CRACK IDENTIFICATION IN A BEAM BY MEASURE OF THE RESPONSE TO WHITE NOISE

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ABSTRACT

The aim of this paper is to inspect the vibrational response of a beam with an edge non-propagating crack by means of stochastic analysis, in order to detect the presence and the location of structural damage. The non-linear behavior of the beam due to the opening and closing of the crack is fully exploited. The non-linearity measure is based on the response evaluation of the beam subjected to a white noise process. Both numerical and experimental investigations regarding a cantilever beam with a crack are reported in the paper.

1 INTRODUCTION

Damage detection by means of non-destructive testing plays an important role in ensuring the integrity of machine elements and structures. The techniques commonly employed are based on vibration measurement, which offers a convenient tool for investigating crack presence and position.

Some research has been devoted in modeling the cracks as always open during vibrations. In these works crack location and amplitude may be detected from alterations in natural frequencies and modes of vibrations as well as amplitude of forced vibrations (Yuen [1], Kisa *et al.* [2]). However, as the cracks usually exhibit non-linear behavior, the need of modeling the crack closure has been widely recognized. In fact, if an insufficient static preload is present, the cracks open and close depending on the vibration direction causing the variation of the physical system parameters such as the stiffness. Qian *et al.* [3] and Ruotolo *et al.* [4] investigated the variation of the modal properties and of the deterministic response under harmonic input caused by the presence of a breathing crack.

Herein a stochastic approach is employed which assumes the probabilistic characteristics of the beam response under stochastic input as an indication of the crack presence and position. Such a choice is motivated by the radical change encountered by some of the response properties as soon as a crack occurs in the beam. In fact, if the beam is undamaged (linear state) and a Gaussian load is applied, the structural response will be Gaussian too. On the contrary, if non-linearity arises, due to the appearance of a crack, the response becomes non-Gaussian. Inspection of the integrity of the structural element may be performed by estimating the non-Gaussianity of the response, through the evaluation of higher order statistics. The non-Gaussianity may be revealed in several ways. The choice of the measure to detect the non-Gaussianity is a crucial point to give clear information on the beam condition when the crack has small depth. For example, Rivola and White [5] have employed a frequency domain procedure based on the measure of the bispectrum for the detection of a crack in a straight beam modeled as a single degree of freedom system. Cacciola *et al.* [6] performed a time domain analysis where the beam is discretized by finite elements in which a so-called closing crack model, with fully open or fully closed crack, is used to describe the damaged element. They evidenced a remarkable variability of the skewness at the rotational degrees of freedom for different crack locations and depths, assessing the capability of this measure to detect both the presence and the position of the crack along the beam even if the crack depth is small.

The present paper briefly summarizes the damage identification procedure proposed by Cacciola *et al.* [6]. Then, new numerical solutions pertaining the case of a base stochastic acceleration applied to a cracked cantilever beam are derived. Furthermore the experimental results gathered from the first tests on a shaking table are discussed. A favorable comparison with predictions from numerical analysis is shown.

2 ANALYTICAL MODEL OF THE CRACKED BEAM

The presence of a crack in the beam, according to the principle of Saint Venant, causes a perturbation of the stress field in the neighborhood of the breach. Such a perturbation is relevant especially when the crack is open and determines a local reduction of the flexural rigidity. On the other hand, when the crack is closed the beam acts, approximately, as a homogeneous beam with no crack. A natural choice is a finite element formulation of the problem. In this case, the properties of the cracked element, changing during the motion, determine the non-linearity of the whole system, being constant the parameter modeling all the other elements. According to several authors [2,3,4] the stiffness matrix is the structural property that is most affected from the breathing of the crack, as damping and mass matrices do not change appreciably during the opening and closure of the crack.

In the following, the finite element model for the cracked beam with an on-edge non-propagating crack, proposed by Qian *et al*. [3] is adopted. Undamaged elements of the beam are modeled by Euler type finite elements with two nodes and two degrees of freedom (transverse displacement and rotation) at each node. The cracked element will be modeled as an undamaged element if the crack is closed whereas it exhibits a more flexible behavior if the crack is open. A non-propagating crack will be considered so that the characteristics of the cracked element in the crack open phases do not change during motion. Then, the problem is of piecewise interval nature in time domain, the closure and opening phases being governed by two different sets of linear equations differing just for the stiffness matrix of the cracked element.

