A MULTI-SCALE APPROACH FOR THE SIMULATION OF CRACKS

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ABSTRACT

A new multi-scale approach for the computation of stress intensity factors is proposed. The two scales of interest are i) the macroscopic scale, which involves a boundary value problem solved by the finite element method (FEM) on the real geometry, and ii) the mesoscopic scale, associated with a local boundary value problem solved by the extended finite element method (X-FEM) on one element at a time of the real geometry, including cracks and singularities. The two scales are linked using a hierarchical or nested approach. The problem solved at the mesoscopic scale by the extended finite element method provides the stress tensor and the tangent stiffness tensor at the integration point of the macroscopic finite element problem. The cracks are represented both at the macroscopic and mesoscopic scales by level sets. As a result, any geometrical singularity can be virtually inserted in the real geometry without need for a conforming mesh. These singularities are not seen but felt by the global FE code through a modified tangent stiffness tensor and a reduced stress at the integration point. The cracked element can be seen as a damaged region. The method has been used with success to determine the K_I stress intensity factor in a standard 2D geometry. An extended "interaction" matrix is defined in order to evaluate the loss of information induced by the hierarchical approach with respect to the reference solution obtained by solving the global problem with the extended finite element method.

1 INTRODUCTION

Industrial procedures for the determination of the total fatigue life of structural components involve the computation of the stress intensity factors (SIFs) K_I , K_{II} and K_{III} together with the knowledge of a law of propagation relating their evolution to the rate of increase of the crack length a.

One of the most commonly used procedures is the following. First, a set of critical loading conditions is determined from a load spectrum. These conditions are analysed independently by using a FE code to identify the regions where the principal stresses are the most severe.

Then pure mode I fracture is assumed to occur in the direction normal to the highest principal stress. At this point, a crack of the smallest detectable length should effectively be inserted in the mesh (micro-mechanical models of crack initiation are still today rarely used) and the stress intensity factors computed with the displacements resulting from the FE simulation. Different techniques are available which are generally cumbersome and time-consuming. Therefore it appears that the most widespread technique consists of evaluating analytically or numerically the SIFs on an equivalent uni-dimensional rod in the framework of the linear elastic fracture mechanics (LEFM). The total life is deduced based on this value of ΔK_I, on an experimental Paris or Forman law and on the load spectrum.

Inserting a crack in the real geometry generally implies the modification of the Computer Assisted Design (CAD) data as well as the adaptation of the mesh. Inserting parameterised cracked meshes under the form of adequately sized boxes is an alternative, but the operation has to be performed iteratively as the crack propagates. Furthermore, imposing the proper conditions at the interface between the parameterised and original meshes can be difficult. Note finally that accounting for mixed mode fracture is extremely important when dealing with complex local stress states.

Moës et al. [1] propose a promising non standard FE techniques based on the partition of the unity property, which allows the occurrence of discontinuities of displacement or strain. New degrees of freedom and shape functions are added in such a way as to account for the singularities, which are modelled by Level Sets, Sethian [2].

The eXtended Finite Element Method (X-FEM) has been shown to provide good approximations of the SIFs and crack paths, and is expected to be widely used in the near future by most of the industrial actors of the sector of Aeronautics. However, in order to be integrated in daily design procedures, the extended finite element and Level Set (LSM) methods have to be introduced in existing commercial FE codes, which involves lots of difficulties. As a result, it is reasonable to assume that the technique will not be available in industrial procedures for fatigue life determination within a short period of time.

Therefore, according to the work of Feyel & Chaboche [3] or Smit et al. [4], it was deemed interesting to implement a hierarchical approach allowing the use of the extended finite element and level set methods at the local scale and take profit of their particular features, e.g. the user-friendly insertion and propagation of a crack, while keeping the classical finite element formulation at the global scale. This is possible in most commercial finite element software via the use of a typical "User Material" interface.

It is worth mentioning again that although this approach renders the use of the X-FEM possible at the *local scale*, it is not a full implementation of the method: the discrete character of the crack is replaced at the macroscopic scale by a modified tangent stiffness, the element containing the tip of- or cut by the crack can be seen as a homogenized damaged medium. Together with the scale transition, some of the mathematical features are lost (e.g., continuity of the crack opening from element to element in the macroscopic mesh) and the "extended" or "non-local" character of the crack is, to a certain extent, neglected.

The present paper is organised as follows: first, the hierarchical approach is presented, with emphasis on the definition of a well posed local boundary value problem (BVP). Then, the method is applied to uni-axially loaded standard 2D cracked samples and the predicted mode I SIF is compared with the corresponding analytical value and with the results obtained by different numerical techniques. Finally a method is proposed to validate the approach by evaluating an "interaction" matrix representative of the loss of information implied by the hierarchical approach compared to the full X-FEM simulation performed on the real geometry.

