CRACKING IN PLAIN CONCRETE: A MULTIFIELD APPROACH

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ABSTRACT

Traditional phenomenological constitutive relationships sometimes fail in the description of mechanical behaviour of plain concrete. In such circumstances more refined models are necessary, which takes into account the multiphase structure of the material. This paper presents a generalised finite element formulation, which incorporates solid and fluid phases together with a temperature field. The model is developed to obtain time-dependent solutions of 2-D cases, such as concrete gravity dams subjected to loading-unloading cycles, non-homogeneous specimens subjected to thermo-mechanical effects, etc. A fully coupled cohesive-fracture discrete model, which includes thermal and hydraulic loads, is adopted to describe crack nucleation and propagation. The evolution of fractures leads to continuous topological changes of the domain and these are handled by systematic local remeshing of the domain and by a continuous change of fluid and thermal boundary conditions. In the adopted approach, cracks may nucleate everywhere depending only on the stress field and propagate along paths and with a velocity of the tip that is a priori unknown. The determination of the crack path and the velocity of the tip propagation represent an important part of the solution, as the temperature and stress fields and allows for correct updating of the domain. Governing equations are firstly presented together with their space discretization. The solution procedure is finally discussed in particular as far as the projection of the solution between two successive meshes is concerned.

1 INTRODUCTION

The overall mechanical behaviour of cementious materials as concrete is the result of their complex mesolevel structure and chemical transformations during the ageing process. Traditionally, all these aspects are not considered and phenomenological relationships are introduced for the description of the mechanical behaviour of such materials. Sometimes this *practical* approach is sufficient to explain and model the observed experiences, but in certain circumstances does not. This is for instance the case of the alkali aggregate reactions, which depend on local temperature, humidity and stress and result in macroscopic cracking of massive concrete. Recent progress in computational multifield mechanics, when applied to cementitious materials opens the perspective to obtain not only better estimates of the mechanical behaviour of concrete structures, but also deeper insigth into the degradation as a result of the interaction of the material components (e.g. Meschke and Grasberger [1]).

The present paper describes a numerical model in which cracking of concrete is caused not only by applied loads, as usual, but also by the pressure of water present from the casting operations and/or percolating from the boundaries (hydraulic fracture) and by hydration heat and/or by environmental temperature changes. In the present formulation fully saturated conditions and cohesive crack behaviour are assumed. Crack can nucleate and propagate everywhere depending on the stress field evolution. This problem is handled according to the procedure described in Secchi, Simoni [2].

2 MATHEMATICAL MODEL

The mathematical model follows from Lewis and Schrefler [3]. Within the framework of Biot theory, non-isothermal, quasi-static conditions, small displacements and displacement gradients are assumed. The mechanical behaviour of the solid is dependent on the effective stress σ'_{ij} defined, following Biot and Willis as:

$$\sigma'_{ij} = c_{ijrs}\left(\varepsilon_{rs} - \delta_{rs}\varepsilon_T\right) - \overline{\alpha} p \delta_{ij}, \qquad \varepsilon_T = \frac{\alpha_s}{3} T \tag{1}$$

ε_{rs} being the total strain tensor, p the fluid pressure, δ_{ij} the Kronecker symbol, $\overline{\alpha}=(1-K_T/K_S)$ Biot's coefficient, which accounts for small volumetric strain due to pressure, K_T the bulk modulus of the

overall skeleton and K_S the averaged bulk modulus of solid grains. ε_T is the strain associated to temperature T changes, according to cubic expansion coefficient α_s. A Green-elastic material is assumed with c_{ijrs} elastic coefficients dependent on the strain energy function W.

