Modeling delamination fracture with frictional contact in orthotropic laminates

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ABSTRACT

Approximate weight functions are derived and validated for isotropic and orthotropic double cantilever beams loaded in mode II. The functions define the stress intensity factor at the crack tip due to a pair of tangential point forces acting on the crack surfaces. They have been deduced using asymptotic matching through finite elements and an orthotropy rescaling technique. The weight functions can be used, along with the related mode I functions [1], to formulate mixed mode fracture problems as integral equations. This approach overcomes the limitations imposed on accuracy by beam/plate theory approximations and it is particularly relevant for those problems where regions of contact and friction and other nonlinear cohesive or bridging mechanisms develop between the crack surfaces on different scales (small and large) as a consequence of near tip processes, structural effects, the action of through-thickness reinforcement or the interaction between multiple delaminations.

1. INTRODUCTION

Contact and friction between the surfaces of delamination cracks are nonlinear mechanisms that strongly control the response to static and dynamic loading of laminated plates. In single delamination fracture, large regions of contact may develop along the delamination surfaces due to geometrical effects (see example of Fig. 1b), or to the action of bridging mechanisms such as those produced by a through thickness reinforcement. In the case of mode I and mixed mode fracture specimens reinforced through the thickness by stitching or z-fibers, for instance, the through-thickness reinforcement may develop actions similar to those of a Winkler elastic foundation, creating multiple regions of contact along the faces of the delamination that may either arrest crack growth (for mode I problems) or substantially modify the mode ratio and crack tip conditions (for mixed mode problems) [2].

In multiple delamination fracture, which is typical of laminated plates subjected to dynamic loading and impact, contact and friction may arise also due to the interaction of the delaminations. Models based on beam and plate theory approximations have shown that these mechanisms strongly affect the propagation of the crack system and the macrostructural response [3].

An example of such behaviors is shown in the diagram of Figure 1. The diagram has been obtained using a model based on the theory of bending of isotropic beams with non-frictional contact between the crack surfaces modeled using a Winkler foundation approximation. Figure 1b refers to the cantilever beam with two equally spaced delaminations of length, a_U and a_L, subjected to a concentrated force P of Fig. 1a. It depicts the relative amount of mode I to mode II, defined in terms of stress intensity factors at the tip of the upper crack by the ratio (K_I/K_{II}) versus the normalized length of the upper crack, a_U/h, for a fixed length of the lower crack a_L = 0.5L.

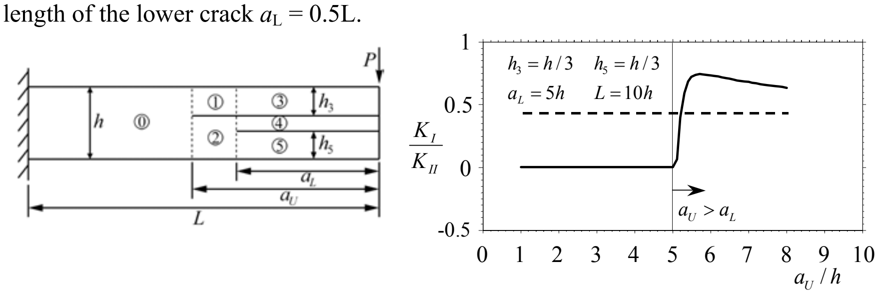


Figure 1: (a) cantilever beam with multiple delaminations; (b) influence of the interaction between delaminations on the mode ratio in a multiply delaminated beam.

In the absence of the lower crack (dashed curve) the upper crack would be in mixed mode conditions with K_I/K_{II} = 0.43. The mode I component is due to a geometrical effect, namely the misalignment of the crack from the mid-plane; the crack would be in pure mode II conditions for $h_3 = h/2$. The presence of a lower crack longer that the upper crack modifies the mode ratio creating regions of contact at the crack tip so that the problem becomes mode II (solid curve). When the upper crack reaches the length of the lower crack there is a sudden transition in the behavior, the amount of mode I increases above that corresponding the single crack solution and then the mode ratio slowly tends to the single crack solution when the effect of the lower crack disappears for large a_U. Also in this range the behavior is strongly dominated by the presence of contact between the crack faces. These effects are accompanied by an amplification of the energy release rate [3].

