A DISCRETE FRACTURE APPROACH FOR DIFFUSSION/ COUPLED ANALYSIS WITH CRACKS AND DISCONTINUITIES

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ABSTRACT

This paper deals with hydro- and hygro-mechanical (HM) coupling analysis of cracks and discontinuities, paying special attention to the modeling of the diffusive behavior. The HM problem is tackled using a new fully integrated approach, which formulates the fracture behavior by means of the same double nodded zero-thickness interface elements for both the mechanical and diffusion analyses. The mechanical behavior of the joint is reproduced by means of a non-linear constitutive law with fracture work softening. An innovative diffusion joint model is presented and analyzed, since it reproduces both longitudinal and transversal flows to the discontinuity through a double-nodded zero-thickness interface element, resulting in an improvement of the existing double-nodded models. Although the mechanical analysis of discontinuities with this kind of zero-thickness interface elements has well established procedures, other types of joint elements are common in the FE analysis of flow through discontinuities. A comparative example is presented to state the range of applicability for the most representative flow elements existing in literature, and to illustrate the good behavior of the proposed double node interface element. The coupled problem is solved following a staggered strategy, where the same FE mesh is used.

1 INTRODUCTION

In many material problems, the mechanical and diffusive behaviors appear combined and influenced reciprocally, leading to the so-called hydro- or hygro-mechanical (HM) coupled problems. The creation and/or progressive opening of discontinuities, as fissures or joints, is a key aspect in these problems, and their presence or development represent an additional coupling factor to take into account. The hydraulic fracture phenomenon is a classical example within the hydro-mechanical coupling field involving the presence of discontinuities, whereas shrinkage-induced cracking in concrete becomes a degradation process of main concern with coupling effects between moisture transport and the mechanical behavior of the concerning material. In this case, cracking turns into a preferential path to accelerate the drying of cementitious materials. During the last two decades some complex numerical models have been developed for the solution of HM coupling in fractured media [1] most combining a Finite Element based formulation for the mechanical analysis with Finite Differences for the diffusion problem along the fracture.

However, HM analyses in fractured media and other similar problems with coupling through interfaces may be efficiently tackled just by means of the FEM provided the use of zero-thickness interface elements with double nodes. The use of this kind of elements has been well established for mechanical analyses since some time already [2], therefore it would seem convenient to be able to use the same double nodded joint in the diffusion problem too. In this work, special attention is given to analyzing the capabilities of a recently proposed interface model with double nodes [3] for diffusion analyses. The formulation is developed in subsequent paragraphs in terms of a general diffusion problem in which h is the potential (hydraulic head in saturated flow or relative humidity in the moisture diffusion problem), K the diffusion conductivities, etc.

2 JOINT ELEMENTS FOR DIFFUSION ANALYSIS

Diffusion through a discontinuity may occur in the longitudinal and normal directions to the joint. The equation governing longitudinal flow may be achieved imposing conservation of fluid mass in the longitudinal direction combined with Darcy or Fick's law, leading to (1D joint):

$$\frac{d}{dx}\left(k_l \frac{dh_{mp}}{dx}\right) + s + q^- + q^+ = \frac{1}{\rho}\frac{\partial}{\partial t}\left(\rho \cdot w\right) \tag{1}$$

where q^- and q^+ are leak off terms between the discontinuity and the porous medium, ρ is the fluid density, w is the local discontinuity width, s is a source term, h_{mp} is the potential at the mid-plane of the joint, and $k_l = w \cdot \hat{k}_l$, with $\hat{k}_l$ the diffusive conductivity of the joint.

The discontinuity may represent an obstacle for the flow in the transversal direction (i.e. it may cause a potential drop due to the transition from a pore system into an open channel and back into a pore system), and therefore we consider a jump in the potential field across the joint $\Delta h_{Tmp} = h_{mp}^{-} - h_{mp}^{+}$ associated to the transverse flow q_t. We consider a law similar to that of Darcy/Fick, but now the total head jump plays the role of the total head gradient:

$$q_t = k_t \cdot \Delta h_{Tmp} \tag{2}$$

where k_t is a transverse hydraulic conductivity coefficient.