The linear momentum balance for the mixture (solid plus water), in weak form, hence containing the natural boundary conditions, may be written as:

$$\int_\Omega \delta\varepsilon_{ij}\, c_{ijrs}\varepsilon_{ij}\, d\Omega - \int_\Omega \rho\, \delta u_i g_i\, d\Omega - \int_\Omega \delta\varepsilon_{ij}\, \overline{\alpha}\delta_{ij} p\, d\Omega - \int_\Omega \delta\varepsilon_{ij}\, c_{ijrs}\frac{\alpha_s}{3}\delta_{ij}\, T d\Omega - \int_{\Gamma_e} \delta u_i t_i\, d\Gamma - \int_{\Gamma'} \delta u_i c_i\, d\Gamma' = 0 \tag{2}$$

where Ω is the domain of the initial boundary value problem, Γ_e is the external boundary and Γ' the boundary of the fracture and process zone. $\delta\varepsilon_{ij}$ is the strain associated with virtual displacement δu_i, ρ the density of the mixture, g_i the gravity acceleration vector, t_i the traction on boundary Γ_e and c_i the cohesive tractions on the process zone as defined in the following.

Forced conditions fixed the field variable values along the constrained boundary and completely define the problem. The fracturing material in the process zone generally undergoes mixed mode crack opening, which is modelled Margolin [4] and Dienes [5] proposals.

As far as water transfer mechanism is concerned, Darcy's law with constant absolute permeability is assumed for the fluid fully saturated medium surrounding the fracture. Within the crack the Poiseuille is assumed to be valid: permeability is not dependent on the rock type or stress history, but is defined by crack aperture only. Incorporating Darcy's law, the weak form of the mass balance equation for water in all the domain, except for the fracture zone, is given by:

$$\int_{\overline{\Omega}} \delta p \left\{ \left(\frac{\overline{\alpha} - n}{K_s} + \frac{n}{K_w} \right) \frac{\partial p}{\partial t} + \overline{\alpha}\, v^s_{i,i} - \left[(\overline{\alpha} - n)\alpha_s + n\alpha_w \right] \frac{\partial T}{\partial t} + \left[\frac{k_{ij}}{\mu_w} \left(- p_{,j} + \rho_w g_j \right) \right]_{,i} \right\} d\Omega - \int_{\overline{\Omega}} (\delta p)_{,i} \left[\frac{k_{ij}}{\mu_w} \left(- p_{,j} + \rho_w g_j \right) \right] d\Omega + \int_{\Gamma_e} \delta p q_w\, d\Gamma + \int_{\Gamma'} \delta p \overline{q}_w\, d\Gamma' = 0 \tag{3}$$

where $\overline{\Omega}$ is the domain of the fluid field, δp is a continuous pressure distribution satisfying boundary conditions, n the porosity, K_w the bulk modulus for liquid phase, α_w the thermal expansion coefficient of water, v^s_i the velocity vector of the solid phase, k_{ij} the permeability tensor of the medium, μ_w the dynamic viscosity of water, ρ_w its density and q_w the imposed flux on the external boundary. In the last term of Eq. (3) $\overline{q}_w$ represents the water leakage flux along the fracture toward the surrounding medium. This term is defined along the entire fracture, i.e. the open part and the process zone.

Incorporating Poiseuille law into the water mass balance equation within the crack results in:

$$\int_{\Omega'} \delta p \left\{ \frac{n}{K_w}\frac{\partial p}{\partial t} + \frac{\partial w}{\partial t} - n\alpha_w \frac{\partial T}{\partial t} + \left[\frac{w^2}{12\mu_w} \left(- p_{,j} + \rho_w g_j \right) \right]_{,i} \right\} d\Omega' - \int_{\Omega'} (\delta p)_{,i} \left[\frac{w^2}{12\mu_w} \left(- p_{,j} + \rho_w g_j \right) \right] d\Omega' + \int_{\Gamma'} \delta p \overline{q}_w\, d\Gamma' = 0 \tag{4}$$

which represents the fluid flow equation along the fracture. In this equation, Ω' and Γ' are the domain and the boundary of the fracture. The last integral in eq. (4) is related to leakage flux into the surrounding porous medium across the fracture borders and is of paramount importance in hydraulic fracturing techniques. It can be expressed by means of Darcy's law using the permeability of the medium surrounding the cracked area and pressure gradient generated by the application of water pressure on the fracture lips. Further investigation is probably necessary for the permeability in correspondence to the fracture tip: surrounding concrete is in fact not completely saturated (fluid lag formation) and the permeability of the first wetting of a partially saturated sample could represent better the real conditions of the fluid field in this zone.

