USE OF AN INVERSE ANALYSIS TO DETERMINE THE ELASTOPLASTIC MATERIAL PROPERTIES FROM INDENTATION TESTS

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ABSTRACT

This work introduces an inverse analysis procedure for identifying the elastoplastic material parameters from indentation tests. The parameter optimisation Levenberg-Marquardt method is used to extract the elastic modulus, yield strength and strain hardening exponent from the indentation load-depth curve. The effectiveness of the proposed procedure is verified using pseudo-experimental (computer generated) data obtained from 2D finite element analyses of the indentation problem. It is shown that a single set of indentation load-depth data may not be sufficient to predict effectively the mechanical properties due to ill-conditioned and non-uniqueness problems. However, these problems are overcome when two different sets of indentation data obtaining from two different indenters e.g. conical and spherical indenters, are used. The results of this work confirm the usefulness and feasibility of the proposed procedure. Although the procedure proposed here is used to predict two unknown parameters, it is also applicable to estimate other physical or mechanical properties where there are more than two unknown parameters.

1 INTRODUCTION

It is well-established that the mechanical properties of materials such as Young's modulus, yield strength and strain hardening behaviour can be deduced from the indentation load versus displacement curves for loading and unloading. For example, the Young's modulus can be calculated from the initial slope of the unloading curve using the methods of Doerner and Nix [1] or that of Oliver and Pharr [2]. Cheng and Cheng [3-4] used the dimensionless and finite element analyses to evaluate the mechanical properties of the materials from conical indentation problems. More recently, Tunvisut et al. [5-6] employed a similar technique to study the indentation behaviour of both uncoated and coated substrates. Their work showed that the elastoplastic properties (e.g. Young's modulus, yield strength and hardening behaviour) could be uniquely identified from the measurement of the peak load, unloading slope of the indentation curve and the residual contact area of indentation. However, the applicability of this method is strongly dependent on accurate measurements of the measured quantities. For instance, a 5% error in the measured residual contact area may result in errors as high as 20% and 60% in the predicted yield strength and hardening exponent, respectively. In practice, the residual contact area is difficult to locate precisely. Therefore, this method may not be appropriate for small load indentations or in situations where piling-up or sinking-in effects may occur. Due to this inaccuracy, there is still a need to seek for a better way of determining the unique material properties directly from the available indentation data.

In this work, an alternative strategy based on finite element and inverse analyses, where unknown properties can be inferred from indentation measurements, is introduced for evaluating the elastoplastic material properties. The necessary steps for implementing this procedure and its effectiveness and feasibility are also discussed.

2 INVERSE ANALYSIS PROCEDURE

The present inverse analysis utilises the Levenberg-Marquardt (LM) method to estimate the unknown material properties. The method is described extensively in the work of Schnur and Zabaras [7] and its use within the context of this work is illustrated by the flow chart shown in Fig.1. Essentially, it processes the experimental data and attempts to obtain the best estimates for unknown state variables based on least-squares theory. The theory of the LM method is based on minimising an error function, Φ, with respect to the parameter, $\mathbf{p}$, as

$$\Phi(\mathbf{p}) = \frac{1}{2}\sum_{i=1}^{m}[r_i(\mathbf{p})]^2 = \frac{1}{2}\mathbf{r}^T\mathbf{r} \tag{1}$$

where **p** is a vector which contains the unknown parameters and m is the number of measurements. The vector **r** is defined as

$$\mathbf{r} = \mathbf{p}_i - \mathbf{p}_{oi} \tag{2}$$

where $\mathbf{p}_i$ and $\mathbf{p}_{oi}$ are the numerically generated and measured data, respectively.

For some simple problems, the solution may be obtained from a closed-form analytical solution. However, in many cases involving complex material systems, the solution can only be obtained from separate numerical analyses. For example, unlike the cases of indentation of elastic or rigid-plastic solids, the indentation of elastoplastic materials is more complicated and has in fact no analytical solution due to additional material parameters [8].

Here, detailed finite element calculations are conducted to simulate indentation process of elastoplastic materials. The elastoplastic materials with Young's modulus, E, and Poisson's ratio, ν, are characterised by a power law relation which is defined as

$$\frac{\tilde{\varepsilon}}{\varepsilon_y} = \frac{\tilde{\sigma}}{\sigma_y} \qquad \text{for } \tilde{\sigma} \le \sigma_y \tag{3}$$

$$\frac{\tilde{\varepsilon}}{\varepsilon_y} = \left(\frac{\tilde{\sigma}}{\sigma_y}\right)^n \qquad \text{otherwise} \tag{4}$$

where $\tilde{\sigma}$ and $\tilde{\varepsilon}$ are the equivalent stress and strain, respectively and $\varepsilon_y = \sigma_y/E$, is the yield strain.