Contact and friction are expected to play an important role also in dynamic delamination fracture and impact problems [4] and work is currently in progress to investigate these mechanisms that have been recently studied for other material systems.

The crack wake nonlinear mechanisms may include near tip processes (e.g. the formation of a craze zone, crack tip contact and friction) and processes acting over the far crack wake (e.g., contact, friction, bridging mechanisms produced by the action of a through-thickness reinforcement). In many cases beam and plate theories have enough accuracy to analyze such problems. However, if the cracks are short or mechanisms acting over different size regions act simultaneously, greater accuracy is needed in the description of the near tip fields. The finite element method offers a convenient numerical alternative to beam theory. However, the method can present difficulties in resolving the crack tip singularity, especially when the boundaries of the contact/friction and bridging/cohesive zones move during crack evolution. Thus motivation exists for reducing these problems to a system of integral equations by means of weight functions. Weight functions have the correct asymptotic forms and therefore offer the greatest control in numerical methods over singularities either at the crack tip or at discontinuities in the nonlinear mechanism in the crack wake. Furthermore, the integral equation formulation reduces plane problems to one-dimensional problems, with distance from the crack tip the only spatial variable. This allows very rapid scanning of large quantities of parametric problems.

In this paper approximate weight functions are derived and validated for isotropic and orthotropic double cantilever beams loaded in mode II. The work is an extension of prior solutions derived for the mode I problem [1]. The weight functions have been derived for the static case and they will be later applied to investigate the mechanisms of contact and friction in plates subject to static loading and to validate solutions based on beam/plate theory approximations. It is expected that the conclusions reached for the static case will be valid also for dynamic fracture problems since the conditions affecting accuracy are mainly geometrical rather than rate dependent.

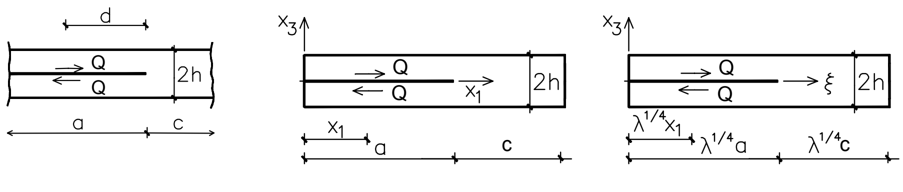
2. MODE II WEIGHT FUNCTION FOR ISOTROPIC DOUBLE CANTILEVER BEAMS

The basic solution used to define the mode II weight function for an isotropic double cantilever beam (Fig. 2b) is the plane elasticity problem of an infinite strip of thickness $2h$ and unit width with a semi-infinite crack loaded by a pair of concentrated forces acting tangentially to the crack surfaces at a distance d from the crack tip (Fig. 2a). The problem was solved by Entov and Salganik in [5]. They applied the Wiener-Hopf technique to a strip arbitrarily loaded by opening and tangential tractions acting along the crack surfaces and used the Dirac delta function to define concentrated forces. The stress intensity factor due to a pair of tangential forces acting per unit width, Q, was expressed by an integral equation of complex variable. From the general solution, Entov and Salganik derived analytical expressions for the two asymptotic limits of small and large d/h. For small d/h the stress intensity factor approaches the exact solution of Irwin for a semi-infinite crack in an infinite sheet:

$$\frac{K_{IIQ}h^{0.5}}{Q} = \sqrt{\frac{2h}{\pi d}} \quad \text{for} \quad d/h \ll 1 \ \text{and} \ d/a \ll 1 \tag{1}$$

For very large *d/h*, the asymptotic limit coincides with the elementary beam theory solution for an Euler-Bernoulli double cantilever beam with built-in ends. The stress intensity factor can then be derived from the energy release rate $\mathcal{G}_{\mathrm{IIQ}}=4Q^2/(E_1 h)$, where E_1 is the Young modulus, and is given by:

$$\frac{K_{\mathrm{IIQ}} h^{0.5}}{Q} = 2 \quad \text{for} \quad d/h >> 1 \tag{2}$$



(a) (b) (c)

Figure 2: (a) infinite strip with semi-infinite crack loaded in mode II; (b) double cantilever beam loaded in mode II by pair of tangential forces Q_i acting at the coordinate x_{1i}; (c) rescaled problem