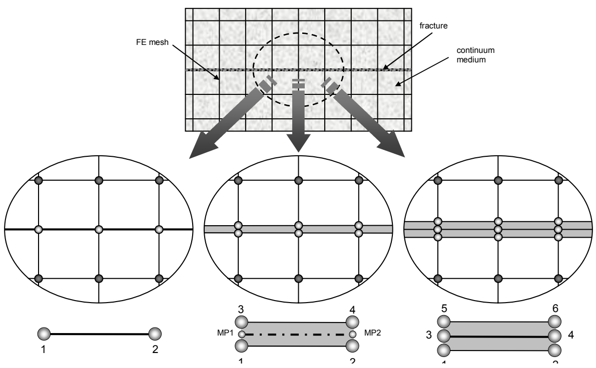

Figure 1: Zero-thickness interface elements for diffusion analyses inserted into the FE mesh

Diffusive flow through discontinuities has traditionally been modeled using special elements of zero-thickness, which we can classify into single, double and triple nodded. Single-node elements are the simplest, and consist of "line" or "pipe" elements which are superimposed on the standard continuum element edges using the same node common to adjacent elements. These elements can only model the longitudinal flow through the discontinuity with a longitudinal conductivity k_l. On the other hand, some authors [4] have included the transversal flow with its transmissivity k_t, and the subsequent localized potential drop across the discontinuity, by using triple-node interface elements. In those, the two nodes of the adjacent continuum elements represent the potentials in the pore system on each side of the interface, and a third node in the middle represents the average potential in the center of the channel represented by the discontinuity. Finally, some authors [5] have proposed double-node interface elements although without considering the influence of the transversal flow, i.e. prescribing the same potential for the two nodes before solving the global system equations. The interface element considered [3] includes the two types of flow, lengthwise and transverse, in the context of a double-nodded geometry. See [3] for the details about the three elements formulation.

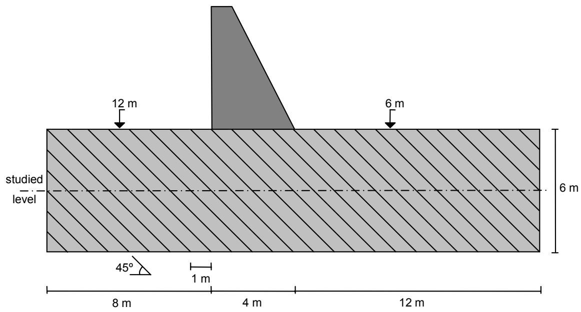

Figure 2: Geometry and boundary conditions of the problem under analysis.

A simple academic case of flow through porous medium with discontinuities is presented and solved by using each of the proposed interface elements, examining the accuracy and efficiency that their use gives, and with the subsequent advantages and disadvantages. The problem consists of a gravity dam laying on a highly fractured medium (Figure 2). The steady-state problem is analyzed. Three different apertures for the joints were considered, w = 0.01, 0.05 and 0.1 *mm*, which according to the widely used "cubic law" [9] give the following values for the longitudinal parameter $k_l = 8.12 \cdot 10^{-10}$, $1.01 \cdot 10^{-07}$ and $8.12 \cdot 10^{-07}$ m^2/s. As an example of the typical results obtained, Figure 3 plots the hydraulic head profile at a level of 3 m downward from the ground surface for a joint aperture of 0.05 mm, and for values of the transversal transmissivity of the joint ranging from 10^{-04} to 10^{-08} 1/s. No reference values have been found in literature for this transversal parameter, however we may think of the transverse conductivity of an equivalent width of continuum medium, which has been given values between 0.001 and 10 *m* in order to cover a wide range of transversal conductivities. Additional plots and results for other values of the apertures can be found in [3].

As it was expected, all the analyses gave very good agreement between the three models when there is a negligible effect of the transversal hydraulic conductivity, i.e. for high values of k_t. This result changes when increasing the effect of this parameter, where the profiles given by the double and triple nodded elements lay far away from the single node model, appearing the related hydraulic head jumps across the fractures. In general, we may admit a good agreement between the double and triple nodded models in the whole range of k_t. In addition, the hydraulic head at the inner node of the triple node element is very close to the average of the values at the boundary nodes of the element, so that the hypothesis used by the double node interface model seems to be a good approximation.

With the aim of checking the overall performance of the system, the total flow rate under the dam was also calculated by means of the three models [see also 13]. The obtained results were as expected, and as we increased the value of k_l the total flow rate increased, and as we decreased the value of k_t the fracture acted as progressively more impervious flow barriers, leading to lower values of the total flow rate under the dam. The flow rate values for double and triple node models were in good agreement.