The stiffness matrix of the undamaged element with rectangular cross section is that given by Bernoulli-Euler theory with Hermite shape functions:

$$\mathbf{k}_u = \begin{bmatrix} Ebh^3/l^3 & Ebh^3/2l^2 & -Ebh^3/l^3 & Ebh^3/2l^2 \\ & Ebh^3/3l & -Ebh^3/2l^2 & Ebh^3/6l \\ & sym & Ebh^3/l^3 & -Ebh^3/2l^2 \\ & & & Ebh^3/3l \end{bmatrix} \tag{1}$$

An explicit expression for the matrix of the cracked element was first derived in [6], where it has been cast in the following form

$$\mathbf{k}_c = \alpha_1 \begin{bmatrix} k_{u_{11}}\alpha_2 & k_{u_{12}}\alpha_2 & k_{u_{13}}\alpha_2 & k_{u_{14}}\alpha_2 \\ & k_{u_{22}}\alpha_3 & k_{u_{23}}\alpha_2 & k_{u_{24}}\alpha_4 \\ & sym & k_{u_{33}}\alpha_2 & k_{u_{34}}\alpha_2 \\ & & & k_{u_{44}}\alpha_3 \end{bmatrix} \tag{2}$$

The plot of the four coefficients is given in Figure 1 for several values of the crack depth.

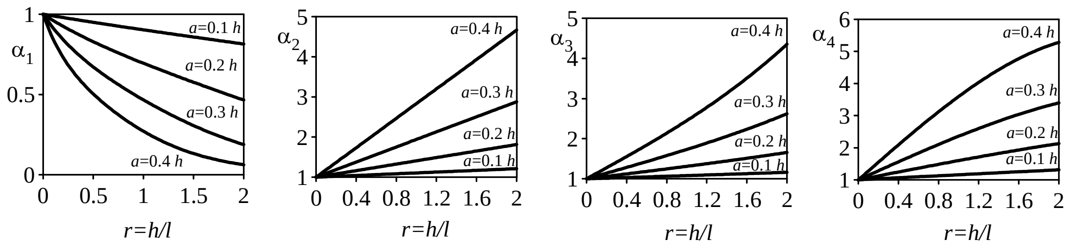

Figure 1: Coefficient values in the stiffness matrix of the cracked element ($r = h/l$ is the ratio between the height and the length of the element; a is depth of the crack).

2.1 Equation of motion

The dynamic response of the beam in the time intervals the crack is closed may be regarded as that of a beam without crack, because the crack interfaces are completely in contact with each other. Under the action of the excitation force, crack opening and closure will alternate as a function of time.

The equations of motion of a cracked beam discretised by N_e finite elements and subjected to a base acceleration $\ddot{u}_g(t)$ can be written adopting the usual symbols as:

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+(\mathbf{K}_u-\gamma\,\Delta\mathbf{K})\mathbf{u}(t)=-\mathbf{M}\boldsymbol{\tau}\ddot{u}_g(t);\;\; \mathbf{u}(0)=\mathbf{u}^0,\quad \dot{\mathbf{u}}(0)=\dot{\mathbf{u}}^0 \tag{3}$$

It has been assumed that the change between fully open and fully closed takes place instantaneously through the two-valued function γ (γ = 1 when the crack is open; γ = 0 when the crack is closed) giving rise to a bilinear-type non-linearity. The change in the global stiffness matrix due to the crack is $\Delta\mathbf{K}=\mathbf{K}_u-\mathbf{K}_c$, being $\mathbf{K}_u$ and $\mathbf{K}_c$ the stiffness matrices of the undamaged and damaged beam, respectively.