When mechanical terms are neglected, internal energy depends on temperature only and is related to heat capacity of the mixture at constant volume C_v. Volume heat sources (s being the strength) are retained and, in the present application, they represent heat production due to hydration of concrete, but, in general they may represent other coupling effects between stress and thermal fields. Source terms may also arise along the boundary are on the contrary dropped. The weak form of the energy balance takes the form

$$\int_\Omega \delta T\, \rho\, C_v \dot{T}\, d\Omega + \int_\Gamma \delta T\, q_i^{conv}\ d\Gamma + \int_\Gamma \delta T\, q_i\, d\Gamma = \int_\Omega \delta T_{,i}\, q_{,i}\ d\Omega + \int_\Omega \delta T\, s\, d\Omega \tag{5}$$

being δT an admissible virtual temperature q_i^{conv} and q_i the convective and imposed heat flux normal to the boundary. Fourier's law is used as constitutive assumption for heat flux (λ_{ij} being the effective thermal conductivity tensor), and Newton's law to represent convective flux (being h the convective heat transfer coefficient and T_∞ the temperature in the far field of the undisturbed surroundings and n_i the outward normal to the boundary),

$$q_i = -\lambda_{ij} T_{,j} \qquad\qquad q_i^{conv} = h\left(T - T_\infty\right) n_i \tag{6}$$

The governing equations (2)-(5) are firstly discretized by means of the Galerkin procedure, then solved simultaneously to obtain the displacement and pressure and temperature fields together with the fracture path. The topology of the domain Ω and boundary change with the evolution of the fracture phenomenon. In particular, the fracture path, the position of the process zone and the cohesive forces are unknown and must be determined during the analysis.

3 DISCRETIZED GOVERNING EQUATIONS AND SOLUTION PROCEDURE

Space discretization of equations (2)-(5), incorporating the constitutive equations, results in the following system of time differential equations (dot represents time derivative) at element level

$$\begin{bmatrix} \mathbf{K} & \mathbf{C_{sg}} & \mathbf{C_{sT}} \\ \mathbf{C_{sg}}^T & \mathbf{S} & \mathbf{C}_{pT} \\ \mathbf{0} & \mathbf{0} & \mathbf{P} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{d}} \\ \dot{\mathbf{p}} \\ \dot{\mathbf{T}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{H}_{th} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{p} \\ \mathbf{T} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{F}} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_p \\ \mathbf{G}_{th} \end{bmatrix} \tag{7}$$

Adopting the usual symbols [3], the submatrices of eq. (7) are:

$$\mathbf{K} = \int_{\Omega_E} \mathbf{B}^T \mathbf{D}\, \mathbf{B}\, dV \tag{7a}$$

$$\mathbf{C_{sg}} = -\int_{\Omega_E} \mathbf{B}^T\, \overline{\alpha}\mathbf{m}\, \mathbf{N}^p\ dV \tag{7b}$$

$$\mathbf{C_{sT}} = \int_{\Omega_E} \mathbf{B}^T \mathbf{D}\mathbf{m} \frac{\alpha_s}{3}\, \mathbf{N}^{th} dV \tag{7c}$$

$$\mathbf{S} = -\int_{\Omega_E} \mathbf{N}^{p^T} \left(\frac{\overline{\alpha} - n}{K_s} + \frac{n}{K_w} \right) \mathbf{N}^p\ dV \tag{7d}$$

$$\mathbf{C_{pT}} = \int_{\Omega_E} \mathbf{N}^{p^T} \left((\overline{\alpha} - n)\, \alpha_s + n\, \alpha_s \right) \mathbf{N}^{th}\ dV \tag{7e}$$