Thus, four material parameters, namely, E, σ_y, n and ν, are required to define the property of elastoplastic materials. For simplicity, the value of Poisson's ratio is assumed to be constant and equal to 0.30. The value of E can also be extracted directly from the indentation records using the Oliver and Pharr's method [1] which relies on fitting the unloading part of the indentation load-depth data. Additionally, the effective modulus which accounts for the elastic deformation which occurs in both the substrate and the indenter is given as

$$E_{eff} = \frac{\pi}{2} \frac{(dF/dh)_{h=h_m}}{\sqrt{A_c}} \tag{5}$$

where A_c and $(dF/dh)_{h=h_m}$ are the contact area and the unloading slope evaluated at maximum indentation load, respectively. The effective modulus is related to the specimen modulus through

$$\frac{1}{E_{eff}} = \frac{(1-\nu^2)}{E} + \frac{(1-\nu^2)}{E_i} \tag{6}$$

where E and ν are the Young's modulus and Poisson's ratio of the specimen, and E_i and ν_i are the same quantities for the indenter.

Accordingly, the problem can now be simplified to determining only σ_y and n. Starting with initial guess values of σ_y and n, the inverse analysis technique based on the LM method is undertaken to evaluate σ_y and n by matching the numerically generated force, $\mathbf{p}_{oi}$, to the measured one, $\mathbf{p}_i$, in the least squares sense. The problem is iterated until the solution converges (see Fig. 1). In this analysis, the indentation depth is specified while the indentation load is the measured parameter. Alternatively, one may choose to measure the depth instead of the load. The indentation of elastoplastic materials using both conical and spherical indenters as shown in Fig. 2 are considered. The computations were conducted with the commercial ABAQUS [9] finite element code.

2.1 CREATION OF DATABASE

Since the LM method compares the measured data with the known solutions, the indentation load-displacement relation for any given combinations of σ_y and n must be available. In this study, this information or reference database is generated by finite element calculations since there is no closed-form solution for indentation of elastoplastic materials. The LM method requires indentation load and its derivatives with respect to σ_y and n at given indentation depths. These

values can be determined by carrying out many finite element simulations for various sets of σ_y and n. However, in order to minimise such computation efforts, only 30 combinations of σ_y-n are chosen as the database points in the simulations. These combinations of the parametric values are
σ_y : 200, 400, 600, 800, 1000, 1200
n : 3, 5, 8, 10, 15, 20

After implementing 30 separate finite element simulations for the combinations of the parametric values, the indentation load-displacement values were obtained through the bi-cubic interpolation function. This interpolation was carried out using a commercial MATLAB code [10].

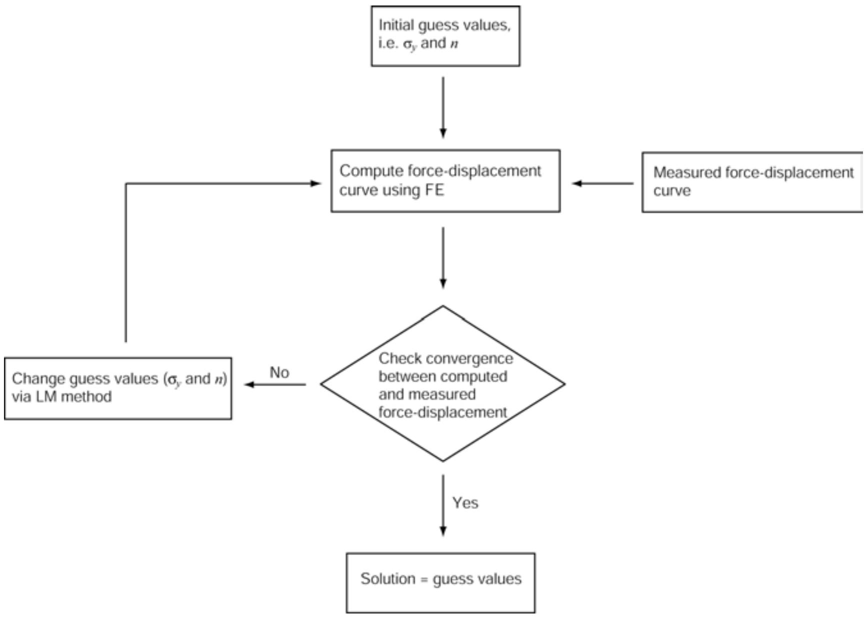

Figure 1: Flow chart for the inverse analysis procedure to determine the mechanical properties from the indentation data

