A TWO PARAMETER $K_{\text{max}}$ AND $\Delta K$ MODEL FOR FATIGUE CRACK GROWTH

D. Kujawski & S. Stoychev
Department of Mechanical and Aeronautical Engineering
Western Michigan University, Kalamazoo, MI 49008, USA

ABSTRACT
In this paper, previously proposed two-parameter ($K_{\text{max}}$ and $\Delta K$) driving force model is adopted for fatigue crack growth analyses for both positive and negative load ratios. It is based on the premise that the damage at the crack-tip process zone is an interplay of two damage mechanisms, namely a monotonic damage due to $K_{\text{max}}$ and a cyclic damage due to $\Delta K$. Fatigue crack growth rate, for a constant amplitude loading, is represented by a new three dimensional crack propagation (CP) table in terms of $K_{\text{max}}$ and $\Delta K$ in accordance with the two parameter model. It is shown that the CP table provides a general representation of crack growth data for constant amplitude loading. Experimental data taken from literature for 7055-T7511 aluminum alloy under various load ratios ranging from -1 to 0.7 were used to illustrate the two parameter approach.

1. INTRODUCTION
In general, load-bearing components/structures experience both an alternating load and a mean load during their service applications. Interactions between the alternating and mean loads on fatigue crack growth behavior are commonly introduced through a load ratio, $R (= \text{min. load} / \text{max. load})$, and an associated modification of the stress intensity factor range, $\Delta K$. This is accomplished by considering different mechanisms that contribute to crack-tip shielding usually, crack closure (Elber, [1]) or residual stresses (Schijve [2], Klesnil and Lucas [3], Kujawski and Ellyin [4]). The vast majority of investigations assume that the crack growth is governed solely by the modified or ‘effective’ $\Delta K$, where the effect of the maximum stress intensity factor, $K_{\text{max}}$, is only accounted indirectly through load ratio, $R$. However, it is well established that the $K_{\text{max}}$ could significantly affect the crack driving force depending on the actual environment. Therefore, Vasudevan, Sadananda and coworkers [5-8] have reconsidered fatigue crack closure and its effects on fatigue crack growth behavior. They demonstrated that there is no significant contribution to crack closure due to residual plastic strain at the crack wake. Further, when asperity- or roughness-induced closure is present, the actual contribution is small, about one quarter of that computed from the experimental compliance measurements. They suggested that load ratio effects on fatigue crack growth behavior should be represented in terms of $\Delta K$ and $K_{\text{max}}$.

In this paper, previously proposed two-parameter ($K_{\text{max}}$, and $\Delta K$) driving force model (Kujawski [9,10], Dinda and Kujawski [11]) is adopted for constant amplitude fatigue crack growth analyses. It is based on the premise that the damage at the crack-tip process zone is caused by an interplay of two damage processes, namely a monotonic damage due to $K_{\text{max}}$ and a cyclic damage due to $\Delta K$. In this approach the mechanical driving force, $\Delta K^*$, is related to the applied values of the stress intensities and is calculated as:

$$\Delta K^* = (\Delta K)^\alpha (K_{\text{max}})^{\gamma - \alpha}$$  \hspace{1cm} (1)
where $\Delta K^+$ is the positive part of the applied stress intensity range and $\alpha$ is considered to be a material parameter. For positive load ratios the $\Delta K^*$ parameter depends on both $\Delta K^+$ and $K_{\text{max}}$. However, for negative load ratios, since $K_{\text{min}} < 0$ will result in $\Delta K^+ = K_{\text{max}} = \Delta K^*$, which means that $\Delta K^*$ is insensitive to negative load ratios. In order to account for negative load ratios, the two-parameter driving force approach is modified by incorporating the total values of $\Delta K$ and $K_{\text{max}}$ and using a new three dimensional crack propagation (CP) table relating $\text{da}/\text{dN}$ to different combinations of the applied $\Delta K$ and $K_{\text{max}}$ values.

