On the investigation of material stability during the simulation of ductile damage in metallic materials

F. REUSCH & B. SVENDSEN
Department of Mechanical Engineering, University of Dortmund, D-44221 Dortmund, Germany

ABSTRACT
This work is concerned with the analysis of stability, loss of stability and the associated bifurcation phenomena in the context of local and non-local simulation of ductile damage in metallic materials at the structural level of finite elements and the material level of observation. From a mathematical point of view, the loss of material stability is caused by the loss of ellipticity of the set of partial differential equations. The development and application of a criterion for loss of ellipticity demonstrates the regularisation of the solution obtained by the non-local Gurson model in comparison to the local description of damage evolution with the classical Gurson model.

1 INTRODUCTION

The numerical analysis of ductile damage and failure in engineering materials is often based on the micromechanical model of Gurson [1, 2, 3] and is quite sufficient for a large number of applications in solid mechanics. However, numerical studies in the context of the finite-element method demonstrate that, as with other such types of local damage models, the numerical simulation of the initiation and propagation of damage zones is not reliable and strongly mesh-dependent. The numerical problems concern the global load-displacement response as well as the onset, size and orientation of damage zones and thus to the reliability of the obtained results [4, 5].

One possible way to overcome these problems with and shortcomings of the local modelling is the application of so-called non-local damage models. In particular, these are based on the introduction of a gradient type evolution equation of the damage variable regarding the spatial distribution of damage and thus the incorporation of a material length scale [6, 7, 8]. Such a non-local formulation of a damage model exhibits a multifield problem, which needs a closer look on the formulation of possible criteria for the preservation of the well-posedness of the underlying constitutive equations and thus the stability of the deformation process and the uniqueness of the obtained solution [10, 11, 12]. The development and application of a criterion for loss of ellipticity is presented and accounts for the regularisation of the solution obtained by the non-local Gurson model [12]. Furthermore the regularizing effects of the non-locality of the damage evolution are investigated and it's effect on the stability of the numerical solution is presented.

2 REVIEW OF THE GURSON MODEL

The constitutive equations for the Gurson model are based on a yield function introduced by Gurson [1] and modified by Needleman and Tvergaard [3]. The yield function is of the form

$$\Phi(\mathbf{T},\sigma_m,f) = \frac{\sigma_v^2(\mathbf{T})}{\sigma_m^2} + 2q_1\, f^*\cosh\left(q_2\,\frac{\mathrm{tr}(\mathbf{T})}{2\sigma_m}\right) - (q_1 f^*)^2 - 1\;. \tag{1}$$

Here, f represents the void volume fraction in the matrix material, and σ_m is the yield stress of the matrix material. Further, $\mathbf{T}$ represents the Cauchy stress tensor, and σ_v the von Mises stress. The parameters q_1 and q_2 were introduced by Tvergaard [2] to improve the numerical results for large values of f.

The effective damage parameter

$$f^*(f) = \begin{cases} f & f \le f_c \\ f_c + k\,(f - f_c) & f \ge f_c \end{cases}\quad,\qquad k = \frac{(f_u^* - f_c)}{(f_f - f_c)}\,,\qquad f_u^* = \frac{1}{q_1} \tag{2}$$

introduced in [2] accounts for the effect of the coalescence of two neighboring voids on the yield behavior. f_c is the critical value of the void volume fraction at which void coalescence occurs. The ultimate value of f_f at which the loss of stress carrying capacity is determined by the parameter k_f.