An explicit expression of K_{IIQ} valid for intermediate values of *d/h* was not derived in [5]. Following the approach used in [1] for the derivation of the stress intensity factor in an orthotropic double cantilever beam loaded in mode I, an approximate formula is proposed here for K_{IIQ} that has the right asymptotic behavior for small and large *d/h* and fits numerical results for intermediate *d/h*. The formula has been derived referring to a strip of finite length and unit width in the plane x_1-x_3, with *a* the length of the crack, x_1 the coordinate which defines the position of the applied forces, $x_1 = a - d$, and *c* the length of the ligament ahead of the crack tip (Fig. 2b):

$$\frac{K_{\mathrm{IIQ}} h^{0.5}}{Q} = 2 + \sqrt{\frac{2h}{\pi d}} - \left[0.559\left(\frac{d}{h}\right)^{0.653} + \exp\left(-1.665\, h/d\right) + 0.5\right]^{-1} \tag{3}$$

The first two terms on the right hand side of Eq. (3) are the two asymptotic limits of Eqs. (1) and (2) and the third term is a connecting function whose shape has been chosen to ensure optimal fit of the finite element results. The relative error between the predictions of Eq. (3) and the finite element results is within 1.5% for all *d/h*, normalized crack lengths $a/h \geq 0.8$ and $c/h > 2$. The error reduces to 0.7% when the crack length is $a/h \geq 1.0$. The dimensionless stress intensity factor of Eq. (3) is shown in the semi-logarithmic diagram of Fig. 3, curve with $\lambda = 1$, along with the two limiting solutions for small and large *d/h.*

Equation (3) has been modified in [6] to describe cracks of any length. The modified expression include changes to the connecting function and the asymptotic limit (1) that in this case is given by Tada's solution for a semi-infinite strip with crack of finite length *a*.

3. MODE II WEIGHT FUNCTION FOR ORTHOTROPIC DOUBLE CANTILEVER BEAMS

The mode II weight function for an isotropic material (3) has been modified to describe a generally orthotropic material by referring to the orthotropic asymptotic limits and applying the orthotropy rescaling technique proposed by Suo [7]. The asymptotic limit for small *d/h* in an orthotropic strip with principal material axes x_1 and x_3 and loaded in mode II coincides with the isotropic solution, Eq. (1). This result has been demonstrated by Sih, Paris and Irwin (*IJF*, 1965) and is applicable to cracked infinite sheets loaded by self-equilibrating loads acting on the crack surfaces. For large *d/h*, the mode II stress intensity factor can be derived from the expression of the energy release rate $\mathcal{G}_{\mathrm{IIQ}}=4Q^2/(E_1 h)$ already used for the isotropic case, which is still an exact elasticity asymptote as $d/h \to \infty$. The limit (2) then modifies as:

$$\frac{K_{\mathrm{IIQ}} h^{0.5}}{Q} = \frac{2\lambda^{1/8}}{\sqrt{n}} \qquad \text{for } d/h \quad >> \ 1 \tag{4}$$

where the two dimensionless material parameters, λ and ρ, are defined in terms of the four elastic constants of plane orthotropic elasticity:

$$\lambda = \frac{E_3}{E_1}, \qquad \rho = \frac{\sqrt{E_1 E_3}}{2\mathrm{G}_{13}} - \sqrt{\nu_{13}\nu_{31}}, \qquad n = \sqrt{\frac{1+\rho}{2}} \tag{5}$$

and E_1 and E_3 are the Young's moduli in the x_1 and x_3 directions, G_{13} is the shear modulus and ν_{13} and ν_{31} are Poisson's ratios in the plane x_1-x_3. Equation (4) has been checked through finite element calculations and has an accuracy of 2% for all $d/h \geq 1.5\lambda^{-1/4}$, $0 \leq \rho \leq 5$ and any value of λ. For a degenerate orthotropic material the accuracy is higher than 1% for all $d/h \geq 0.8\lambda^{-1/4}$.

