MICRO-MACRO SCALE TRANSITION FOR DAMAGED VISCOELASTIC PARTICULATE COMPOSITES

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ABSTRACT

The complex problem of micromechanically based constitutive description of composite time-dependent materials for which damage consists in grain-matrix debonding is treated starting from the homogenization framework initially proposed by Christoffersen [1]. The localization relations as well as the homogenized stress are established and discussed for a fixed state of damage (i.e. for a given actual number of open and closed cracks) and using the hypothesis of no sliding on closed crack lips. The basic formulation is completed by a second stage leading to a thermodynamically consistent and simplified formulation of the model eliminating some superfluous damaged-induced strain like variables related to open cracks and replacing a full set of relaxation internal variables by a single global tensorial variable.

1 INTRODUCTION

This paper deals with a two step scale transition, for modelling the anisotropic damage behaviour of viscoelastic particulate composites, starting from the methodology initially proposed by Christoffersen [1] for elastic bonded granulates. By means of a specific geometrical and kinematical description giving much attention to the granular character, this methodology offers an advantage of accounting - in a direct manner – for initial morphology and internal organization of a large class of particulate composites. The recent extension of the method, performed by Nadot-Martin et al. [2] for composites involving a viscoelastic matrix, has confirmed its efficiency since it allows to account for genuine viscoelastic interactions between constituents and for their macroscopic consequence, the "long range memory" effect. The present contribution attempts to extend further the technique in presence of damage by grain-matrix debonding. Section 2 recapitulates the mean ingredients and results of the scale transition with special attention paid to coupling between damage - embodied by two damage tensors involving granular aspects and emerging from the scale transition - and viscoelasticity (see also Nadot-Martin et al. [3]). In Section 3, a complementary localization-homogenization procedure appears crucial to ensure both the thermodynamic consistence and further applicability of the model final formulation.

2 VISCOELASTIC SCALE TRANSITION IN PRESENCE OF DAMAGE: MEAN TOOLS AND RESULTS

2.1 Micro-mechanical analysis and behaviour

Figure 1 (a) shows a close-up schematic for grains separated by matrix layers according to the scheme initially proposed by Christoffersen [1] for sound, i.e. undamaged, particulate composites. The grains are considered as polyhedral; any two of them are interconnected by a thin material layer of a given uniform thickness. The grain-layer interfaces are characterized by their orientation ($\underline{\mathbf{n}}^{\alpha}$ for the αth layer). Some granulometry is accounted for through the vectors linking grain centroids ($\underline{\mathbf{d}}^{\alpha}$ for the αth layer). The damage by grain/matrix debonding is incorporated in the form of material discontinuities (cracks) located at grain/layer interfaces (see Figure 1 (b)).

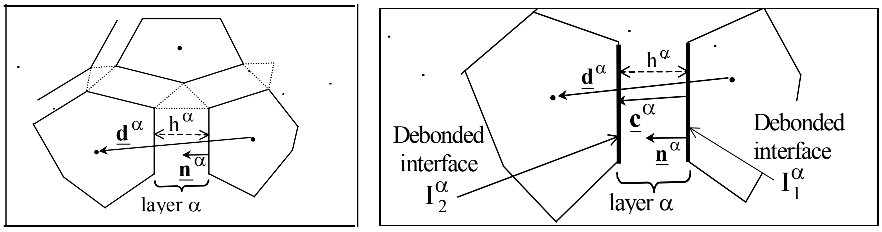

Figure 1: (a) Two neighbouring grains with an interconnecting material layer according to Christoffersen [1]. (b) A layer with cracks at its boundaries.