$$\mathbf{P} = \int_{\Omega_E} \mathbf{N}^{th^T} \left[(1-n) \rho_s C_s + n \rho_w C_w \right] \mathbf{N}^{th}\ dV \tag{7f}$$

$$\mathbf{H}_p = -\int_{\Omega_E} \left(\nabla \mathbf{N}^p \right)^T \frac{\mathbf{k}}{\mu_w} \nabla \mathbf{N}^p\ dV \tag{7g}$$

$$\mathbf{H}_{th} = \int_{\Omega_E} \left(\nabla \mathbf{N}^{th}\right)^T \boldsymbol{\lambda} \nabla \mathbf{N}^{th}\, dV - \int_{\Omega_E} \left(\nabla \mathbf{N}^{th}\right)^T \rho_w C_w \mathbf{q} \mathbf{N}^{th}\, dV \tag{7h}$$

$$\mathbf{G}_p = \int_{\Omega_E} \mathbf{N}^{p^T} q_E\, dV + \int_{\Gamma_E} \mathbf{N}^{p^T} q^{\Gamma}\, d\Gamma + \sum \mathbf{N}^{p^T} Q - \int_{\Omega_E} \left(\nabla \mathbf{N}^{p}\right)^T \frac{\mathbf{k}}{\mu_w} \nabla\left(\rho_w \mathbf{g} h\right) dV \tag{7i}$$

$$\mathbf{G}_{th} = -\int_{\Gamma_E} \mathbf{N}^{th^T} q^e\, d\Gamma - \int_{\Omega_E} \mathbf{N}^{th^T} s\, dV \tag{7l}$$

$$\dot{\mathbf{F}} = \int_{\Omega_E} \mathbf{N}^T \dot{\mathbf{f}}\, dV + \int_{\Gamma_E} \mathbf{N}^T \dot{\mathbf{t}}\, d\Gamma + \int_{\Gamma_{E\,crack}} \mathbf{N}^T \dot{c}\, d\Gamma \tag{7m}$$

In Eq. (7f) $\dot{c}$ represents the cohesive traction rate and is different from zero only if the element has a side on the lips of the fracture Γ_{Ecrack}. Given that the liquid phase is continuous over the whole domain, leakage flux along the opened fracture lips is accounted for through eq. (7d) together with the flux along the crack. Finite elements are in fact present along the crack, as previously stated, which account only for the pressure field and hove no mechanical stiffness. In the present formulation, non-linear terms arise through cohesive forces in the process zone and permeability along the fracture. Further nonlinear terms could be related for instance to the dependence of medium permeability on porosity and porosity on volumetric strain.

Global equations are assembled in usual way and can be integrated in time by means of the generalized trapezoidal rule [3]. This yields the algebraic system of discretized equations, written for simplicity in a concise form as

$$\mathbf{A}_{n+1} \mathbf{x}_{n+1} = \mathbf{V}_n + \mathbf{Z}_{n+1} \tag{8}$$

being

$$\mathbf{x}_{n+1} = \begin{bmatrix} \mathbf{d} \\ \mathbf{p} \\ \mathbf{T} \end{bmatrix}_{n+1}$$

$$\mathbf{A}_{n+1} = \begin{bmatrix} \mathbf{K} & \mathbf{C}_{sp} & \mathbf{C}_{sT} \\ \mathbf{C}_{sp}^T & \mathbf{S} + \theta \Delta t \mathbf{H}_p & \mathbf{C}_{pT} \\ \mathbf{0} & \mathbf{0} & \mathbf{P} + \theta \Delta t \mathbf{H}_{th} \end{bmatrix}_{n+1}$$

$$\mathbf{V}_n = \begin{bmatrix} \mathbf{K} & \mathbf{C}_{sp} & \mathbf{C}_{sT} \\ \mathbf{C}_{sp}^T & \mathbf{S} - (1-\theta)\Delta t \mathbf{H}_p & \mathbf{C}_{pT} \\ \mathbf{0} & \mathbf{0} & \mathbf{P} - (1-\theta)\Delta t \mathbf{H}_{th} \end{bmatrix}_n \begin{bmatrix} \mathbf{d} \\ \mathbf{p} \\ \mathbf{T} \end{bmatrix}_n \tag{9}$$

$$\mathbf{Z}_{n+1} = \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}_{n+1} - \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}_n + \Delta t(1-\theta) \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_p \\ \mathbf{G}_{th} \end{bmatrix}_n - \Delta t\, \theta \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_p \\ \mathbf{G}_{th} \end{bmatrix}_{n+1}$$

As usual, n represents the time station and θ the time discretization parameter. Implicit integration is used in the following applications.

