

PHYSICS OF SUB-CRITICAL CRACK GROWTH IN A FIBROUS MATERIAL: EXPERIMENTS AND MODEL.

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ABSTRACT

We are interested in slow rupture processes observed when a material is submitted to a constant load below a critical rupture threshold. It is well known that the delay time (or lifetime) of the material before complete macroscopic rupture strongly depends on the applied stress. Thermodynamics has slowly emerged as a possible framework to describe delayed rupture of materials since early experiments have shown temperature dependence of lifetime with an Arrhenius law. On the other hand, efforts are made to describe slow rupture dynamics from rheological properties of the material such as viscoelasticity and plasticity.

To shed light on this problem, it is important to compare experiments and models to distinguish between the different theoretical descriptions. For this purpose, we have studied experimentally the slow growth of a single crack in a fibrous material made of fax paper. Specifically, we have observed that the crack grows by steps of various sizes whose distribution is rather complex and evolves as a function of the crack length. In spite of this complexity, a statistical average of the growth dynamics reveals a very simple behaviour.

We show that a model of thermally activated dynamics is able to reproduce many experimental observations. In particular, we show that the average dynamics is in good agreement with the experimental data. In addition, we find that the distribution of step sizes follows sub-critical point statistics with a power law and a stress-dependent exponential cut-off diverging at the critical rupture threshold. The exponent of the power law predicted by the model (3/2) seems to be slightly too large. Leaving the exponent as a free parameter gives a value 1.23 ± 0.1 . We stress that the material heterogeneity appears in the model only as a characteristic mesoscopic length scale. The fact that a simple model of thermally activated crack dynamics is able to reproduce with a good accuracy our experimental findings may open new perspectives in the description of slow rupture dynamics.

1 INTRODUCTION

Understanding fracture in solid materials is paramount for a safe engineering design of structures. Nevertheless, phenomenology is still very present in fracture modelling and many efforts are still needed to obtain a better physical picture. A puzzling observation is the slow rupture of a material when stressed below a critical threshold. This is often observed when applying a varying stress like in fatigue tests but also when applying a constant external stress. Then, the delay time before rupture (or lifetime of the material) strongly depends on the applied stress. Thermodynamics has slowly emerged as a possible framework to describe delayed rupture of materials since early experiments have shown temperature dependence of lifetime with an Arrhenius law (Brenner [1], Zhurkov [2]). From the determination of the activation energy of rupture in elastic materials, statistical physics models have recently proposed several predictions for lifetime (Golubovic [3], Pomeau [4], Buchel [5], Roux [6], Scorretti [7]) as well as for the average dynamics of a slowly growing crack (Santucci [8]). Efforts are also made to describe slow rupture dynamics from rheological properties of the material such as viscoelasticity and plasticity (Langer [9], Chudnovsky [10]).

Therefore, it is important to make quantitative comparisons between experiments and models in order to be able to differentiate between theoretical descriptions. To give new insight into this problem, we present in this paper an experiment on slow crack growth in a fibrous material and

recall briefly the predictions of a model of thermally activated dynamics which gives qualitative and quantitative explanation of the experimental results. In particular, we show that the average dynamics is in good agreement with the experimental data. In addition, we have observed in the experiments that the crack growth progresses by steps of various sizes whose distribution is rather complex and evolves as a function of the crack length. This behaviour can be explained modelling the material as an elastic square lattice where the crack is pinned by elastic traps and grows due to thermally activated stress fluctuations in an energy landscape with multiple metastable states [13]. The model predicts statistical distribution of step sizes in very reasonable agreement with the experiments and has the typical functional form obtained for sub-critical point statistics. We stress that the material heterogeneity appears in the model only as a characteristic mesoscopic length scale. The effect of disorder in the material properties and the rheological behaviour have not been explicitly included in this simple model.