The grains are assumed isotropic and linear elastic. The matrix occupying each elementary layer α is considered as viscoelastic and isotropic according to a thermodynamically consistent internal variable representation (Nadot-Martin et al. [2]). The dissipative process related to viscoelastic relaxation is accounted for via the symmetric, strain-like, tensorial internal variable $\boldsymbol{\gamma}$. The free energy per unit volume and correspondingly the total stress, are decomposed into two terms, a reversible one, function of the total strain $\boldsymbol{\varepsilon}$, and a viscous one, function of $\boldsymbol{\gamma}$. The reversible and viscous stresses are obtained by partial derivation of the free energy respectively with respect to $\boldsymbol{\varepsilon}$ and $\boldsymbol{\gamma}$. The evolution of $\boldsymbol{\gamma}$ which can be interpreted as 'delayed elastic' strain is given by the law (2) employing, for simplicity, a single relaxation time υ:

$$\mathrm{w}(\boldsymbol{\varepsilon},\boldsymbol{\gamma}) = \frac{1}{2}\,\boldsymbol{\varepsilon} : \mathrm{L}^{(e)\ell} : \boldsymbol{\varepsilon} + \frac{1}{2}\,\boldsymbol{\gamma} : \mathrm{L}^{(v)} : \boldsymbol{\gamma} \qquad \boldsymbol{\sigma} = \boldsymbol{\sigma}^{(r)} + \boldsymbol{\sigma}^{(v)} = \mathbf{L}^{(e)\ell} : \boldsymbol{\varepsilon} + \mathbf{L}^{(v)} : \boldsymbol{\gamma} \tag{1}$$

$$\dot{\boldsymbol{\gamma}} + \frac{1}{\upsilon}\boldsymbol{\gamma} = \dot{\boldsymbol{\varepsilon}} \quad ; \; \boldsymbol{\gamma}\,(\mathrm{t}=0) = 0 \qquad \mathrm{d}^{(v)} = \boldsymbol{\sigma}^{(v)} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\gamma}}) = \frac{1}{\upsilon}\,\boldsymbol{\gamma} : \mathbf{L}^{(v)} : \boldsymbol{\gamma} \geq 0 \tag{2}$$

$\mathbf{L}^{(e)\ell}$ and $\mathbf{L}^{(v)}$ are respectively the fourth-order tensors of elastic and viscous stiffness for the matrix.

2.2 Local problem approach and expression of overall stress

The material discontinuities and relative displacement jumps are incorporated in a compatible way with the Christoffersen's original kinematical description. The latter is defined by four assumptions for the local displacement field that are recalled below (see Nadot-Martin et al. [2] for relative analysis). The kinematics of grain centroid is characterized by the global displacement gradient $\mathbf{F}$. The grains are supposed homogeneously deformed and the corresponding displacement gradient $\mathbf{f}^0$ assumed to be common to all members of the statistically representative volume element (SRVE). Each layer is subject to a homogeneous displacement gradient, proper to the layer α and noted $\mathbf{f}^{\alpha}$. Local disturbances at grain edges and corners are neglected in regard to thinness of the layers. In addition to imply the displacement jump linearity, the previous assumptions, and more precisely the second one regarding $\mathbf{f}^0$ as common to all grains, impose only two possible configurations for a layer α: either its two boundaries are cohesive, or they are both debonded. The second configuration is described with two mean displacement discontinuity vectors of opposite signs. By taking into account relative conditions of displacement jump, the displacement gradient $\mathbf{f}^{\alpha}$ for a debonded layer α (see Figure 1 (b)), is given by (Nadot-Martin et al. [3]):

$$\mathrm{f}_{ij}^{\alpha} = \mathrm{f}_{ij}^{0} + \left(\mathrm{F}_{ik} - \mathrm{f}_{ik}^{0}\right)\mathrm{d}_{k}^{\alpha}\,\mathrm{n}_{j}^{\alpha}/\mathrm{h}^{\alpha} + \mathrm{f}_{ij}^{\alpha D} \qquad \left\langle \mathrm{b}_{i}^{\alpha} \right\rangle_{I_1^{\alpha}} = -\left\langle \mathrm{b}_{i}^{\alpha} \right\rangle_{I_2^{\alpha}} = -\frac{1}{2}\,\mathrm{f}_{ij}^{\alpha D}\,\mathrm{c}_{j}^{\alpha} \tag{3}$$

with h^{α} the thickness of the layer and $\underline{\mathbf{c}}^{\alpha}$ the vector connecting the centres of two opposite interfaces. For a layer α whose both interfaces are cohesive $\mathbf{f}^{\alpha}$ is obtained by using the continuity of displacements on the grain/layer interfaces:

$$f_{ij}^{\alpha} = f_{ij}^{0} + \left(F_{ik} - f_{ik}^{0}\right) d_k^{\alpha}\, n_j^{\alpha}/h^{\alpha} \tag{4}$$