Because of the continuous variation of the domain as a consequence of the propagation of the cracks, also the boundary and the related mechanical conditions change. Only the forced boundary conditions need to be imposed explicitly, as the natural ones are accounted for by the weak statement of

the governing equations. To this scope, the part of the boundary where forced boundary conditions do not change during the analysis is firstly detected and the related conditions are imposed on eq. (8). For the remaining boundary, updated at each time step, boundary conditions are imposed by means of the Lagrange multiplier method. This requires the system (8) to be amended with a set of equations that embody the constraint conditions and change dynamically as the solution proceeds. This allows not only to represent the moving cracks, but the application of varying external loads, for instance a varying level of dammed water in a reservoir.

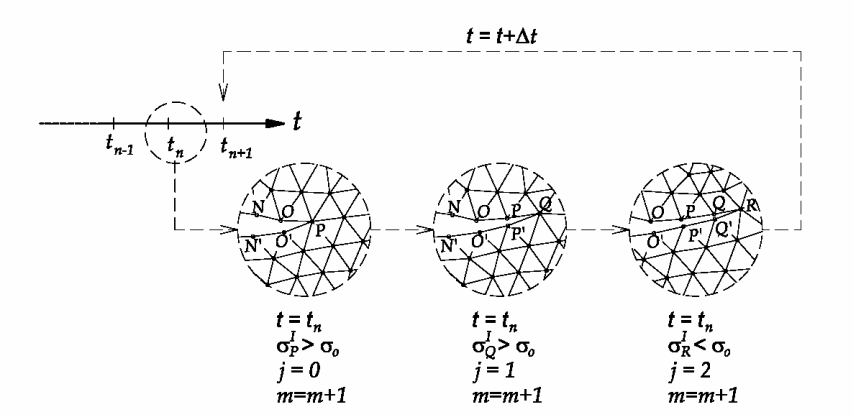

Figure 1: Multiple advacing fracture step at the same time station.

At each time station t_n, j different tip advancements are possible (Fig. 1). Their number in general depends on the chosen time step increment Δt and adopted crack length increment a. In the following, index m represents the total number of calls of the nonlinear algorithm solver (Newton Raphson in the applications) and $\mathcal{T}(m)$ is a function giving the time corresponding to eah value of index m.

For solution purposes, at a generic iteration Eq. (8) is rewritten as

$$\mathbf{R}^{k}_{m+1} = \mathbf{A}^{k}_{m+1}\,\mathbf{x}^{k}_{m+1} - \mathbf{V}_{m} - \mathbf{Z}^{k}_{m+1} \tag{10}$$

$\mathbf{R}^k$ being the residuum at the current iteration. Differentiation of the residuum with respect to the unknowns yields

$$\mathbf{R}^{k}_{m+1,x} = \mathbf{A}^{k}_{T} = \mathbf{A}^{k}_{m+1,x}\,\mathbf{x}^{k}_{m+1} + \mathbf{A}^{k}_{m+1} - \mathbf{Z}^{k}_{m+1,x} \tag{11}$$