2 DESCRIPTION OF THE HIERARCHICAL APPROACH

Finite element codes require a *bilinear operator* at each integration point to assemble the global stiffness matrix of the FE problem $\mathbf{K}_{cc}$ in $\mathbf{K}_{cc}u_c = F_c$. In the case of elasto-statics, this *bilinear operator* is the 4th order Hooke tensor C_{ijkl}, linking the 2nd order strain tensor ε_{ij} to the 2nd order stress tensor σ_{ij} via the relationship $\sigma_{ij}=C_{ijkl}\varepsilon_{kl}$. In the framework of a static non-linear analysis, this operator is the tangent or elasto-plastic stiffness tensor $H_{ijkl} = d\sigma_{ij}/d\varepsilon_{kl}$. The tensors σ_{ij} and H_{ijkl} can be derived from semi-analytical models or may be evaluated by solving a local FE problem on a representative volume element (RVE). In this case, the characteristic length of the local problem solved at each integration point is very small with respect to the characteristic length of the global problem. This is the core of *micro-macro* approaches.

In the present paper, the local problem is solved by the extended finite element method. The material is assumed to remain elastic. At this point, the main difference comes from the fact that the homogenised stiffness tensor is calculated by applying independent loading cases to a non-representative volume element. It is a *meso-macro* approach (see Figure 1).

The cracks are defined both at the macroscopic and mesoscopic scales. They are first represented by a pair of normal and tangent level sets evaluated at the nodes of the macroscopic/global mesh. If an element of the global mesh is found to be cut through by, or to con-

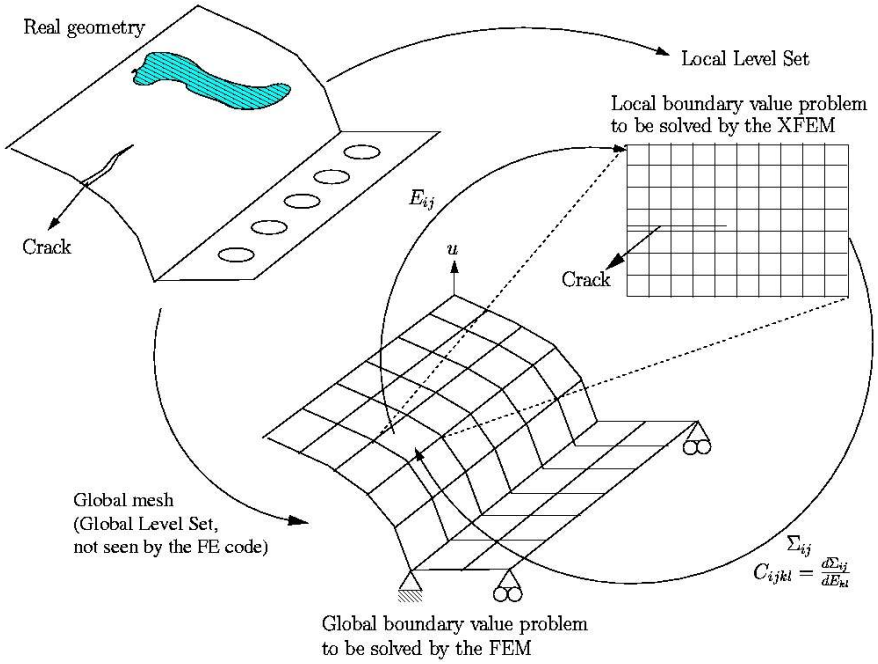

Figure 1: The FEM-XFEM hierarchical approach

tain the tip of a crack, a local mesh is defined and the level sets are evaluated at the local nodes.

The homogenized stiffness tensor is evaluated by imposing either fully cinematically constrained boundaries (i.e. linear displacements) or fully statically constrained boundaries (i.e. uniform tractions). The first condition is too constraining and implies crack closure, as a consequence, the homogenised stiffness is overestimated. Therefore, the condition of uniform traction is more appropriate and even leads to conservative results. In 2D, three sets of independent loading conditions are necessary to determine the homogenised stiffness tensor. The stress is determined by multiplying the strain supplied by the FE code by the computed stiffness tensor. The stiffness tensor is the same at each Gauss point of one macroscopic element while the computed stress depends on the strain given by the FE code. The problem of a completely cut element in the global mesh can also be treated analytically.