Based on the requirement that the macroscopic plastic work equals the microscopic one, the evolution relation for the equivalent inelastic strain ε_m in the matrix material becomes

$$\dot{\varepsilon}_m = \frac{\mathbf{T} : \mathbf{D}_p}{(1-f)\sigma_m} \; . \tag{3}$$

The Gurson yield function determines in particular the associated form

$$\mathbf{D}_p = \frac{\partial \Phi}{\partial \mathbf{T}} \tag{4}$$

of the flow rule (1) relevant here. The evolution of the void volume-fraction f follows the equation

$$\dot{f} = \dot{f}_{gr} + \dot{f}_{nuc}. \tag{5}$$

For a plastically incompressible matrix material and ignoring the elastic contribution to the void volume-change the growth of the existing voids is described as

$$\dot{f}_{gr} = (1-f)\,\mathrm{tr}(\mathbf{D_p}) \; . \tag{6}$$

The plastic strain-controlled nucleation is described as

$$\dot{f}_{nuc} = A\,(\varepsilon_m)\,\dot{\varepsilon}_m \; , \quad A(\varepsilon_m) = \frac{f_n}{\sqrt{2\pi s_n^2}} \, \exp\left(-\frac{(\varepsilon_m - \varepsilon_n)^2}{2\, s_n^2}\right) \; . \tag{7}$$

so that the nucleation strain follows a normal distribution with the mean value ε_n and the standard deviation s_n.

3 A GRADIENT TYPE MODIFICATION TO THE EVOLUTION EQUATION OF POROSITY

In order to achieve a more realistic model damage process behavior, to minimize mesh dependence, and to account for the influence of a material lengthscale on the material behavior, a large number of non-local extensions to the modelling of rate-independent material behavior have been proposed in the last 15 years. For example, integral-type modifications of the Gurson model have been presented in Tvergaard and Needleman [2], Klingbeil and Brocks [4] and other works. More recently, a gradient-based extension of the Gurson model has been discussed and implemented by Ramaswamy and Aravas [6].

In contrast to this, the extension of local Gurson-based damage modelling pursued here [7, 8] retains the original Gurson modelling of f as based on the evolution relation (5). Indeed, the non-local extension presented here is based on a scalar-valued continuum damage field d. The incorporation of this field into the existing Gurson-based framework is based on the assumption that the effect of d on the yield behavior due to ductile damage is the same as that of f^* on this behavior in the local Gurson-based modelling of Needlemann and Tvergaard [3]. On this basis, d is incorporated into the current model via the form

$$\Phi_d\,(\mathbf{T}, \sigma_m, d) = \frac{\sigma_v^2(\mathbf{T})}{\sigma_m^2} + 2q_1\, d^* \cosh\left(q_2 \frac{\mathrm{tr}(\mathbf{T})}{2\sigma_m}\right) - (q_1 d^*)^2 - 1 \tag{8}$$

for the yield function Φ formally analogous to the Gurson form (1). Here, $d^*(d)$ is defined analogously to $f^*(f)$ in (2). Representing a quantity which describes in an effective fashion the transition from the undamaged to the damage state, the field d is modelled here as a continuum microstructural field or generalized phase field via the thermodynamic approach [9] to the modelling of such structure. Specializing this general approach to the case of quasi-static conditions and a single scalar-valued microstructural field representing phenomenological damage, one derives in particular the basic evolution relation

$$\dot{d} = c\nabla^2\dot{d} + \dot{f} \; . \tag{9}$$

The parameter c describes the strength of the influence of the gradient, and has dimensions of length squared. In the context of ductile damage, the non-local process of void coalescence is associated with this term [8]. For the solution of (9) in the context of an initial-boundary-value problem the Neumann boundary condition

$$\frac{\partial \dot{d}}{\partial \mathbf{n}} = \nabla\dot{d} \cdot \mathbf{n} = \mathbf{0} \tag{10}$$

of the initial configuration of the material under consideration is applied.

