TOWARDS A PHYSICAL DAMAGE VARIABLE FOR A HETEROGENEOUS QUASI-BRITTLE MATERIAL

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ABSTRACT
Damage mechanics models have been applied to a variety of materials for many years because of the elegant way the models handle progressive degradation and failure. Damage models can be developed with a varying degree of sophistication, can provide insight into different types of material behavior, and can be tailored to a wide range of applications. A weakness of traditional damage models is the phenomenological nature of the damage variables. Damage variables must be established from laboratory measurements, which can become expensive depending on the model sophistication. The focus of this work is a first step towards defining damage in terms of physical microstructural changes. Small concrete specimens were subjected to cyclic compressive loading while simultaneously being scanned by a synchrotron-based tomography system. The tomography system allowed us to examine internal structural changes at high spatial resolution. Damage-induced changes in specimen stiffness were evaluated using recorded load-deformation response, while internal damage was measured using an array of 3D image processing techniques. Preliminary measurements suggest a power law relationship between the damage variable and measured microcrack surface area.

1 INTRODUCTION
Continuum damage mechanics is a powerful tool to represent materials that exhibit microstructural changes in the form of distributed cracking or void growth. Damage models have been developed over the past 40 years at varying levels of sophistication, depending on the particular phenomenon the models are intended to depict. Regardless of the level of sophistication, damage is represented by a damage variable that represents the extent of cracking in the solid, and the corresponding change in material stiffness.

While the usual continuum damage approach is intended to represent changes in microstructure that result from various damage regimes, there traditionally has been relatively little effort to link damage variables to the specific microstructural features that lead to the model behavior [1]. Damage is generically associated with a somewhat tenuous definition of “cracks”, and it is applied as a purely empirical, if not phenomenological parameter [2]. Examples of quantitative links between crack density and effective elastic parameters, while valuable, tend to be simplistic in assumptions about crack distributions in order to make the problem analytically tractable (e.g. [3]).

Recent advances in three-dimensional imaging of internal structure have opened up the possibility of more firmly grounding a physical basis for damage variables. The rationale for moving in this direction is fairly simple. Damage mechanics can have little application to assessment of existing structures unless there is some way to determine damage variables from in situ measurements of the material’s condition state. If damage variables are based on some physically real parameter, efforts can be made to develop methods to measure that parameter to determine a quantifiable damage state.

The goal of the work described in this paper was to explore ways in which measured internal cracking and the corresponding measured changes in material stiffness can be presented in a damage mechanics framework. This may be thought of as a necessary step towards quantitatively linking physical microstructure to a useful damage parameter. In previous work [4,5] we examined...
fracture energy in three dimensions. The question then was whether the fracture energy of concrete is indeed a nonlinear process given that we are now able to make fairly comprehensive 3D measurements of complete crack areas. In this work the same data is examined in a damage context rather than a fracture context.

2 EXPERIMENTAL METHODS

A technique called x-ray microtomography [6] was used to produce three-dimensional images of internal structure. The technique uses high intensity x-rays from a synchrotron source that are transmitted through the specimen and captured digitally. The specimen is rotated such that a large number of images at different view angles are made. This sequence of images then undergoes a tomographic reconstruction, where complete three-dimensional spatial information about the internal structure of the specimen is obtained. 3D data is typically represented as a series of cross-sectional “slices,” but can also presented as a 3D rendering as shown in Figure 1.

For the tomographic scanning described here, a small compression load frame was constructed so that scanning could take place while the specimen was under load. The frame included a load cell that measured applied force, and a pair of LVDTs that measured specimen deformation.

The loading protocol was as follows. An initial scan of an undamaged specimen was made prior to loading. Then the specimen was loaded to a prescribed deformation, and a subsequent scan was made. This procedure was repeated for several cycles as illustrated in Figure 2. In each of the successive scans, progressively more damage appears in the specimen. This progressive damage can be clearly seen in the images of Figure 3, which represents roughly the same portion of the specimen at different deformation levels.