3 APPLICATION

As a preliminary case, the method has been used to compute the K_I stress intensity factor in a uniaxially loaded standard 2D sample (Figure 3). The dimensions are $a/L = 0.25$ and $L/H = 0.1$.

A structured mesh of 4-noded quadrangles with linear shape functions and 8 Gauss points is used to model the macroscopic sample. In the hierarchical approach, each quadrangle is in turn considered as a geometry on which a local BVP must be solved. Each quadrangle is then meshed and recursively refined with 3-noded linear triangles (the total number of degrees of freedom is 368, including the enriched dofs). Only the Heaviside function and the first crack tip enrichment ($r^{1/2} * \sin(t/2)$) are used to enrich the basis of the shape functions, which is the core of the X-FEM. As far as a bi-dimensional case is concerned, three independent loading cases are imposed using statically constrained boundaries. Completely cracked quadrangles lead to a loss of major

symmetry of the stiffness tensor. As a result non-symmetric bilinear operator should be given to the host FE code. The cases of non-symmetric and of zero (i.e. the material looses all its strength) Hooke tensors have been considered, showing differences of less than 1% in the calculated mode I stress intensity factor.

The stress intensity factor is calculated on basis of the displacements of the nodes of the global mesh. A domain integral is used to compute the energy release rate at the crack tip and the interaction integral is used to derive the SIFs. The gradients of the level sets are used to build the local coordinates system. The details can be found for instance in Moës et al. [5].

The pure mode I SIF $K_{I,MS}$ (MS yields for Multi-Scale) is calculated for various levels of mesh refinement (defined as the number of elements along x in the global mesh) and plotted in Figure 4 as a function of the computational time. In this figure, $K_{I,MS}$ is also compared to

- the corresponding analytical value $K_{I,A}$ given by

$$K_{I,A} = \Sigma_0 \cdot \sqrt{a} \cdot f\left(\frac{a}{L}\right), \tag{1}$$

where

$$f\left(\frac{a}{L}\right) = 1.99 - 0.41\frac{a}{L} + 18.7\left(\frac{a}{L}\right)^2 - 38.48\left(\frac{a}{L}\right)^3 + 53.85\left(\frac{a}{L}\right)^4 . \tag{2}$$

Note that H must be greater than $2L$;

- the value $K_{I,X}$ computed with the stand alone XFEM-Xcrack code using the Heaviside and the four crack tip enrichment functions;
- the value $K_{I,k}$ calculated by setting to zero the stiffness of all the elements cut or touched by the crack (this is similar to element killing).

All the methods converge when the global mesh is refined. It must be pointed out that the local refinement in the hierarchical approach has a beneficial effect on the resulting value of the mode I SIF. The theoretical value is approached with coarser meshes.

The total number of dofs involved in the solution of the mesoscopic problem lead to a limited computational cost. Even in 3D cracked geometries, the computation time remains limited as the procedure only affects the elements which contain a part of the crack front.

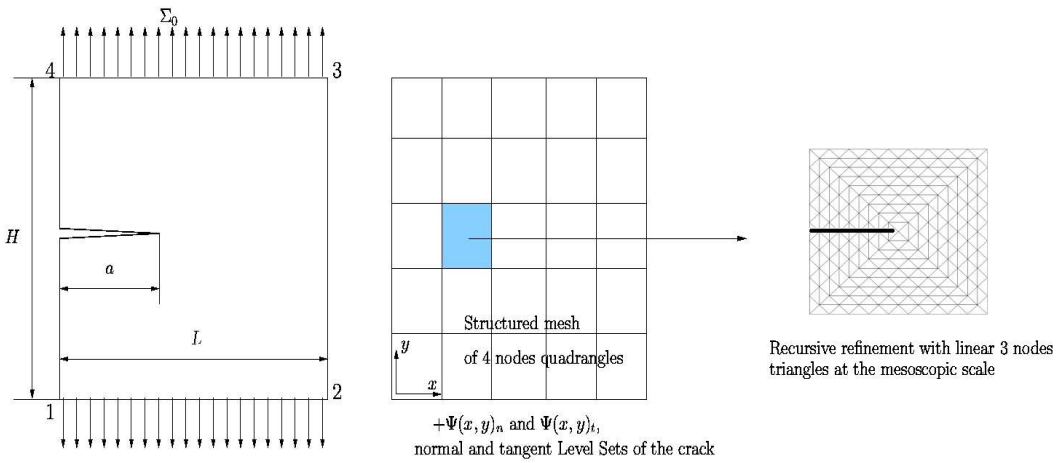

Figure 3: A standard 2D sample submitted to uni-axial loading.

