) = -1.0, -7.0, -13.0 and $-\infty$ in Table 1, this simplest traction-separation law gives $\delta_s(k)$ = 0.23, 0.67, 2.09, and 14.82 nm, respectively. The length scale of the crack-tip opening displacement $\delta_s(k)$ in glass agrees, at least in the order of magnitude, with the experimental data of Wiederhorn et al. [17]. Overlaying the two sections taken from each surface of the crack in the glass specimen, shows that they replicate one another

TABLE 1: Averaged values of test parameters and fracture toughness for glass and PMMA.

Specimen and k ratio	B_0 mm	W_0 or $2W_0$ mm	T/σ	$(dc/dt)_S$ m / s	K_{Is} MPa m$^{1/2}$
Silicate-based glasses					
C(T) $k = 0$	10.0	120	6.0	$5\cdot10^{-5}$	0.50
C(LC) $k = 0$	3.0	250	…	$5\cdot10^{-5} - 1\cdot10^{-4}$	0.37
ML(IP-C) $k = 0$	4.2 – 4.8	750	- 1.0	$2\cdot10^{-5}$	0.52 / 0.53[a]
$k = -6$			- 7.0	$2\cdot10^{-4}$	0.54 / 0.91[a]
$k = -12$			- 13	$6\cdot10^{-4}$	0.59 / 1.60[a]
MT(EP) $k = -\infty$	4.2 – 4.8	750	$-\infty$	$1\cdot10^{-3}$-$1\cdot10^{-4}$	0 / 4.26[a]
PMMA of TOSP grade					
C(T) $k = 0$	8.0	85	6.0	$1\cdot10^{-5}$-$1\cdot10^{-4}$	0.90
C(LC) $k = 0$		85	…	$5\cdot10^{-2}$	0.60
M(T) $k = 0$		100	- 1.1	…	0.80
PS(T) $k = 0$		100	- 0.4	$5\cdot10^{-4}$	0.83
MM(T-TC) $k = 0$		100	- 1.3[b]	…	1.04
$k = 1$			- 0.3[b]		1.00
$k = 2$			0.7[b]		0.97
$k = 4$			2.7[b]		0.92

[a] The value obtained using equation (1).
[b] The value determined for the square plate ($W_0 = H_0$ in Fig.1).

to an estimated accuracy of ±2 nm. Such a strong increase in $\delta_s(k)$ with decreasing constraint level is similar to the constraint effect on δ_c for HY80 steel presented by Hancok and Cowling [18].

4 CONCLUDING REMARKS

Considering the results obtained in the context of both practical application and fundamentals of fracture mechanics, the following conclusions may be drawn: (i) There are the pressing demands for developing a general concept of Mode I cracking in high-constrained and low-constrained crack geometries; (ii) The UM, contrary to the CM, can cope, in a reasonable manner, with the description of brittle fracture at the extremely low level of constraint, that is, under uniform uniaxial compression. It is pertinent to cite here the well-substantiated assertion from [11] "…the theories of Griffith and Irwin are incapable of proper treatment of the biaxial effect"; (iii) The plane strain fracture toughness characteristics of brittle, quasi-brittle, and ductile materials may change with constraint in a roughly similar manner.

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