The supplementary term $\mathbf{f}^{\alpha D}$ in eqn (3) represents the specific contribution of two microcracks located at the boundaries of the debonded layer considered. In view of (3)-(4), strain as well as rotation in any layer depends on its geometrical features. Such a dependence allows to account for microstructure effects on deformation mechanisms of the matrix. Due to the assumption neglecting interlayer zones, the transmission through grains-and-layers assembly is strongly involving grains as expressed through the presence of $\mathbf{f}^{0}$ in eqns (3)-(4).

By considering the SRVE loaded on its boundaries by uniform tractions represented by a given macroscopic stress and using the generalized Hill lemma (i.e. the principle of macro-homogeneity in the presence of damage), one may prove that (Nadot-Martin et al. [3]):

$$\Sigma_{ij} = \left\langle \sigma_{ij} \right\rangle_V = \frac{1}{|V|} \sum_{\alpha} t_i^{\alpha} d_j^{\alpha} \qquad , \quad t_i^{\alpha} = \sigma_{ki}^{\alpha} n_k^{\alpha} A^{\alpha} \tag{5}$$

$|V|$ represents the volume of grains and layers excluding interlayer zones, A^{α} is the projected area of the αth layer and $\underline{\mathbf{t}}^{\alpha}$ represents the total force transmitted through the interfacial layer, $\boldsymbol{\sigma}^{\alpha}$ being the average stress in this latter. Although the first averaging is “classically” exploited the second one remains specific to the Christoffersen-type approach: stresses are seen from a granular viewpoint as forces transmitted from grain to grain by layers acting as contacts zones.

The following consists in searching $\mathbf{f}^{0}$ as such a way that the real stress field, namely this associated to the strain field by local constitutive laws, satisfies eqn (5). All the grains have identical moduli noted $\mathbf{L}^{0}$. The mechanical properties of the matrix ($\mathbf{L}^{(e)\ell}$, $\mathbf{L}^{(V)}$, υ) are considered as homogeneous, namely the same for all layers. Consequently, as $\boldsymbol{\varepsilon}^{\alpha} = \mathrm{Sym}.\mathbf{f}^{\alpha}$ is uniform over the αth layer, the corresponding relaxation $\boldsymbol{\gamma}$ introduced by eqn (2) is also uniform, it is noted $\boldsymbol{\gamma}^{\alpha}$. Furthermore, the average stress $\boldsymbol{\sigma}^{\alpha}$ in eqn (5) becomes a uniform field for the αth layer and $\underline{\mathbf{t}}^{\alpha}$ is the mean force transmitted through its first interface I_1^{α}. For a debonded layer α, two cases are considered. When the cracks located at its boundaries are open then $\underline{\mathbf{t}}^{\alpha} = \underline{\mathbf{0}}$. When they are closed it is supposed that no sliding is allowed so that $\underline{\mathbf{t}}^{\alpha}$ is integrally transmitted. The solution is then:

$$\begin{aligned} f_{ij}^{0} = {} & \left(Id^{l} - B'^{-1} : A'\right)_{ijkl} F_{lk} - B'^{-1}_{ijuv} L^{(v)}_{mukl} \left\{ \frac{1}{|V|} \sum_{\alpha'} \Pi_{vm}^{\alpha'} \gamma_{lk}^{\alpha'} A^{\alpha'} h^{\alpha'} + \delta_{vm} \frac{1}{|V|} \sum_{\beta} \gamma_{lk}^{\beta} A^{\beta} h^{\beta} \right\} \\ & - B'^{-1}_{ijuv} L^{(e)\ell}_{mukl} \left\{ \frac{1}{|V|} \sum_{f} \Pi_{vm}^{f} \varepsilon_{lk}^{fD} A^{f} h^{f} + \delta_{vm} \frac{1}{|V|} \sum_{\beta} \varepsilon_{lk}^{\beta D} A^{\beta} h^{\beta} \right\} \quad ; \quad \boldsymbol{\varepsilon}^{\beta D} = \mathrm{Sym}\mathbf{f}^{\beta D},\ \boldsymbol{\varepsilon}^{fD} = \mathrm{Sym}\mathbf{f}^{fD} \end{aligned} \tag{6}$$