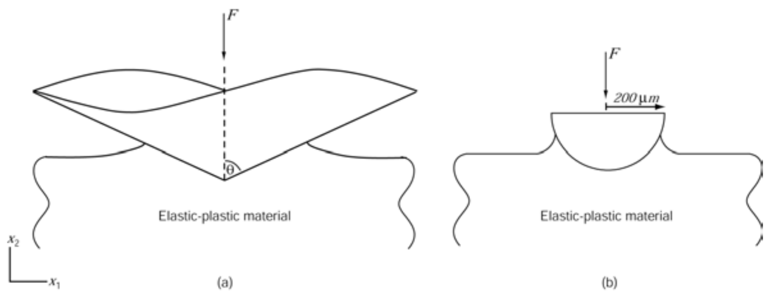

Figure 2: Schematic illustration of indentation problem using (a) conical indenter (b) spherical indenter

3 Results and Discussions

3.1 Single indenter procedure

The inverse analysis uses the experimentally measured load at several depth increments to seek for unknown parameters. Before applying this method in a real experiment, it is necessary to verify the accuracy of the procedure. This task is carried out with simulated experimental data generated from the finite element solutions. The simulation allows us to make direct observations of estimated solutions with respect to actual solutions. To do this, an arbitrary pair of σ_y and n, with values of 800 MPa and 10 respectively, is chosen for the material model and the indentation process using a conical indenter is first simulated. Ten displacement increments ranging from 0.5-8 μm with their corresponding loads were selected for the study.

Using these simulated data as input, the LM algorithm outlined in previous section is performed. Reference data is supplied to the algorithm. Seven different initial guess values are assigned to the unknown parameters. Figure 3 demonstrates the converging trends of the chosen sets of initial guess values i.e. sets A-F. The arrows indicate the direction of convergence. The results show that set A with initial guess values of σ_y = 300 MPa and n = 3 never approaches to the actual values, while sets B and C with the initial guess values prescribed in the figure converge to the values of σ_y = 565 MPa and n = 3 and σ_y = 756 MPa and n = 7, respectively. However, all other remaining sets D, E, G and F converge to the actual values of σ_y = 800 MPa and n = 10. The converging results of the initial guess values of sets B and C were used to simulate the indentation load-depth curves. Figure 4 compares the simulated results obtained from the converging values of sets B and C with the actual simulated data. It is seen clearly that three materials with different uniaxial behaviour can produce almost identical indentation curves. This indicates that single measurement data may lead to the problem with non-uniqueness. This non-uniqueness problem has also been reported in the work of Tunvisut [11].

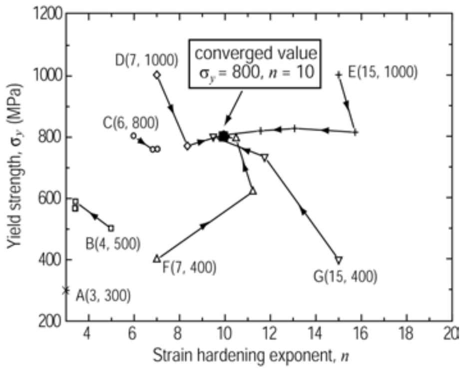

Figure 3: Converging trends of 7 chosen initial guess values for single indenter procedure

These results confirm that initial guess values play an important role in the convergence condition. However, in real tests, it would be impossible to determine whether the final predicted values of σ_y and n are representative or close to the actual values of the tested material. Unlike the finite element simulation, the actual solutions are not known a priori in real test. This means, in order for the inverse procedure to be useful, the problems with "ill-posed" or "ill-conditioned" (i.e. not able to achieve convergence) and "non uniqueness" must be improved so that all initial guess values should lead to the actual solutions. One way to enhance the accuracy is to supply more measurement data to the LM algorithm as discussed next.