A true benefit of the digital data produced by the 3D scans is that we are able to employ quantitative image processing techniques to make actual measurements of internal changes in the material. In this work, two basic image processing steps were taken. First, the images were segmented to separate solid material from void or crack space. This is accomplished by establishing a pixel intensity threshold above which all pixels are labeled as solid, while the remaining are labeled as void. The effect of this is illustrated in Figure 4.
The second step of the image processing is to identify each individual void space for subsequent measurement. This accomplished by examining each dark voxel (a “volume element” – analogous to a 3D pixel), and considering whether adjacent voxels are also dark. If they are, then they are considered part of the same object (crack or void). Once an object is identified its volume, surface area, and other traits can be measured. For this work a fast connected components code, “connect” was used [7]. Crack surface area is determined by counting the number of black voxel faces that are in contact with white voxels, and multiplying that number by the unit voxel area.

3 EXPERIMENTAL MEASUREMENTS AND DAMAGE ANALYSIS
Small mortar specimens were prepared to mimic the mortar phase of a conventional concrete mix. The specimens were 4 mm diameter cylinders 4 mm in height. The small size was dictated by the constraints of the synchrotron x-ray source. The small specimens did allow a relatively high spatial resolution of 6 microns per pixel. The results presented here include only two specimens due to limited access to a shared facility.

Experimental measurements of internal cracks as well as the corresponding changes in specimen stiffness between scans are presented in Table 1. The table shows crack data measured using the 3D digital image analysis described above. As is seen in the table, the small specimens contain very large numbers of crack of void objects. However as can also be seen, if the voids of one or two voxels are removed, the number falls quickly. It should be noted here that in this work no distinction is made between initial void or pore space and crack space. In the damage formulation described below only the change in void space is considered. The assumption here is that damage manifests itself in these images as an increase in void space. Thus separation of cracks from other pores is not necessary.
3.1 Damage Formulation

In this work, because of the extensive heterogeneity in the mortar specimens, (and for the sake of simplicity!) isotropic damage is assumed. Thus, scalar damage variable, $D$, is employed such that the reduced elastic modulus due to damage can be expressed as:

$$ E = E_1 (1 - D), $$  
(1)

where $E_1$ is the elastic modulus of the undamaged specimen. In this work, the elastic modulus was measured for each load cycle, thus the damage variable can be determined by manipulating equation (1):

$$ D = 1 - \frac{E}{E_1}. $$

(2)

$D$ was computed for the specimen of Table 1, and is presented in Table 2, along with cumulative changes in the number of cracks and the total crack area.

Because the ultimate goal is to link the damage variable to measured cracking, a function is sought that relates $D$ to some measurable crack parameter. As a first attempt, following work such as Hutchinson and Jensen [8], crack density is used. Thus, for a specimen of fixed volume, the damage variable is defined by:

$$ D = f(n), $$

(3)

where $n$ is simply the number of cracks. However, in the specimens scanned here, the crack size distribution was fairly large, ranging from a few microns (at the limit of detection) to several thousand microns across. Also, as can be seen in Table 2, even though there is extensive damage in between scans 4 and 5, (as shown by the significant increase in $D$), there was not a significant corresponding increase in the number of cracks. There was however, a more significant increase in

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<th>Table 2. Damage Variables and Crack Measurements</th>
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total crack area, in between those two scans, indicating crack enlargement in addition to formation of new cracks. It seems reasonable then that total crack area might be a better indicator of damage than simply crack density. Thus, the damage variable, might be better represented by:

\[ D = f(A) \]  

(4)

where \( A \) is the increase in crack surface area. Figure 5 illustrates this for the specimen data presented in Table 2. While no attempt is made to make any kind of fit, the plot suggests a potential power law relationship.

4 DISCUSSION AND CONCLUSION

The work described in this paper represents a preliminary attempt to base a scalar damage variable on a measurable microstructural parameter. Clearly there are no fundamental conclusions to be drawn. Rather the work is intended to spark interest in establishing underlying relationships between the changing material microstructure, and the corresponding compliance changes that are a manifestation of damage. A simple scalar damage variable was used here, however higher order tensor representations are certainly possible. A more sophisticated analysis of the tomographic images could include spatial orientations of individual crack surfaces, giving a physical basis to various micromechanics-based damage models. It should be noted, however that in situ field measurement of this kind of quality may not be available in the near future.

![Figure 5. Plot of damage variable as a function of accumulated crack surface area.](image-url)
5 REFERENCES