with $\Pi_{ij}^{\alpha} = \delta_{ij} - d_i^{\alpha}\, n_j^{\alpha}/h^{\alpha}$ and where $\mathbf{A}'$, $\mathbf{B}'$, degraded by the presence of damage, are defined by:

$$A'_{ijkl} = \left\langle L^{(e)}_{ijkl} \right\rangle_V - L^{(e)\ell}_{mjkl}\left(\delta_{im} - D_{im}\right) \qquad\qquad A_{ijkl} = \left\langle L^{(e)}_{ijkl} \right\rangle_V - L^{(e)\ell}_{ijkl} \tag{7}$$

$$B'_{ijkl} = A_{ijkl} - L^{(e)\ell}_{mjkl}\left(\delta_{im} - D_{im}\right) + L^{(e)\ell}_{mjnl}\left(\overline{T}_{imkn} - \overline{D}_{imkn}\right) \tag{8}$$

$$D_{ij} = \frac{1}{|V|}\sum_{\beta} d_i^\beta n_j^\beta A^\beta\,,\quad \overline{D}_{ijkl} = \frac{1}{|V|}\sum_{\beta} d_i^\beta n_j^\beta d_k^\beta n_l^\beta A^\beta/h^\beta\,,\quad \overline{T}_{ijkl} = \frac{1}{|V|}\sum_{\alpha} d_i^\alpha\, n_j^\alpha\, d_k^\alpha\, n_l^\alpha\, A^\alpha/h^\alpha \tag{9}$$

Superscripts α, α', β and f denote summations over respectively all layers, layers either cohesive or with closed cracks, layers with open cracks only and layers with closed cracks only. The form of eqn (6) represents a remarkable decomposition into a reversible term, a viscous one $\mathbf{f}^{0(v)}$ function of internal variables γ^α for α=1,…,N - with N being the total number of layers inside the SRVE - and a damage-induced one $\mathbf{f}^{0(d)}$ involving the full set $\{\boldsymbol{\varepsilon}^{kD}\} = \{\boldsymbol{\varepsilon}^{fD}\} \cup \{\boldsymbol{\varepsilon}^{\beta D}\}$. These three contributions depend on the damage state through the tensors $\mathbf{D}$ and $\overline{\mathbf{D}}$ (see $\mathbf{A}'$, $\mathbf{B}'$). The same can be done for $\mathbf{f}^\alpha$ after employing eqns (3) and (4). The local strain field is then given by:

$$\boldsymbol{\varepsilon}(\underline{y}) = \mathbf{C}(\underline{y},\mathbf{D},\overline{\mathbf{D}}) : \mathbf{E} + \boldsymbol{\varepsilon}^{(v)}(\underline{y}) + \boldsymbol{\varepsilon}^{(d)}(\underline{y}) + \begin{cases} \boldsymbol{\varepsilon}^{\alpha D} & \text{for } \underline{y} \in \text{debonded layer } \alpha \\ \mathbf{0} & \text{elsewhere} \end{cases} \tag{10}$$

$$C_{ijkl}(\underline{y},\mathbf{D},\overline{\mathbf{D}}) = \begin{cases} C^0_{ijkl}(\mathbf{D},\overline{\mathbf{D}}) = (\mathrm{Id} - \mathrm{Id} : B'^{-1} : A')_{ijkl} & \text{for } \underline{y} \in \text{grains} \\ C^\alpha_{ijkl}(\mathbf{D},\overline{\mathbf{D}}) = \mathrm{Id}_{ijkl} - \mathrm{Id}_{ijuv}\,(B'^{-1} : A')_{vmkl}\,\Pi^\alpha_{mu} & \text{for } \underline{y} \in \text{layer } \alpha\ \forall\alpha \end{cases} \tag{11}$$