Degenerate orthotropic material ($\rho = 1$)

The stress intensity factor due to a pair of tangential forces in a degenerate orthotropic strip, characterized by $\rho = 1$, is determined as an exact extension of the isotropic result (3) through the orthotropy rescaling technique of Suo [7]. According to this technique, the x_1 axis of the original problem is rescaled as $\xi = \lambda^{1/4} x_1$; furthermore, the geometry and boundary conditions are rescaled in such a way that the crack length becomes $\lambda^{1/4}a$, the forces applied per unit width Q remain unchanged and the stress intensity factor at the crack tip becomes $\lambda^{-1/8}K_{\mathrm{IIQ}}$. For a degenerate orthotropic material, the rescaled problem in the plane ξ–x_3 is governed by the same equations of the original problem in an isotropic solid. Consequently, the rescaled stress intensity factor takes the form:

$$\frac{K_{\mathrm{IIQ}} h^{0.5}}{Q} = 2\lambda^{1/8} + \sqrt{\frac{2h}{\pi d}} - \left[0.559\lambda^{-1/8} \left(\lambda^{1/4} \frac{d}{h} \right)^{0.653} + \lambda^{-1/8} \exp\left(-1.665\lambda^{-1/4}\, h/d\right) + 0.5\lambda^{-1/8} \right]^{-1} \tag{6}$$

Equation (6) includes the result expected for very small d/h, tending asymptotically to Irwin's solution of Eq. (1), which is independent of the orthotropy. For large d/h, Eq. (6) approaches the limiting solution of Eq. (4) with $\rho = n = 1$. Note that this upper limit as well as the connecting function given by the third term on the right hand side of Eq. (6) depend on the orthotropy of the material through λ. The dimensionless stress intensity factor of Eq. (6) is depicted in Fig. 2 for different values of λ. The dashed curves show limiting solutions.

Equation (6) has been validated through finite element calculations for λ varying in the range $0.025 \leq \lambda \leq 1.0$. The equation applies to beams with an uncracked ligament $c/h > 2\lambda^{-1/4}$. Equation (10) is correct for all d/h provided the crack length a/h is higher than a limit value which the rules of orthotropy rescaling sets equal to $a/h = 0.8\lambda^{-1/4}$ for a 1.5% accuracy. Finite element calculations show that this is in fact a conservative limit for all cases with $\lambda < 1.0$. As for the isotropic case, Eq. (6) has been modified in [6] to describe cracks of any length.

Generally orthotropic material

The stress intensity factor in generally orthotropic ($\lambda \neq 1$ and $\rho \neq 1$) double cantilever beams must approach the known asymptotic solutions, Eqs. (1) and (4) for large and small d/h. Solutions for intermediate values of d/h require rigorous analyses of the problem which have been performed through finite element calculations. Only the effect of ρ needs to be calibrated numerically since the dependence of λ is known analytically (Eq. 6) and in materials with cubic symmetry, $\lambda = 1$, the plane elasticity problem is controlled by ρ alone.

Based on these observations, the following expression is proposed as a canonical approximation to K_{IIQ} for the degenerate orthotropic material, derived by modifying Eq. (6) to have the right asymptotic behavior and fit the finite element results:

$$\frac{K_{\mathrm{IIQ}} h^{0.5}}{Q} = \frac{2\lambda^{1/8}}{\sqrt{n}} + \sqrt{\frac{2h}{\pi d}} - \left[0.559\lambda^{-1/8} \left(\lambda^{1/4} \frac{d}{h} \right)^{0.653} + Y(\rho)\lambda^{-1/8} \exp\left(-1.665\lambda^{-1/4} h/d\right) + 0.5\sqrt{n}\lambda^{-1/8} \right]^{-1} \tag{7a}$$

where: $$Y(\rho) = 1 - 0.105(\rho - 1) + 0.01(\rho - 1)^2 \tag{7b}$$

The validity of Eq. (7a) has been checked through finite element calculations for λ and ρ in the range $0.025 \le \lambda \le 1$ and $1 \le \rho \le 5$ with errors always less than 1.5 % for all crack lengths higher than $a/h = 1.2\lambda^{-1/4}$. As for the isotropic case, Eq. (7) has been modified in [6] to describe cracks of any length.

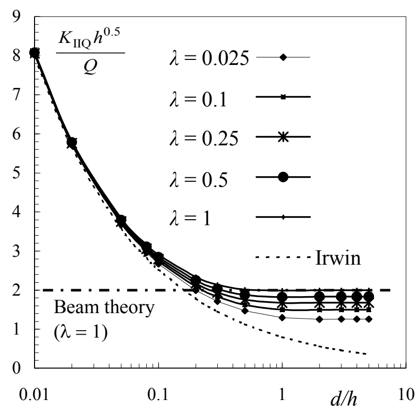


Figure 3: dimensionless stress intensity factor in a degenerate orthotropic double cantilever beam loaded in mode II. Influence of λ on the solution.