4 FINITE ELEMENT IMPLEMENTATION

The given formulation is governed by the equilibrium condition and the implicit averaging equation which sets the relation between the local and the non local damage field. For a given body force $\mathbf{f}$ and a boundary load $\mathbf{t}$, this set of equations reads:

$$\begin{array}{l} \int\limits_{\Omega} \eta_u \cdot (\nabla \cdot \mathbf{T})\, d\Omega = 0 \\ \int\limits_{\Omega} \eta_d \left(\dot{d} - \dot{f} + c^2\nabla\dot{d}\right) d\Omega = 0 \end{array} , \tag{11}$$

where a domain Ω in R^n (n=1,2,3) is considered, with a boundary Γ with a unit outward normal $\mathbf{n}$. The classical boundary condition applies for $\mathbf{T} \cdot \mathbf{n} = \mathbf{t}$ on Γ for the stress equilibrium, while the Neumann boundary condition $\nabla d \cdot \mathbf{n} = 0$ on Γ is adopted for the damage balance equation.

Considering the finite element discretisation $B \approx \bigcup_e B_e$ of B into a finite number of elements $B_1, B_2, \ldots$ the displacement vector $\mathbf{u}$ and the damage field d and the corresponding weighting functions $\eta_\mathbf{u}$ and $\eta_\mathbf{d}$ can be discretized in each element Ω_e:

$$\begin{array}{l} \mathbf{u}_{|\Omega_e} = [\mathbf{N_u}\hat{\mathbf{u}}]^{\mathrm{T}} \wedge \eta_{\mathbf{u}|\Omega_e} = [\mathbf{N_u}\hat{\eta}_\mathbf{u}]^{\mathrm{T}} \\ d_{|\Omega_e} = \mathbf{N}_d\hat{\mathbf{d}} \wedge \eta_{d|\Omega_e} = \mathbf{N}_d\hat{\eta}_\mathbf{d} \end{array} . \tag{12}$$

The columns $\hat{\mathbf{u}}$ and $\hat{\mathbf{d}}$ contain the nodal values of the displacements and the global damage field respectively. The arrays $\hat{\eta}_\mathbf{u}$ and $\hat{\eta}_\mathbf{d}$ contain the nodal values of the weighting functions. $\mathbf{N_u}$ and $\mathbf{N_d}$ are arrays of the interpolation functions for $\mathbf{u}$.

Discretizing the weak forms of (11) over the entire domain leads to the set of equation

$$\begin{array}{l} 0 = \int\limits_{\Omega_e} \mathbf{B}_u^{\mathrm{T}}\sigma d\Omega - \int\limits_{\Gamma_e} \mathbf{N}^{\mathrm{T}}\mathbf{t} d\Gamma \\ 0 = \int\limits_{\Omega_e} \mathbf{N}^T \left(\dot{d} - \dot{f}\right) + \left(\mathbf{B}_d^{\mathrm{T}} c\nabla^2 d\right) d\Omega \end{array} \tag{13}$$

where $\mathbf{B_u}$ represents the matricial strain operator and $\mathbf{B_d}$ represents the gradient operator for the global damage field. The corresponding element nodal forces are

$$\mathbf{f}^u_{e,int} = \int\limits_{\Omega_e} \mathbf{B}_\mathbf{u}^{\mathrm{T}}\mathbf{T} \; , \quad \mathbf{f}^d_{e,int} = \int\limits_{\Omega_e} \mathbf{N}_d^{\mathrm{T}} \left(\dot{d} - \dot{f}\right) + \mathbf{B}_d^{\mathrm{T}} c\nabla\dot{d} \tag{14}$$

Finally the following global system of equations can be obtained for an iteration j of a standard Newton-Rahpson-iteration:

$$\left[\begin{array}{cc} \mathbf{K}^{uu}_{(j)} & \mathbf{K}^{ud}_{(j)} \\ \mathbf{K}^{du}_{(j)} & \mathbf{K}^{dd}_{(j)} \end{array}\right] \left[\begin{array}{c} \delta\hat{\mathbf{u}} \\ \delta\hat{\mathbf{d}} \end{array}\right] = \left[\begin{array}{c} \mathbf{f}^{u}_{ext,(j+1)} \\ \mathbf{0} \end{array}\right] - \left[\begin{array}{c} \mathbf{f}^{u}_{int,(j)} \\ \mathbf{f}^{d}_{int,(j)} \end{array}\right] . \tag{15}$$