$$\varepsilon^{(v)}_{ij}(\underline{y}) = \begin{cases} \varepsilon^{0(v)}_{ij}(\{\gamma^\alpha\},\mathbf{D},\overline{\mathbf{D}}) = \mathrm{Id}_{ijkl}\, f^{0(v)}_{lk} \\ \varepsilon^{\alpha(v)}_{ij}(\{\gamma^\alpha\},\mathbf{D},\overline{\mathbf{D}}) = \mathrm{Id}_{ijuv}\, f^{0(v)}_{vm}\, \Pi^\alpha_{mu} \end{cases} \quad \varepsilon^{(d)}_{ij}(\underline{y}) = \begin{cases} \varepsilon^{0(d)}_{ij}(\{\boldsymbol{\varepsilon}^{kD}\},\mathbf{D},\overline{\mathbf{D}}) = \mathrm{Id}_{ijkl}\, f^{0(d)}_{lk} & \text{grains} \\ \varepsilon^{\alpha(d)}_{ij}(\{\boldsymbol{\varepsilon}^{kD}\},\mathbf{D},\overline{\mathbf{D}}) = \mathrm{Id}_{ijuv}\, f^{0(d)}_{vm}\, \Pi^\alpha_{mu} & \forall \text{ layer } \alpha \end{cases} \tag{12}$$

with $\mathrm{Id}_{ijkl} = \frac{1}{2}(\delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{jk})$. At last, the overall (average) stress is derived from (5):

$$\boldsymbol{\Sigma} = \left[\left\langle \mathbf{L}^{(e)} \right\rangle_V - \mathbf{A} : \mathbf{B}'^{-1} : \mathbf{A}'\right] : \mathbf{E} + \boldsymbol{\Sigma}^{(v)}(\{\gamma^\alpha\},\mathbf{D},\overline{\mathbf{D}}) + \boldsymbol{\Sigma}^{(d)}(\{\boldsymbol{\varepsilon}^{kD}\},\mathbf{D},\overline{\mathbf{D}}) \tag{13}$$

$$\boldsymbol{\Sigma}^{(v)} = \mathbf{A} : \mathbf{f}^{0(v)}(\{\gamma^\alpha\},\mathbf{D},\overline{\mathbf{D}}) + \mathbf{L}^{(v)} : \frac{1}{|V|}\sum_\alpha \gamma^\alpha\, A^\alpha\, h^\alpha\,,\quad \boldsymbol{\Sigma}^{(d)} = \mathbf{A} : \mathbf{f}^{0(d)}(\{\boldsymbol{\varepsilon}^{kD}\},\mathbf{D},\overline{\mathbf{D}}) + \mathbf{L}^{(e)\ell} : \frac{1}{|V|}\sum_k \boldsymbol{\varepsilon}^{kD}\, A^k\, h^k \tag{14}$$