2 EXPERIMENTAL SET-UP

The experimental system we are going to focus on is a two-dimensional sheet with a macroscopic initial crack submitted to a constant load. This geometry is very useful to follow the crack advance using direct observation while this would be difficult in a three-dimensional geometry because a roughening instability of the crack front line usually occurs. We have used a sheet of fax paper (width $w=21\text{cm}$, length 24cm , thickness $e=50\mu\text{m}$) for which a natural mesoscopic length scale is the fibre size. Scanning electron microscopy has revealed a size distribution of fibres between $4\mu\text{m}$ and $50\mu\text{m}$, with an average $20\mu\text{m}$. In order to obtain reproducible results, the fax paper was kept in a controlled low level humidity at least one day at 10%, and also during the experiment at 5%. In these conditions, the paper Young modulus is $E=3.5\text{GPa}$. The paper sheet is mounted on a tensile machine with both ends attached with glue tape and rolled several times over rigid tubes. The crack is initiated at the centre of the sheet using a calibrated blade. The force F applied to the sample by the tensile machine is measured by a force gage and is perpendicular to the crack direction which corresponds to a crack opening in a mode I configuration. During an experiment, the crack grows and a feedback mechanism keeps F constant with a precision 0.1 to 0.5N and a typical time response 10ms. As a consequence, the stress amplitude at the crack tip increases due to stress concentration effects and the motion of the crack accelerates. A high resolution and high speed digital camera (Photron Ultima 1024) is used to follow the crack growth. Image analysis is performed to extract the length of the crack projected on the main direction of propagation. Although the crack actually follows a sinuous trajectory, its projected length gives the main contribution to the stress intensity factor which measures the amplitude of stress divergence near the tip and verifies: $K\sim\sigma L^{1/2}$, with σ the external constant stress applied to the sheet and L the projected crack length. The stress σ is estimated from F and the area A of a cross-section of the sheet, A being approximatively constant: $\sigma=F/A$. Due to the small thickness of the paper a slight buckling occurs but it has been shown that the scaling with stress and crack length is not significantly modified (Riks [11]). On the other hand, finite width corrections on stress intensity factor have been taken into account (Lawn [12]).

3 AVERAGE GROWTH

A typical growth curve is shown on Figure 1. It clearly appears that the crack does not grow smoothly: essentially, there are periods of rest where the crack tip does not move and periods where it suddenly opens and advances of a certain step size s . We have extensively studied the

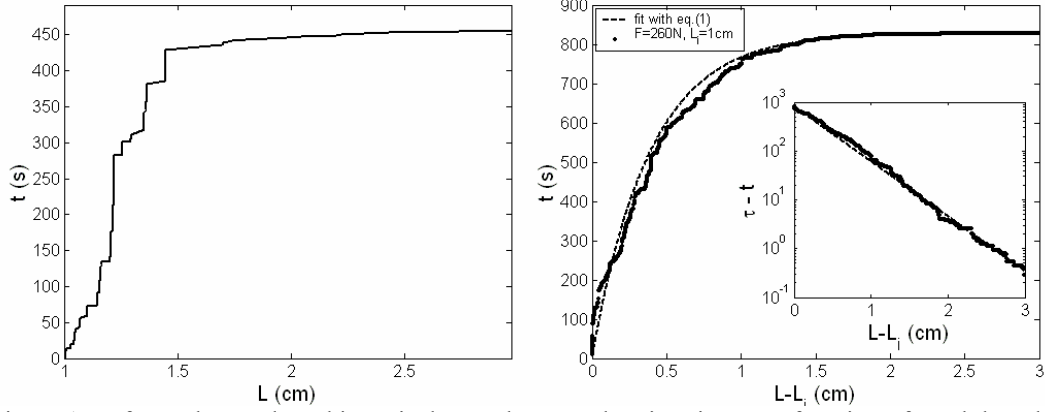


Figure 1: Left: as observed on this typical growth curve showing time as a function of crack length ($F=260\text{N}$, $L_i=1\text{cm}$), the crack grows by steps after periods of rest. Right: the statistical average of 11 growth curves obtained with the same values of F and L_i . The dashed line is equation (1) with $\tau=828\text{s}$ and $\delta=0.38\text{cm}$. The log-lin plot in insert shows that the model remains good close to the rupture time.

growth varying the initial crack length ($1\text{cm}<L_i<4\text{cm}$) and the applied force ($140\text{N}<F<280\text{N}$), equivalent to an initial stress intensity factor K_i between $2.7\text{MPa}\cdot\text{m}^{1/2}$ and $4.2\text{MPa}\cdot\text{m}^{1/2}$. The resulting measured lifetime varied from a few seconds to a few days depending on the value of the applied stress or the temperature. Even for the same experimental conditions (same stress, initial crack length and temperature) a strong dispersion in lifetime was observed as expected in a model of thermally activated growth. Furthermore, this model predicts an average growth dynamics with an approach of lifetime following the functional form (Santucci [8]):

$$t = \tau \left(1 - \exp\left(-\frac{L - L_i}{\delta}\right) \right) \quad (1)$$

In Figure 1, we show an example of statistically averaged growth dynamics and show that equation (1) is a good model for our experiment.

4 STATISTICAL PROPERTIES

We would like now to look more closely at the step size statistics. It is commonly observed that the crack velocity is an increasing function of the stress intensity factor. Thus, it is natural to look at the step statistics for a given value of the stress intensity factor. In practice, we have determined the cumulative step size distributions $P(s)$ for various range of stress intensity factors. Figure 2 shows a loglog plot of $1-P(s)$ determined from all the data we have collected. Typically, 700 data points are used to obtain each distribution. Two regimes are observed. For small step sizes, the distribution does not depend on the value of the stress intensity factor, while for larger step sizes there is a cut-off size increasing with the stress intensity factor. In practice, the toughness of the material, i.e. its critical stress intensity factor $K_c=6.5\pm 0.05\text{MPa}\cdot\text{m}^{1/2}$, has been obtained as the value of K beyond which the probability to detect a jump vanishes.