The biggest advantage of using CP table comes from the fact that the ‘$\alpha$’ parameter from Eq. (1) is no longer needed.

2. CRACK PROPAGATION TABLE

First, a combination of 25x25 values of $\Delta K$ and $K_{\text{max}}$ parameters is constructed, according to Fig. 1.

![Fig. 1 A table representation of $\Delta K$ and $K_{\text{max}}$ parameters.](image)

According to Fig.1, both parameters, $\Delta K$ and $K_{\text{max}}$, are equally spaced in log-log coordinate system between the threshold, $K_{\text{th}}$, and fracture toughness, $K_{\text{IC}}$, values. For a given combination of $\Delta K$ and $K_{\text{max}}$, for every element of this table is related to a corresponding $\text{da}/\text{dN}$. Thus, fatigue crack growth rate, $\text{da}/\text{dN}$, for constant amplitude loading is represented by a new three dimensional crack
propagation (CP) table in terms of the $K_{\text{max}}$ and $\Delta K$, in accordance with the two parameter model. This CP table can be represented graphically as a three dimensional surface. The values of the crack growth rates for any combination of $\Delta K$ and $K_{\text{max}}$ are typically extrapolated from experimental data conducted at constant load ratios, $R$. For variable amplitude loading conditions, only the transient crack growth effects have to be modeled, using the two parameter approach.

3. COMPARISON WITH EXPERIMENTAL DATA
Fatigue crack growth data for 7055-T7511 aluminum alloy taken from Ref. [12] with load ratios, $R$, ranging from -1 to 0.7 were selected for analysis. The data have been plotted using three methods for crack growth rate representation. These methods are:

1. conventional log(da/dN) versus log($\Delta K$);
2. constant da/dN lines in log($\Delta K$) versus log($K_{\text{max}}$) coordinates;
3. three dimensional CP table.

The plots on Fig. 2 (a) and (b) depict the obtained correlation results corresponding to the methods 1 and 2, respectively.

Fig. 2 Fatigue crack growth data [12] of 7055-T7511 aluminum alloy as a function of (a) $\Delta K$ and (b) $\Delta K$ and $K_{\text{max}}$ for constant da/dn.

Fig. 2 (b) indicates that a linear dependence, with two different slopes, exists among log($\Delta K$), log($K_{\text{max}}$) and any particular da/dN = constant. These two different slopes represent the $\Delta K$ and $K_{\text{max}}$ dominated regimes for $R > 0.5$ and $R < 0.5$, respectively.

These two regimes are separated by a transition point, where the slope changes. Other possible relationships in terms of changes in the position of the transition point or in the log($\Delta K$) versus
log($K_{\text{max}}$) slopes are currently under investigation. The observations allow fatigue crack growth to be extrapolated beyond the experimental data to complete the CP table, as is shown in Fig. 3. The grey area on Fig. 3 corresponds to the experimental data whereas the black one corresponds to the calculated values. Once a proper extrapolation is done, $da/dN$ for any combinations of $\Delta K$ and $K_{\text{max}}$ can be easily determined from the 3-D plot or by CP table lookup.

Fig. 3 Fatigue crack growth data [12] of 7055-T7511 aluminum alloy as a function of $\Delta K$ and $K_{\text{max}}$.

4. CONCLUSION
In this paper, previously proposed two-parameter ($K_{\text{max}}$ and $\Delta K$) driving force model is adopted for fatigue crack growth analyses for both positive and negative load ratios. Fatigue crack growth rate, $da/dN$, for a constant amplitude loading, is represented by a new three dimensional crack propagation (CP) table in terms of $K_{\text{max}}$ and $\Delta K$ in accordance with the two parameter model. It is shown that the CP table provides a general representation of crack growth data for 7055-T7511 aluminum alloy under various load ratios ranging from -1 to 0.7. For variable amplitude loading conditions, only the transient crack growth effects have to be modeled, using the two parameter approach.
Acknowledgements
This investigation was supported by the Office of Naval Research under grant N00014-01-1-0952.

References