2.3 Discussion

The strain for any point in grains or layers depends through the term $\boldsymbol{\varepsilon}^{(v)}$ on the full set of relaxations $\{\gamma^\alpha\}$. The internal variable γ^α representing the αth layer's memory, such a dependence clearly indicates viscoelastic interactions between the full set of matrix layers and the set of grains in the SRVE. This dependence directly results from the term $\mathbf{f}^{0(v)}$ which, by means of (3) and (4) appears in the expression of $\mathbf{f}^\alpha$ and therefore in that of the strain field. The structure of $\mathbf{f}^{0(v)}$ (see the second term in (6)) indicates strong complexity of local viscoelastic interactions. Secondly, the strain field depends on damage through the tensors $\mathbf{D}$ and $\overline{\mathbf{D}}$ but also through the term $\boldsymbol{\varepsilon}^{(d)}$ involving the full set $\{\boldsymbol{\varepsilon}^{kD}\} = \{\boldsymbol{\varepsilon}^{fD}\} \cup \{\boldsymbol{\varepsilon}^{\beta D}\}$ related to the effect of any kind of cracks (open or closed) inside the SRVE. The latter dependence results from the term $\mathbf{f}^{0(d)}$ (see the third term in (6)). In particular, for a debonded layer α one may distinguish two kinds of contribution of damage on its strain: a “local” one, $\boldsymbol{\varepsilon}^{\alpha D}$, related to the microcracks located at its own boundaries and a “non local” one, $\boldsymbol{\varepsilon}^{\alpha(d)}$, involving the effect of the whole set of microcracks inside the SRVE. The overall stress is split into a reversible and a viscous part influenced by open cracking related damage through $\mathbf{D}$ and $\overline{\mathbf{D}}$ and completed by a damage-induced stress $\boldsymbol{\Sigma}^{(d)}$. The first terms of $\boldsymbol{\Sigma}^{(v)}$ and $\boldsymbol{\Sigma}^{(d)}$, namely $\mathbf{A} : \mathbf{f}^{0(v)}$ and $\mathbf{A} : \mathbf{f}^{0(d)}$ in eqn (14), correspond respectively to the macroscopic consequence of viscoelastic interactions and damage “non-local” effects. Damage "non-local" effects lead also to the quadractic dependence of the total stress on **D** (see (13)). It is already seen that $\mathbf{A}'$, $\mathbf{B}'$ and therefore the reversible moduli are only degraded by the open cracks via **D** and $\overline{\mathbf{D}}$. This is due to the assumption of no sliding on closed crack lips. These two tensors are natural candidates for macroscopic damage variables. Being tensorial by nature, they allow to account for damage induced anisotropy. Moreover, the moduli may be recovered with crack closure showing that the model is potentially capable of describing unilateral effects (see Nadot-Martin et al. [3] for more details).

3 A COMPLEMENTARY LOCALIZATION-HOMOGENIZATION ANALYSIS

The homogenized stress (13) conveys a full set $\{\boldsymbol{\varepsilon}^{kD}\}$ in addition to the damage tensors **D** and $\overline{\mathbf{D}}$. Remarking in (6) that $\mathbf{f}^{0(d)}\left(\{\boldsymbol{\varepsilon}^{kD}\},\mathbf{D},\overline{\mathbf{D}}\right)=\mathbf{f}^{0(d)1}\left(\{\boldsymbol{\varepsilon}^{\beta D}\},\mathbf{D},\overline{\mathbf{D}}\right)+\mathbf{f}^{0(d)2}\left(\{\boldsymbol{\varepsilon}^{fD}\},\mathbf{D},\overline{\mathbf{D}}\right)$, one may discern that the respective contributions in $\boldsymbol{\Sigma}^{(d)}$ (see (14)) of open and closed cracks are clearly additive:

$$\boldsymbol{\Sigma}^{(d)}\left(\{\boldsymbol{\varepsilon}^{kD}\},\mathbf{D},\overline{\mathbf{D}}\right)=\underbrace{\mathbf{A}:\mathbf{f}^{0(d)1}+\mathbf{L}^{(e)\ell}:\frac{1}{|V|}\sum_{\beta}\boldsymbol{\varepsilon}^{\beta D}A^{\beta}h^{\beta}}_{\boldsymbol{\Sigma}^{(d)1}\left(\{\boldsymbol{\varepsilon}^{\beta D}\},\mathbf{D},\overline{\mathbf{D}}\right)}+\underbrace{\mathbf{A}:\mathbf{f}^{0(d)2}+\mathbf{L}^{(e)\ell}:\frac{1}{|V|}\sum_{f}\boldsymbol{\varepsilon}^{fD}A^{f}h^{f}}_{\boldsymbol{\Sigma}^{(d)2}\left(\{\boldsymbol{\varepsilon}^{fD}\},\mathbf{D},\overline{\mathbf{D}}\right)} \qquad (15)$$