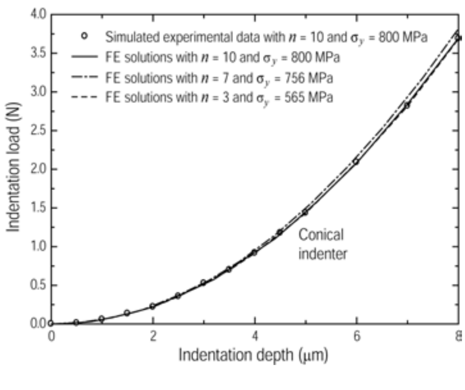

Figure 4: Illustration of a non-uniqueness problem in the indentation analysis

3.2 DOUBLE INDENTER PROCEDURE

As mentioned earlier, the primary objective of this work is to establish a method that requires minimal experimental effort. Although additional measurements from residual contact area using optical methods can certainly improve the LM method, it is not considered due to its difficulty in implementation and is prone to large human errors. Instead, other measurements from a spherical indenter of radius 200 μm is considered. It is assumed that different indentation load-depth responses can be obtained from such the indenter and they are sufficient to improve the accuracy of the method.

As in the conical indenter, the database for spherical indentation is created by implementing separate finite element simulations with the same combinations of σ_y and n as those in the single indenter procedure. Prior to running the LM method using a combination of the conical and spherical indentation data, the convergence behaviour of the spherical data alone was inspected. Similar results were observed to that of the conical indenter. Although the analysis using the spherical indenter is more stable compared to the conical indenter, the problem with non-uniqueness is still not solved. This confirms that single indenter data are not adequate to obtain good convergence behaviour and overcome the problem with non-uniqueness.

The combined indenter procedure is performed with the LM algorithm by reformulating the objective function as defined in eqn. (1). With the additional measurement data, the dimensions of vectors and matrices increase. Similar to the single indenter procedure, the reference data for both indenters are supplied initially. After initial guess values of σ_y and n, the program processes the two loads of the first increment from both indenters simultaneously. These values are then used to update the guess values until the actual solutions are obtained.

As in the case of the single indenter, the accuracy of double indenter procedure is evaluated by checking the convergence results of the sets containing initial guess values which diverge or result in non-uniqueness i.e. choosing sets A, B and C mentioned in previous section. The convergence results are depicted in Figure 5, where a dramatic improvement can be observed. This figure clearly illustrates that all initial guess values now converge to the actual values in less than 10 iterations. These findings suggest that one can assign any initial values for σ_y and n in the LM method to obtain very accurate estimates of σ_y and n. The result supports the usefulness and appropriateness of the proposed method. The feasibility of this method was also tested by changing the actual values of σ_y and n and repeating the above procedure. Similar convergence behaviour to the results shown in Figure 5 was obtained.

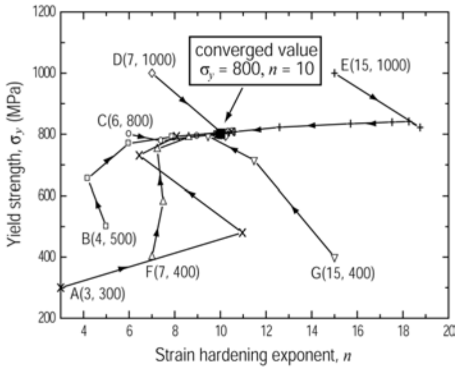

Figure 5: Converging trends of 7 chosen initial guess values for double indenter procedure

4 CONCLUSIONS

This work proposes a procedure based on finite element and inverse analyses to predict mechanical properties of elastoplastic materials from indentation tests. The proposed procedure is shown to be one of the means to maximise the usefulness of the indentation data. As in many inverse analyses, it is found that the model initially has problems with "ill-conditioned" and "non-uniqueness" when single indenter procedure is used. However, these problems are overcome when a double indenter procedure is introduced (i.e. two different sets of data from two different indenters). The feasibility studies also show that this procedure is well suited for extraction of elastoplastic mechanical properties from the indentation tests because it processes over multiple increments to best estimate the unknown parameters with the experimentally measured data.

The most important issue in the inverse analysis is how to identify the accuracy of the final estimated values in real tests where actual solutions are unknown. Although there is no direct method to determine the accuracy, one should be cautious by repeating the procedure for several times with different initial guess values. If the estimates are similar in all analyses, then it is likely that those estimated values are close to the actual solutions. It is also worth noting that although only two unknown parameters are predicted in this present work, the proposed method could easily be modified to extract the mechanical properties where there are more than two unknown parameters e.g. materials with rate-dependent. The use of this procedure to determine the mechanical properties of thin films is being investigated. These results will be presented in future work.

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