CREEP - DAMAGE INTERACTION IN CONCRETE STRUCTURES

Gilles PIJAUDIER-CABOT1, Mirvat OMAR1, Ahmed LOUKILI1, and Yann LE PAPE2
1R&DO – GeM, Ecole Centrale de Nantes, France
2EDF R&D, Moret sur Loing, France

ABSTRACT
The serviceability of concrete structures is a coupled problem in which creep and damage are coupled. In this paper, we start with experimental investigations on concrete notched beams that involve tertiary creep. We consider residual capacity tests on bending beams and investigate the evolution of size effect due to creep. We show that the fracture characteristics, namely the fracture energy and the size of the fracture process zone, decrease when creep is applied prior to failure. A candidate constitutive relation is presented in the second part. It is a damage-based model in which the relationship between the effective stress and the strain follows a generalised Maxwell chain. The influence of creep on damage growth can be adjusted in order to avoid a coupled effect between creep and damage that is too strong when damage is function of the total strain. Several test-computations are discussed including creep tests at several load levels on bending beams, similar to those predicted by Zhou and Hillerborg. In order to describe the variation of the size of the fracture process zone due to creep observed in the experiments, the internal length in the model should decrease. A first approach is proposed in the closure.

1 INTRODUCTION
In common practice, it is usually assumed that concrete exhibits a linear visco-elastic response for low load levels and that the instantaneous mechanical response of concrete is elastic. For high load levels, deviation from linearity of the creep behaviour of concrete is expected. Under high-sustained loads, cracks grow and interact with visco-elasticity. Some experimental and analytical results concerning non-linear creep can be found in the literature (see among others Bazant, 1988, Gettu and Bazant 1992, Mazzotti and Savoia 2003, Rüsch et al. 1957, Rüsch 1958) but much remains to be learned.

Modelling non linear creep of concrete is of fundamental importance for severely loaded structures as creep may decrease the material strength with increasing time, for instance in the evaluation of the safety of structural components such as nuclear power plants. Actually, these structures may be exposed to high stresses for a long time, due to what damage develops. Therefore, it is of great interest to devise methods for evaluating the evolution of their safety margin with time. This problem has drawn the attention of some authors, among them Rüsch and co-workers (1957, 1958), who have carried out the first experiments in order to determine the effect of continuous loading duration on the resistance and deflections of concrete specimens under compression tests. They found that the capacity of a specimen subjected to creep loads seems to be 70 to 80% of that observed in short time compression tests.

The main results of a set of experimental creep tests on concrete three points bend specimens at different load levels are presented first. The objective was to investigate the influence of the load level on the residual capacity of bending beams. For each creep test, a different load level has been applied, starting from low levels (36% of peak load) where linear visco-elasticity applies, to high levels (80% of peak load) where tertiary creep causing failure at a finite time can be observed. Size effect test data, before and after creep at 80% of the maximum load were also obtained.
The second part of the paper deals with a coupled creep - damage constitutive relation. It is based on the standard effective stress approach. The growth of damage is defined as a function of a total strain invariant, from which a portion of the creep strain is removed in order to achieve a coupling between creep and damage that is not too strong. The evolution of the size effect test data shows that the internal length in the damage model should decrease as the amount of creep strain increases. This effect is included in the constitutive relations too.

2 EXPERIMENTAL STUDY

Standard concrete, same as the one tested by Granger (1995) has been used in this experimental program. The specimens were made with this mix which consisted of ordinary Portland cement CPA-CEMII 42.5, fine sand with a maximum size of 5 mm, crushed gravel of size 5 to 25 mm, a super-plasticiser agent (Glenium 21) and water. This mixture is characterised by a water-cement ratio of 0.56 and a slump of 4 cm.

The creep tests were performed on frames that apply a constant load on flexural beams. The frames have a capacity ranging from 5 to 50 KN and can accommodate geometrically similar specimens of three different sizes. The specimens tested have the same thickness of 10 cm. The smallest (D1) is 10 cm high and 35 cm long. The medium size (D2) is 20 cm high and 70 cm long and the largest (D3) is 40 cm high and 140 cm long. In the present study, we shall consider only basic creep test data. The curing conditions (3 months at 100% RH and 20°C) guaranteed to avoid early age autogeneous shrinkage so that basic creep could be measured only. Figure 1 compares the displacements due to basic creep for the smaller specimens (size D1). It shows the influence of the load level on the basis creep evolution versus the elapsed time.

Figure 1: Basic creep displacement of notched beams D1 for 36%, 60%, and 80% of peak load.

As expected, increasing the applied load increases the basic creep magnitude. Moreover, creep develops very fast in the first days of loading and stabilises after a few weeks for the two lower loading levels. For the specimen loaded at 80% of the peak load, basic creep increases rapidly.

In order to investigate the variation of the residual capacity due to creep, comparison specimens cast at the same time than those subjected to creep were kept under the same conditions of temperature and relative humidity. After 60 days of loading, beams subjected to 80% of the maximum load were removed from the creep frames and then immediately subjected to three point
bending loading up to failure with a constant loading rate. The results were then processed according to Bazant's size effect law.

Figure 2 shows the size effect test data for the two sets of beams, with and without creep. We can observe a shift of the data points to the right when creep is applied prior to the fracture tests. Upon creep, the behaviour of the beams is closer to linear elastic fracture mechanics. Irwin's length decreases and the parameter $d_0$ in the size effect law decreases too. This shows that the size of the fracture process zone decreases.

Figure 2: Residual capacity of unotched beams of dimension D2.