In (15), $\{\boldsymbol{\varepsilon}^{fD}\}$ acquire the status of macroscopic internal variables accounting for the distortion due to the blockage of closed cracks inside the SRVE and $\boldsymbol{\Sigma}^{(d)2}$ appears as the corresponding residual stress. Let examine now the status of $\{\boldsymbol{\varepsilon}^{\beta D}\}$. At the microscopic level, $\boldsymbol{\varepsilon}^{\beta D}$ represents, for a layer β, the "local" contribution on its strain of the open cracks located at its own boundaries. It seems natural to think that the crack opening depends on the total strain **E** and therefore $\boldsymbol{\varepsilon}^{\beta D}$ also, so that each $\boldsymbol{\varepsilon}^{\beta D}$ cannot *a priori* be considered as a macroscopic variable independent of **E**. This is confirmed when noting that reversible moduli in (13) have not all the symmetries required for effective moduli tensor suggesting that $\boldsymbol{\Sigma}^{(d)1}$ must depend, through $\{\boldsymbol{\varepsilon}^{\beta D}\}$, on **E**. Moreover, the representation (13) contains the whole set $\{\gamma^{\alpha},\alpha=1\cdots N\}$. Considering the possible large number of layers contained in the SRVE, this is not really the most convenient form for engineering applications. In order to get an applicable model it is assumed that a single second-order symmetric strain-like tensorial variable $\boldsymbol{\Gamma}$ can be substituted for the set $\{\gamma^{\alpha}\}$ to describe the relaxation state of the composite. The purpose of this section consists then in establishing two kinds of relations, the first one : $\boldsymbol{\gamma}^{\alpha}=\boldsymbol{\gamma}^{\alpha}(\boldsymbol{\Gamma})$ for α=1,…,N as it was done by Nadot-Martin et al. [2] for the sound material but here should be done in the presence of damage and, simultaneously, the second : $\boldsymbol{\varepsilon}^{\beta D}=\boldsymbol{\varepsilon}^{\beta D}(\mathbf{E},\boldsymbol{\Gamma})$ for each layer β with open cracks at its boundaries. These relations are not *a priori* postulated so that the advanced strategy can be viewed as a complementary "localization" analysis involving two aims. First, it appears from thermodynamic requirements that it is necessary to clarify the status of local strains $\boldsymbol{\varepsilon}^{\beta D}$. Then, concerning the relaxation internal variables, their aggregated expression is principally motivated by the model's applicability. To reach these two aims simultaneously, thermodynamic framework is used as a guide. The solution is given by considering **D** and $\overline{\mathbf{D}}$ as parameters related to the damage configuration considered. In this way, the expression of the overall viscoelastic dissipation with respect to **E** and **Γ** opens a way to determine the above mentioned relations. They are searched as such a way that $\Sigma=\langle\sigma\rangle_V=\dfrac{\partial\langle w\rangle_V}{\partial\mathbf{E}}+\dfrac{\partial\langle w\rangle_V}{\partial\boldsymbol{\Gamma}}$. The problem is solved by postulating the linearity of $\boldsymbol{\gamma}^{\alpha}=\boldsymbol{\gamma}^{\alpha}(\boldsymbol{\Gamma})$

and the decomposition of $\boldsymbol{\varepsilon}^{\beta D}=\boldsymbol{\varepsilon}^{\beta D}(\mathbf{E},\boldsymbol{\Gamma})$ in two parts respectively linear in $\mathbf{E}$ and $\boldsymbol{\Gamma}$ and independent of $\{\boldsymbol{\varepsilon}^{fD}\}$. After some manipulations, it follows:

$$\varepsilon_{ij}^{\beta D}=-\frac{1}{2}\,\mathrm{Id}_{ijmu}\,d_v^{\beta}n_m^{\beta}/h^{\beta}\left(\left(B'^{-1}:A'\right)-K'\right)_{uvkl}\left(E+\Gamma\right)_{lk}+r_{ij}^{\beta D}\quad \forall \text{ layer } \beta \text{ with open cracks} \tag{16}$$