During crack closure, compression acts on the crack interface and the strain at the opposite edge is positive (tensile) and, conversely it is negative (compressive) during crack opening. Accordingly, the opening and closure of the crack on one edge of the beam can be detected by the sign of strain in the opposite edge. Assuming Hermite interpolation for the deformed shape of the cracked element, both for the open and the close phase, and under the circumstance the crack is located in the midpoint of the *i*-th element, then the sign of the strain is equal to the sign of the difference between the rotation at the nodes of the cracked element. Then, to determine the state of the crack it is sufficient to evaluate the slopes ϕ_i and ϕ_{i+1} of the response deformation at the nodes *i* and *i*+1, closest to the crack. In particular, if the slope is positive when counter clockwise, the condition of closing is $\phi_{i+1} > \phi_i$..

A convenient numerical procedure to solve eqn. (3) was presented by Cacciola and Muscolino [7] and will be adopted in the numerical applications.

3 USE OF STOCHASTIC ANALYSIS FOR CRACK DETECTION

It is well known from stochastic mechanics that a linear system driven by Gaussian stationary stochastic process undergoes steady state Gaussian stationary oscillations. This means that only first and second order statistical information are sufficient to completely describe the stationary stochastic response of the system. Moreover such statistics, namely the mean vector and the covariance matrix of the degrees of freedom, can be easily determined and the exact solution is available in the case the forcing function is a white noise.

If the system is non-linear the stochastic response under Gaussian input is not Gaussian anymore. Then, the presence of non-Gaussianity in the response may be employed to assess the presence of damage in the structure. This means that higher order measures such as skewness (third order), kurtosis coefficients (fourth order), higher order cumulants or statistical moments may provide details about the response that the conventional second order statistics cannot.
The major issue is the choice of the right parameters to be estimated so to detect the non-Gaussian behavior of the structural response. For the case under study of a beam with an edge crack, the opening and closure of the crack causes the non-Gaussianity of the nodal displacement and rotation processes. However, if the crack depth is small in comparison with the beam height then such non-Gaussianity may be overlooked. For these reasons it is important to select the proper higher order statistics at given measure points, so that these are much sensitive to the crack presence. In the following, the skewness coefficient γ_3 and the kurtosis coefficient γ_4 of the stationary response at nodal degrees of freedom will be considered:

$$\gamma_3 = \frac{k_3}{k_2^{3/2}}; \qquad \gamma_4 = \frac{k_4}{k_2^2} \tag{4}$$

where k_2, k_3 and k_4 are the second, third and fourth cumulants of the generic nodal degree of freedom u, namely,

$$k_2 = E[(u-\mu_u)^2] = \sigma^2; \;\; k_3 = E[(u-\mu_u)^3]; \;\; k_4 = E[(u-\mu_u)^4] - 3\sigma^4 \tag{5}$$

being $E[\bullet]$ the mean operator, μ the mean value and σ^2 the variance.

The skewness coefficient quantifies the asymmetry of the probability density function present in the response due to non-symmetric non-linearity in the stiffness matrix of the cracked element. The kurtosis coefficient is a measure of whether the data are peaked or flat relative to a Gaussian distribution. That is, data sets with a high kurtosis tend to have a distinct peak near the mean, decline rather rapidly, and have heavy tails. Data sets with low kurtosis tend to have a flat top near the mean rather than a sharp peak.

Under white noise Gaussian stationary excitation, the skewness and kurtosis coefficients vanish if the system is linear, i.e. the beam is undamaged, whereas the coefficients do not vanish if the system is non-linear, i.e. a crack is present. As the crack depth increases, the coefficient moduli, generally, increase too due, to more pronounced non-Gaussian behaviour.
The evaluation of the shape indices γ_3 and γ_4 requires the application of stochastic analysis so to get the cumulants needed. Monte Carlo simulation will be adopted to this aim, and the numerical and experimental results pertaining a cantilever beam under with noise Gaussian base motion are reported in the next section.