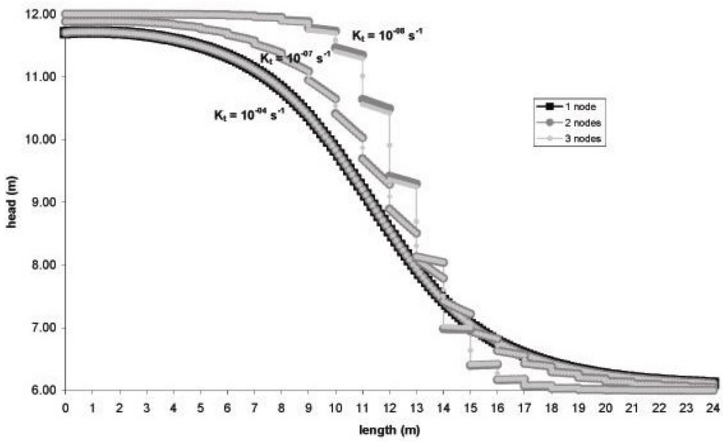

Figure 3: Hydraulic head distribution at level 3m with $k_l = 1.01 \cdot 10^{-07}$ m^2/s

3 H-M COUPLING ANALYSIS

The HM problem is tackled using a staggered strategy, which makes use of exactly the same FE mesh for both the diffusion and the mechanical problems, and where an iterative procedure leads to the solution of the coupled problem.

The approach used for mechanical fracture analysis follows the so-called discrete crack approach, where every discontinuity is represented individually by means of zero-thickness interface elements with double nodes that are equipped with appropriate constitutive laws. Interface behavior is reproduced by means of a work-softening elasto-plastic constitutive law, and it is formulated in terms of the stresses on the interface plane $\boldsymbol{\sigma} = [\sigma_n \quad \sigma_t]^T$, normal and shears, and the corresponding normal and tangential relative displacements $\mathbf{u} = [u_n \quad u_t]^T$. The interface constitutive model is that described and analyzed in detail in [6], and used in many other analyses [7] involving crack opening and propagation.

The solution of the mechanical problem gives as output the nodal displacement distribution, which allows us to calculate the joint aperture that will influence the diffusion analysis in terms of a variation of the diffusive conductivity of the joint (k_l) and its storage capacity ($\dot{Q}$) [8]. Subsequent solution of the diffusion problem with the new aperture results in a new nodal distribution of the potential, and therefore of fluid pressure or moisture content which, in turn, is used to modify the following mechanical analysis. In this way, the loop is closed leading to a new cycle of calculation till convergence is reached (Figure 4).

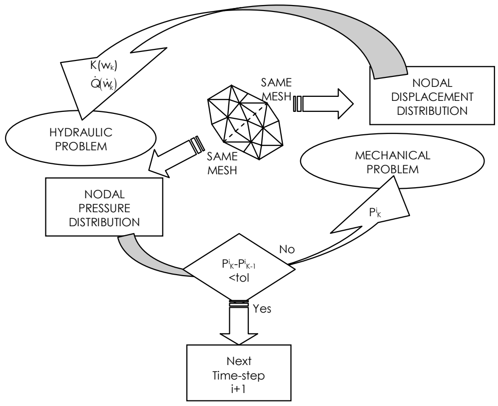

Figure 4: Staggered procedure scheme.

4 CONCLUDING REMARKS

The proposed double nodded zero-thickness interface elements seems to perform well and lead to valid results in a "standard" flow analysis, and, by extension, to similar diffusion problems governed by the same equation. This element has several advantages in front of the existing formulations. It is a simpler model than those using triple nodes, with fewer degrees of freedom. On the other hand, despite having double nodes, it does not lead to indetermination thanks to the introduction of a transverse flow governed by the transversal hydraulic conductivity k_t. Nevertheless, in some specific diffusion analyses and in transient regime the performance of the double nodded element must still be tested. From the HM point of view, the same FE mesh may be used for mechanical and diffusion analyses with the subsequent advantages.

On going work focuses on the numerical implementation and verification of this model to solve numerically HM coupled problems, with application to the cases of hydraulic fracture in rock and shrinkage cracking in concrete specimens.

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