$$\gamma_{ij}^{\beta}=\left[-\,\mathrm{Id}_{ijvu}\left(\left(B'^{-1}:A'\right)-K'\right)_{uvkl}+\mathrm{Id}_{ijkl}\right]\Gamma_{lk}+\gamma_{ij}^{\beta Res}\quad \forall \text{ layer } \beta \text{ with open cracks} \tag{17}$$

$$\gamma_{ij}^{\alpha'}=\left[-\,\mathrm{Id}_{ijmu}\,\Pi_{vm}^{\alpha'}\left(\left(B'^{-1}:A'\right)-K'\right)_{uvkl}+\mathrm{Id}_{ijkl}\right]\Gamma_{lk}+\gamma_{ij}^{\alpha' Res}\quad \text{for others layers} \tag{18}$$

where $\mathbf{K}'$ is a complex structural tensor depending on $\mathbf{A}$, $\mathbf{A}'$ and $\mathbf{B}'$ given by eqns (7)-(8) and on their equivalents $\mathbf{A}^{(v)}$, $\mathbf{A}'^{(v)}$ and $\mathbf{B}'^{(v)}$ in which $\mathbf{L}^{(v)}$ replaces $\mathbf{L}^{(e)\ell}$. In view of (16), the strain induced in a layer by the open cracks at its interfaces is controlled by the macroscopic state variables $\mathbf{E}$ and $\boldsymbol{\Gamma}$, $\mathbf{D}$ and $\overline{\mathbf{D}}$ the damage parameters (through $\mathbf{A}'$ and $\mathbf{B}'$) but also by the geometrical features of the layer β under consideration. The constant $\mathbf{r}^{\beta D}$ represents a residual strain induced in this layer by a residual opening of the cracks at its boundaries when $\mathbf{E}=\boldsymbol{\Gamma}=\mathbf{0}$. In eqns (17) and (18), the constants $\boldsymbol{\gamma}^{\beta Res}$ and $\boldsymbol{\gamma}^{\alpha' Res}$ correspond to non perfectly relaxed states of the layers when the composite is relaxed (for $\boldsymbol{\Gamma}=\mathbf{0}$). In the limit cases where there is no open crack inside the SRVE, i.e. for only closed cracks or for the sound material, $\boldsymbol{\varepsilon}^{\beta D}$ and $\boldsymbol{\gamma}^{\beta}$ are nulls and one may observe that the expression (18) of $\boldsymbol{\gamma}^{\alpha}$ for any layer corresponds, as expected, to that obtained for the sound material by Nadot-Martin et al. [2]. This result constitutes a proof of the pertinency of the two-stage complementary localization-homogenization approach. With eqns (16)-(18), one may formulate the whole model, giving local and global responses of the composite, in terms of state variables $\mathbf{E}$, $\boldsymbol{\Gamma}$, $\{\boldsymbol{\varepsilon}^{fD}\}$ and damage parameters $\mathbf{D}$ and $\overline{\mathbf{D}}$. The model comprises the evolution law of $\boldsymbol{\Gamma}$ obtained by averaging the local evolution rule (2) over all elementary layers. The homogenized degraded elastic and viscous moduli have then all the symmetries available.

4 CONCLUSION

A non-classical homogenization method that constitutes an extension of the Christoffersen-type approach for both viscoelasticity and damage by grain/matrix debonding is presented. It leads to the natural emergence of two macroscopic damage tensorial variables, involving granular aspects, in order to describe moduli degradation, induced anisotropy and unilateral effects. These variables - in addition to the textural tensor $\overline{\mathbf{T}}$ (see (9)) related to initial morphology and internal organisation of constituents - allow to account, in a general 3D context, for coupling of primary anisotropy with the damage-induced one. A complementary localization-homogenization is advanced in order to express the damage-induced strains related to open cracks as functions of macroscopic state variables and simultaneously to replace the set of relaxation variables by a single internal variable. Such an analysis appears crucial to obtain a thermodynamically consistent and applicable formulation of the model. Further works include the detailed treatment of unilateral effects i.e. opening/closure transition modelling and damage evolution.

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