4. VALIDATION OF THE PROPOSED WEIGHT FUNCTIONS

The stress intensity factors derived in the previous section allow for the definition of the stress intensity factor, K_{IIq}, at the tip of the crack in double cantilever beams subject to generic distributions of tangential tractions, $q(x_1)$, acting along the crack faces:

$$K_{\mathrm{IIq}} = \int_{a_0}^{x_{1f}} h_{\mathrm{II}}(x_1, a, \lambda, \rho)\, q(x_1)\, \mathrm{d}x_1 \tag{8}$$

where $h_{\mathrm{II}}(x_1, a,\lambda,\rho)$ is the mode II weight function of the problem, $h_{\mathrm{II}}(x_1, a,\lambda,\rho) = K_{\mathrm{IIQ}}/Q$ and a_0 and x_{1f} are the limits of the loaded region.

For normalized crack lengths $a/h > \lambda^{-1/4}$ in isotropic and orthotropic materials the validity of the proposed weight functions has been checked through a comparison with the numerical solution obtained by He and Evans [8] for the energy release rate in an orthotropic End Notched Flexural specimen (a three-point bending beam of thickness $2h$, length L and with a mid-plane delamination of length a). The approximate expression for the energy release rate was determined in [8] by fitting finite element results with an accuracy of 1% for $a/h > \lambda^{-1/4}$ and $1 \le \rho \le 10$. The corresponding dimensionless stress intensity factor is:

$$\frac{K_{\mathrm{II}\tau}}{\tau h^{0.5}} = \frac{2\lambda^{1/8}}{\sqrt{n}} \frac{a}{h} \left(1 + Y(\rho)\lambda^{-1/4} \frac{h}{a}\right) \quad \text{for } a/h > \lambda^{-1/4} \tag{9}$$

$$Y(\rho) = 0.209 + 0.064(\rho - 1) - 0.00266(\rho - 1)^2$$

where τ is the tangential stress that would be generated at the mid-plane of the specimen in the absence of a crack ($\tau = 3P/8h$, with P the external load) . The relative error between Eq. (9)

and the stress intensity factor calculated using the weight function of Eq. (7) is lower than 0.5% for all crack lengths $a/h > \lambda^{-1/4}$ in isotropic and degenerate orthotropic materials and always lower than 1.5% in generally orthotropic materials.

For isotropic materials, the validity of Eq. (3) and its modified form valid for crack of any length [6], is also confirmed by a comparison with the weight function derived by Fett and Munz [9] for an edge cracked plate subject to a uniform distribution of tangential tractions, τ, acting along the crack faces. Fett gives two solutions for different ranges of the crack length that are given below in a dimensionless form:

$$\frac{K_{II\tau}}{\tau h^{0.5}} = 2\frac{a}{h} + 0.431 \quad \text{for } a/h \geq 0.467 \quad ; \quad \frac{K_{II\tau}}{\tau h^{0.5}} = 1.1215\sqrt{\frac{\pi a}{h}} \quad \text{for } a/h < 0.467 \qquad (10)$$

Both solutions are given with an accuracy of more than 2%. Eq. (10) leads to maximum deviations of less than 0.6% from Eq. (3) when $a/h \geq 0.8$ and maximum deviations of less than 1.5% from the modified form of Eq. (3) in the range $a/h \leq 0.3$ [6]. Larger errors up to 4% are found in the range $0.3 < a/h < 0.8$, where however Eqs.(3) and its modified form were found to agree very well with our finite element calculations.

5. CONCLUSIONS

Approximate weight functions have been derived and validated for isotropic and orthotropic double cantilever beams loaded in mode II. The weight functions are the essential prerequisite for integral equation formulations of fracture problems in which contact, friction and other crack wake mechanisms are present on small and large scales. The weight functions will allow equally accurate and convenient solution of crack initiation problems, in which the crack length remains smaller than or comparable to the laminate half-width; and large-scale bridging problems, in which bridging effects, including the action of contact and friction, may extend over zones many tens of times the laminate half-width.

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