If the capacity matrix is independent of the unknowns, i.e. $\mathbf{S}_{,\mathbf{x}} \cong \mathbf{0}$, and permeability can be assumed as constant during the iteration process, i.e. $\mathbf{H}_{,\mathbf{x}} \cong \mathbf{0}$, the coefficient matrix is also independent of the unknowns, i.e. $\mathbf{A}_{m+1,\mathbf{x}} \cong \mathbf{0}$, and the tangent matrix $\mathbf{A}_T$ becomes

$$\mathbf{A}^{k}_{T} = \mathbf{A}_{m+1} - \mathbf{Z}^{k}_{m+1,x} \tag{12}$$

The last term of Eq. (12) depends the cohesive forces only, which simply results in an updating of the stiffness matrix. The required increment of the unknowns $\Delta\mathbf{x}^k$ is simply obtained as

$$\Delta\mathbf{x}^{k} = -\mathbf{A}^{k\,-1}_{T}\,\mathbf{R}^{k} \tag{13}$$

Particular care must be used in handling the term $\mathbf{V}_n$ of eq. (8). It results from time discretization and represents the effects of the unknowns calculated at time station n on the solution at station n+1.

Let us consider the solution on the initial domain Ω_0, which is calculated on the initial spatial discretization (mesh) $\mathcal{M}_0$ by means of eq. (8). It is assumed that no fracture nucleation or propagation occurs, hence the mesh $\mathcal{M}_0$ is always referred to. At m+1, t= $\mathcal{T}(m)$ let the conditions for nucleating or propagating of fracture/s be attained at least in one point. Fracture/s evolution requires topological changes in the domain $\Omega_m \equiv \Omega_0$ and the boundary is consequently updated by introducing one or more new

nodes {N} as previously discussed. A discretization of the new domain Ω_{m+1} is then required for the numerical solution and the new mesh $\mathcal{M}_{m+1}$ is built. The solution of Eq. (8) is now sought using mesh $\mathcal{M}_{m+1}$, whereas the solution at the previous step m, hence term $\mathbf{V}_m$, is known on $\mathcal{M}_m$ ($\equiv\mathcal{M}_0$). Transfer operators based on element shape functions, when applied to nodal values of the field variables of $\mathcal{M}_m$ and $\mathcal{M}_{m+1}$, are not sufficient to guarantee the fulfilment of the system (8) at time station n on the new mesh. In the present approach, the projection is directly applied to forces $\mathbf{V}_m$, as defined by Eq. (9), requiring them to be mechanically equivalent between the two meshes. Not only a local balance is in this way obtained, but a high computational efficiency, independent on and increasing with order of approximations. This mapping can be written as

$$\widetilde{\mathbf{V}}_m(\Omega_{m+1}) = \aleph\left(\mathbf{V}_m(\Omega_m)\right) \tag{14}$$

being $\aleph$ a suitable consistent transfer operator. Then, terms $\mathbf{x}_m$ are recalculated on domain Ω_m, i.e. using the new mesh $\mathcal{M}_{m+1}$ It is necessary to recall that domain Ω_{m+1} presents updated boundary conditions, i.e. a nucleated or advanced fracture and nodes doubled along the last advancement of the fracture, whereas in the previous mesh continuity was present along the sides of the advanced part of the fracture. Multipoint constraints are introduced in correspondence of the master-slave nodes {N} immediately before the apex node, eliminating in this way the discontinuity of the field variables created by the latest advancement of the crack (eg. nodes P and P' in Fig. 1 when j=1). The only discontinuities allowed are the ones present at time station n, if present. The domain Ω_m is hence recreated with the discretization $\mathcal{M}_{m+1}$ and the relative (known) boundary conditions are fixed. The solution is then repeated until convergence of the Newton-Raphson procedure is obtained. This results in the determination of unknown $\mathbf{x}_m$ on mesh $\mathcal{M}_{m+1}$. Multipoint constraints are then relaxed and the solution procedure continues.

The recalculation step is not strictly necessary and could be dropped. In the applications it is however made in order to avoid the diffusion of numerical errors ensuing from the updating of the mesh and projection of the variables. In fact it guarantees that discretized governing equations are fulfilled at time station t_n also on the mesh $\mathcal{M}_{m+1}$. Further, mesh $\mathcal{M}_{m+1}$ is finer than the previous one, hence it allows for the definition of error measure and/or to perform convergence checks of the solution.

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