We have shown in a previous paper that it is possible to obtain the (non-cumulative) probability distribution of step size in a model of thermally activated crack growth (Santucci [13]). The main

conclusion is that the probability distribution has the form of sub-critical statistics in percolation theory:

$$\rho(s) = N \frac{\exp(-s/\xi)}{s^\alpha} \quad (2)$$

where $\alpha=3/2$, $\xi^{-1}=(\sigma_c-\sigma_m)^2V/(2EkT\lambda)$ and N is a normalization constant, the expression of which will not be considered in the rest of the paper. Here, T is the temperature, k is the Boltzmann constant, V is a volume and λ is of the order of the fiber size in the material. The stresses σ_m and σ_c are respectively the stress at the crack tip and the critical bulk stress. We estimate the stress σ_m near the crack tip by assuming that the fiber size acts as a cut-off length preventing divergence and writes $\sigma_m=K/\lambda^{1/2}$. The only unknown in the model is the volume V . This is the volume at which the statistical stress fluctuations are large enough to trigger rupture.

Although the model is able to reproduce many important features of the step size distribution, it is possible to empirically improve the model. We are going to do this changing only one parameter in the distribution, that is the exponent α . Leaving two parameters free, α and V , and setting $\lambda=50\mu\text{m}$, we obtain a constant value for the exponent : $\alpha=1.23\pm 0.1$. Note that this exponent is only slightly lower than the exponent $3/2$ predicted by Santucci [13]. Now, leaving only V free and setting $\alpha=1.23$, we find the curves shown in Figure 2 (left). The adjustment is good for 2 orders of magnitude in $1-P(s)$. For larger step sizes, it is clear from the scatter in data points that there is not enough measurements which might be an explanation for the failure of the model. More interesting are the values obtained for the volume V . We observe that this parameter is of the order of cubic nanometer, but we also find that it decreases as the stress intensity factor becomes closer to the critical stress intensity factor. Such a behaviour is reminiscent of recent experimental observations in rupture of glass where the nucleation size of cavities at the nanometer scale seem to be a decreasing function of the crack speed, and thus is a decreasing function of the stress intensity factor (Bouchaud [14]).

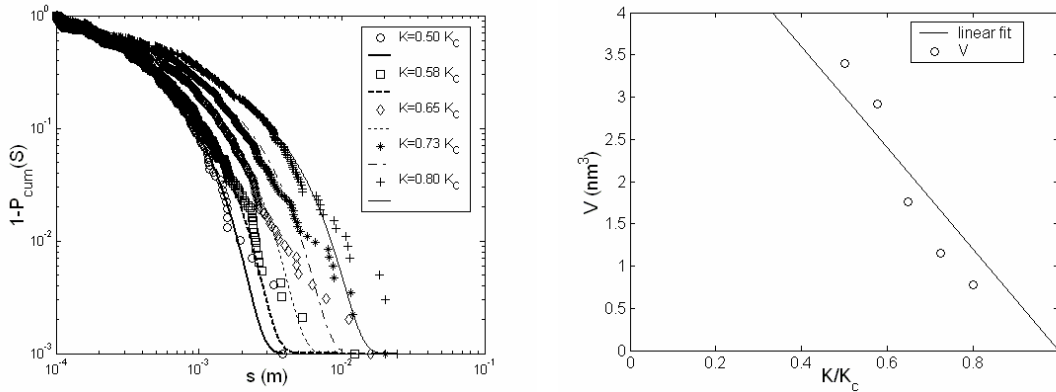


Figure 2: Left: The cumulative step size distribution $P(s)$ is plotted as $1-P(s)$ versus step size s . The lines correspond to a fit obtained by numerical integration of equation (2) with $\alpha=1.23$. Right: The volume V is the only remaining fit parameter. It shows a tendency to decrease with K . The straight line is just a guide for the eye, assuming the volume goes to zero at $K=K_c$.

5 CONCLUSION

As a conclusion, we have shown that crack dynamics in fibrous materials such as paper is a complex and rich statistical process. It is confirmed that the step size distribution is well-described by a percolation-like distribution. It is rather surprising that the sub-critical behaviour of the crack growth comes from a simple model of thermally activated crack dynamics. We have shown that the lack of accuracy in the prediction of the step size distribution is greatly improved if one reduces the value of the exponent from 1.5 to 1.23. More theoretical work needs to be done to understand if this approach is relevant.

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