3 CREEP-DAMAGE MODEL

Creep is modelled with visco-elasticity, in the form of a generalised Maxwell chain. The constitutive law is written with the help of a relaxation function $R$:

$$\sigma'(t) = R(t_0, t) \cdot E (t_0) + \int_{t_0}^{t} R(\tau, t) \cdot \dot{\varepsilon} (\tau) \cdot d\tau.$$  

$\sigma'$ is the effective stress applied to the Maxwell chain, $t_0$ is the age of loading. Note that the formulation is one-dimensional here. The relaxation function is:

$$R(\tau, t) = E_0 (\tau) + \sum_{\mu = 1}^{n} E_{\mu} (\tau) \cdot \exp (-\tau_{\mu} (t - \tau)) .$$  

$\tau_{\mu}$ and $E_\mu$ are the relaxation time and the modulus of elasticity of each branch $\mu$ in the generalised Maxwell model respectively. In general $E_\mu(t_0)$ can be calculated by the method of least squares. As for the parameters $\tau_{\mu}$, however, they cannot be calculated from measured creep data but must be suitably chosen in advance (Bazant, 1988).

In order to incorporate damage, which is a degradation of the overall elastic stiffness, into the time dependent creep functions, we use the effective stress approach (Omar et al. 2004):

$$\sigma = \sigma' (1 - d) .$$  

It is a classical approach, in which we consider a time independent growth of damage. The damage evolution law follows the classical relationship by Mazars (1984), coupled to an integral non local approach. Mechanical damage is controlled by the weighted average of the equivalent strain $\tilde{\varepsilon}$:
\[ \mathcal{E}(x) = \frac{1}{V_r(x)} \int_{\Omega} \psi(x-s) \tilde{\mathcal{E}}(s) ds \quad \text{with} \quad V_r(x) = \int_{\Omega} \psi(x-s) ds, \quad (4) \]

and the weight function \( \psi(x-s) \) is:

\[ \psi(x-s) = \exp \left( -\frac{4\|x-s\|^2}{l_c^2} \right). \quad (5) \]

The equivalent strain is:

\[ \tilde{\mathcal{E}} = \sqrt{\sum \epsilon_i^2}, \quad \text{with} \quad \epsilon_i = \epsilon_i(1-\alpha) + \alpha(C^{-1}\sigma_i), \quad (6) \]

where the subscript \( i \) denotes the principal strain components, \( C^{-1} \) is the inverse of the elastic (initial) stiffness of the material, and \( \alpha \) is a model parameter. When it is equal to zero, damage is a function of the total strain (including the creep strain). For values of this parameter larger than zero, a portion of the creep strain is deducted from the total strain prior the equivalent strain is computed.

Figure 3: Creep deformation of a tensile bar.

Figure 3 shows a typical creep response of a one dimensional bar. Note that tertiary creep can be described, including failure after some finite load duration. Such a result cannot be obtained with a series arrangement between visco-elasticity and damage or plasticity. Figure 4 shows the influence of the parameter \( \alpha \) for a monotonic loading that is displacement controlled. We have shown on this figure the rate effect due to creep too. When \( \alpha \) is taken equal to 0, the strongest possible effect of creep on damage growth is observed. When \( \alpha \) increases, the coupled effect becomes milder. The maximum load increases and the displacement at peak load increases too.
Zhou and Hillerborg (1992) have considered creep tests up to failure on notched bending beams. The beam were loaded to a constant level and then the time to failure was computed. Figure 5 shows how the present model can qualitatively reproduce this experiment. Again, the influence of the parameter $\alpha$ that controls the coupled effect between damage and creep is shown. For small values of $\alpha$, creep failure is observed for loads that are below 70% of the maximum load (measured under quasi-static loading without creep effect). For $\alpha=0.65$, creep failure does not occur for applied loads that are lower than 70% of the maximum one. This is more consistent with our experiments which does not exhibit creep failure at such low load levels.

**Figure 5:** Failure lifetimes for notched bending beams.

4 CLOSURE

At present, calibration of the proposed constitutive relations and comparisons with the presented test data is in progress. Qualitatively, the model provides some consistent responses. The present computations, however, rely on a beam theory and size effect cannot be described. A full implementation into a 3D finite element code is required for that. This is necessary in order to
examine the deviations on the size effect plot due to creep that the constitutive relation is capable of. Already, the variation of the internal length due to creep can be envisioned. We start with the non-local damage model with evolving internal length proposed by Pijaudier-Cabot et al. (2004). The value of the internal length is modified in the weight function:

$$\psi(x-s, \vec{\varepsilon}(s)) = \exp\left(-\frac{4\|x-s\|^2}{l_c^2(\vec{\varepsilon}(s))}\right), \text{with} \ l_c(\vec{\varepsilon}) = \beta f(\vec{\varepsilon}) + l_{c0}.$$ (7)

The internal grows from $l_{c0}$ to $l_{c0} + \beta$ as the function $f$ is in fact the local value of damage. A possible modification of this evolution of the internal length, which of course remains to be tested and analysed, could be:

$$l_c(\vec{\varepsilon}) = \beta \frac{\vec{\varepsilon} - \vec{\varepsilon}_\beta}{\vec{\varepsilon}} f(\vec{\varepsilon}) + l_{c0},$$ (8)

where $\vec{\varepsilon}_\beta$ is an invariant of the creep strain (total strain minus the strain computed according to the damage model without creep). Constructed like this, the growth of the creep strains would decrease the value of the internal length.

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