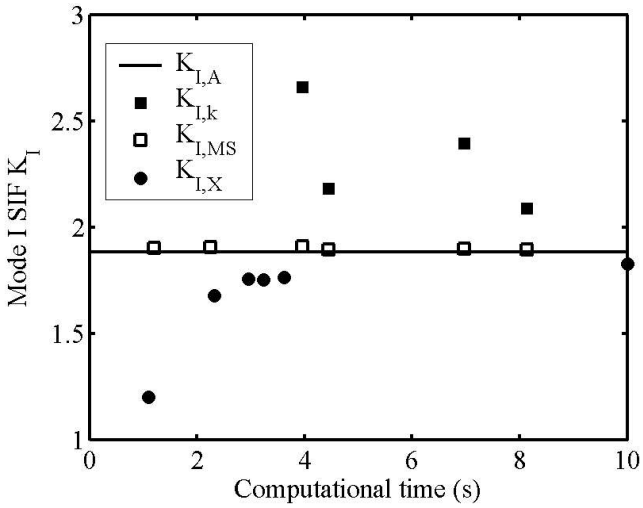

Figure 4: Comparison of the values of the stress intensity factor K_I calculated using various methods as a function of the computational time.

The method must now be validated extensively in 2D and 3D towards comparison with i) analytical results and ii) detailed FE and XFE simulations, for all the modes (including mixed modes) of fracture.

4 DEFINITION OF AN INTERACTION MATRIX

Let us get into the details of the linear system generated by the extended finite element method in order to solve a BVP featuring a discontinuity of displacement, i.e. a crack:

$$\begin{pmatrix} \mathbf{K}_{cc} & \mathbf{K}_{ce} \\ \mathbf{K}_{ec} & \mathbf{K}_{ee} \end{pmatrix} \cdot \begin{pmatrix} u_c \\ u_e \end{pmatrix} = \begin{pmatrix} F_c \\ 0 \end{pmatrix}, \tag{3}$$

where u_c and u_e are the classical and the enriched degrees of freedom, respectively. $\mathbf{K}_{cc}$ is the stiffness matrix of the classical finite element problem, $\mathbf{K}_{ee}$ is the stiffness matrix linking the enriched dofs together, while $\mathbf{K}_{ce}$ and $\mathbf{K}_{ec}$ induce a coupling between the classical and the enriched dofs. It is assumed that $F_e = 0$, i.e. there are no forces applied on the enriched dofs. Eliminating the enriched dofs in eqn (3) leads to the system of eqn (4)

, (4)

which corresponds to a *classical* FE problem with a particular global stiffness matrix. After [6], the "interaction matrix" **D** was defined as the difference between the $\mathbf{K}_{cc}$ matrix (for an ideally elastic material and no discontinuities) and the current stiffness matrix of the *classical* FE problem. From eqn (4), $\mathbf{D}_X$ is naturally identified as $\mathbf{K}_{ce}\mathbf{K}_{ee}^{-1}\mathbf{K}_{ce}^{T}$. The matrix $\mathbf{D}_X$ induces additional constraints between the classical dofs which are not linked in $\mathbf{K}_{cc}$, so that the effect of the crack is naturally extended to several elements (this is the "non-local" character of the crack).

In the hierarchical or multi-scale approach, the terms of $\mathbf{K}_{cc}$ are affected directly while assembling it. Indeed, the local homogenized Hooke operator leads to a lower stiffness for the cracked medium than for the bulk material. No expression of the interaction matrix $\mathbf{D}_{MS}$ is

available. However, its value could be calculated as $\mathbf{D}_{MS} = \mathbf{K}_{cc} - \mathbf{K}_{MS}$ and compared to $\mathbf{D}_X$.

The comparison between $\mathbf{D}_{MS}$ and $\mathbf{D}_X$ will lead to the quantitative evaluation of the loss of information (among them and to certain extent, the non-local character of the crack) associated with the use of the present multi-scale approach, with respect to the stand alone extended finite element method taken as reference.

5 CONCLUSION

A new FEM-XFEM nested / hierarchical meso-macro approach was proposed for the computation of stress intensity factors. Although this method does not consist of the rigorous implementation of the XFEM in the commercial FE codes, it permits the arbitrary introduction of cracks in a real geometry without the need for re-meshing. The method provided a good approximation of the mode I SIF for a crack in a 2D standard geometry. The method must be validated on a large number of 2D and 3D cases. The concept of "interaction matrix" was proposed to quantify the discrepancy between the result of the present multi-scale approach and the standard X-FEM reference solution.

6 ACKOWLEDGEMENTS

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