5 MATERIAL STABILITY

One of the motivations for the incorporation of higher gradients of the damage field variable field is to overcome the pathologies encountered in computations which are related to the ill-posed character of the boundary problem. As is well-known, this ill-posedness is due to the loss of the ellipticity of the governing partial differential equation, represented via the local rate form of the momentum equilibrium

$$\mathrm{div}\, \dot{\mathbf{T}} = 0 \; . \tag{16}$$

The loss of ellipticity is commonly investigated by a wave propagation analysis, as demonstrated for coupled multifield problems in [11, 10, 12]. This is based on the harmonic exponential form

$$\dot{\mathbf{u}}\,(\mathbf{x}) = \dot{\mathbf{u}}_0 \exp\left(ik\mathbf{n}\cdot\mathbf{x}\right) \tag{17}$$

for the velocity. Here, $\dot{\mathbf{u}}_0$ denotes the amplitude of the displacement jump, $\mathbf{n}$ the direction of the wave propagation, i represents the imaginary number, and k represents the wavenumber. We then investigate if the homogenous state admits a bifurcation into a solution of planar wave-type based on this approach and the incremental equations of linear momentum together with the consistency condition. In this context the original homogeneous solution becomes unstable if

$$\det(\mathbf{n}\cdot\mathbf{C}\cdot\mathbf{n}) = \mathbf{0} \tag{18}$$

holds. Here, $\mathbf{C}$ is the tangent operator for the incremental stress update equation

$$\dot{\mathbf{T}} = \mathbf{C} : \dot{\varepsilon} \tag{19}$$

of the local Gurson model.

The non-local extension of the Gurson model is governed by the equilibrium condition (16) and the implicit averaging equation of the damage evolution, which sets the relation between the local and the non local damage evolution (17). Thus, the analysis of the uniqueness is based on the two harmonic forms

$$\dot{\mathbf{u}}\,(\mathbf{x}) = \dot{\mathbf{u}}_0 \exp\left(ik\mathbf{n}\cdot\mathbf{x}\right)\;, \quad \dot{\mathbf{d}}\,(\mathbf{x}) = \dot{\mathbf{d}}_0 \exp\left(ik\mathbf{n}\cdot\mathbf{x}\right) \tag{20}$$

for the velocity and damage rate. Here, $\dot{d}_0$ represents the characteristic continuum damage rate in the material. Using these, one gains together with the incremental update procedure

$$\dot{\mathbf{T}} = \mathbf{C}^{\sigma\varepsilon} : \dot{\varepsilon} + C^{\sigma d}\dot{d}\;, \quad 0 = \mathbf{C}^{d\varepsilon} : \dot{\varepsilon} + C^{dd}\dot{d} \tag{21}$$

and finally the condition

$$\det(\mathbf{n}\cdot\mathbf{C}_{\mathbf{ud}}\cdot\mathbf{n}) = \mathbf{0} \tag{22}$$

for loss of ellipticity of the non local damage model. The operator $\mathbf{C}_{\mathbf{ud}}$ for the coupled-field-problem depends on the material parameter c and the wavenumber k of the harmonic forms (20).

Such a condition for loss of ellipticity for the local (22) and the non local Gurson model (18) allows easily the investigation of the solution stability of any material point in a structure for each loading increment during a finite element analysis.

6 RESULTS

In this section, the behavior of the above gradient-type extension of Gurson-based ductile damage modelling is compared with the corresponding local modelling in the context of a two-dimensional model problem. To this end, a two-dimensional specimen (Fig. 1) is subject to displacement-controlled loading via a prescribed end displacement under plane-strain conditions. The specimen is discretized using plane bilinear quadrilateral elements with an additional degree of freedom representing the nodal damage values d. In order to trigger damage development in the specimen, the initial void volume fraction in the structure is assumed to be slightly larger (10%) in the element shown in grey in Fig. 1 than in the surrounding elements. The simulations to be discussed are based on specific values for the local model parameters: the value of f_0 is chosen very large here in order to reach significant damage growth for comparatively small boundary displacements.

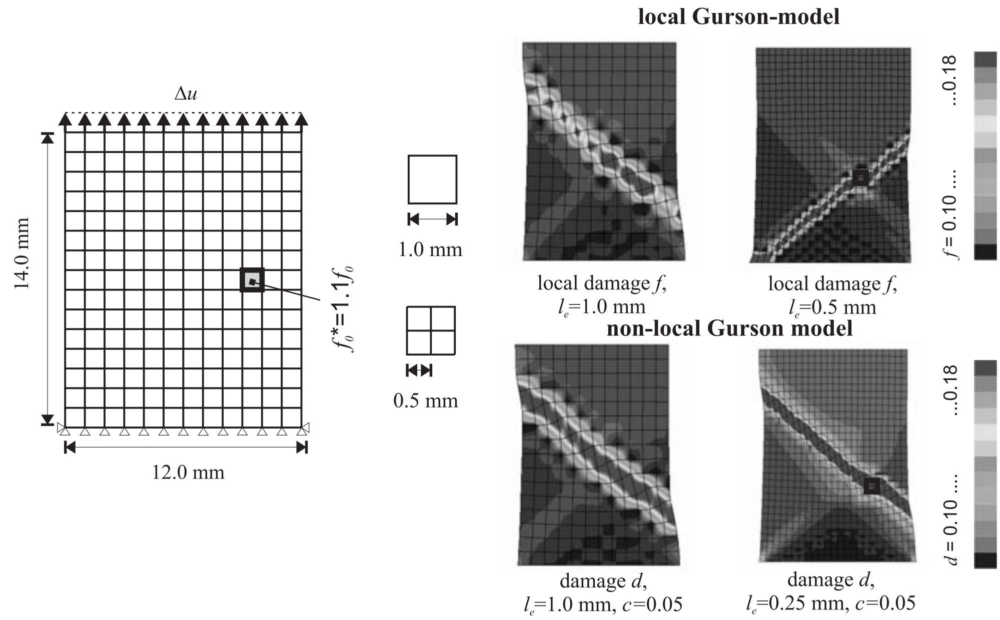

Figure 1: Two-dimensional model problem with initially heterogeneous void distribution and damage band development at the final deformation state for the local and non-local Gurson model ($c = 0.05$).

Further the material parameter $q_1 = 1.0$, $q_2 = 1.5$, $f_0 = 0.1$, $f_c = 0.1$, $f_f = 0.18$, $f_u^* = 0.6$, $f_n = 0.05$, $s_n = 0.3$ and $\varepsilon_n = 0.1$ have been chosen. Otherwise, the values $E = 210000$ MPa, $\nu = 0.3$, and $\sigma_m = 450$ MPa typical of steels have been used. In order to focus on ductile damage and reach ductile failure as quickly as possible, isotropic hardening is neglected here ($H = 0.0$) for linear ideal-plastic hardening. For the non-local Gurson model the parameter c describing the strength of the influence of the gradient was $c = 0.05$.

On this basis, attention is focused on the effects of varying the element edge-length for fixed values of the non-local parameter. To this end, results for two edge-lengths $l_e = 1.0$ and 0.5 mm are presented and discussed in what follows.

Fig. 1 shows the damage and shear-band development at the final deformation state for the simulation with the local Gurson model ($c = 0$) and the non-local Gurson model ($c = 0.05$). As expected, for the local Gurson model damage is concentrated here in a shear band triggered by the initial inhomogeneity and as expected, the width of this band is strongly influenced by the corresponding element size. In contrast, the comparable results for the use of the non-local model ($c = 0.05$) displayed below are largely mesh-independent. Indeed, practically the same damage-band width is predicted and the orientation of the damage band is the same.

Application of the criterion for loss of ellipticity from from (17) and (20) can be used to investigate the regularization of the solution obtained by the non-local Gurson and the local Gurson-model.

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