4 NUMERICAL AND EXPERIMENTAL RESULTS

In this section the results deriving from the analysis of the random vibration of the cantilever beam depicted in Figure 2 are shown. It has been assumed that the beam is subjected to a ground acceleration modeled by a white noise process with power spectral density $S_{\ddot{u}_g} = 0.0031 m^3/s^2$. The finite elements model and the pertinent mechanical and geometrical parameters are also reported in Figure 2. A joined experimental-numerical analysis has been conducted. First, the

$E = 6.9 \times 10^4$ MPa
$\rho = 2800$ kg/m^3
$L = 420 mm$
$h = 15 mm$
$b = 30 mm$

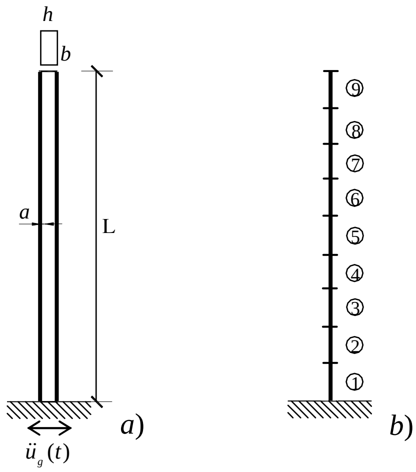

Figure 2: Cantilever beam (a) and finite elements model (b)

damping ratio and the first two natural frequencies have been determined leading to $\zeta = 1.1\%$, $\omega_1 = 68.2\text{Hz}$, and $\omega_2 = 427.4\text{Hz}$, respectively. The numerical step-by step procedure proposed by Cacciola and Muscolino [7], adopting a time integration step $\Delta t = 0.000488\text{s}$ and duration 8s, has been used for the numerical analysis. For various crack depths and positions the random response of the beam has been investigated via a pertinent Monte Carlo study. The skewness coefficients γ_3 have been determined using eqns. (5) and (6). The results relative to the rotations of the joints are reported in Figure 3. It is observed that a distinct jump occurs in the rotation skewness, located at the position of the crack (shaded in gray in Figure 3). The analysis of the results clearly shows that the size of the jump is related to the crack depth.

To validate the proposed procedure experimental tests have been conducted. The study is still in progress and partial results are herein summarized. The experimental set-up is composed by an Unholtz-Dickie vibrating system (slip table and amplifier model TA250), Bruel&Kjaer model 4382S accelerometers with Nexus amplifier and a National Instruments PCI-MIO 16XE10 board for driving the signal to the amplifier. The rotations have been measured using a couple of accelerometers distant 3cm each other. The crack depth is 0.5h and it is located in the middle of the beam. Changing the location of the accelerometers the random response have been measured in various points. Specifically, the time history of the rotations have been measured in two point located under and above the crack, respectively. The skewness and the probability density function of the rotations have been subsequently determined in a Monte Carlo fashion, exploiting the ergodicity of the response process. In Figure 4 the probability density function of the measured response is shown. Figure 4a confirm the Gaussianity of the input process, while the probability density functions of the rotations depicted in Figures 4b and 4c reveals the non Gaussianity of the response owed to the presence of the crack. Remarkably, the skewness coefficients of the rotations under and above the crack are respectively $\gamma_3 = 0.0589$ and $\gamma_3 = -0.3483$, in agreement with the results from the finite element analysis.

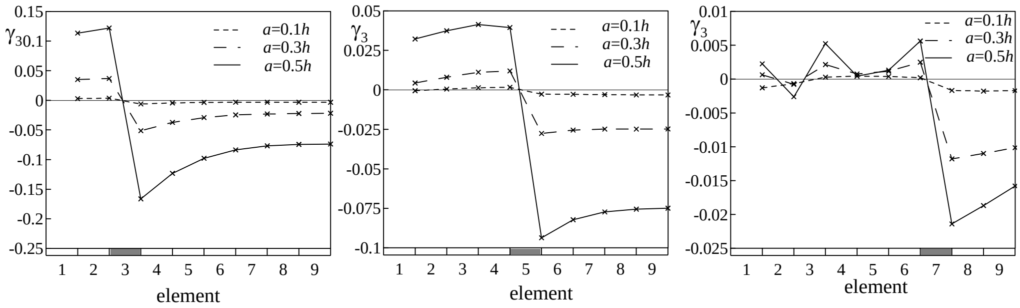

Figure 3: Skewness coefficents for various crack depths and positions

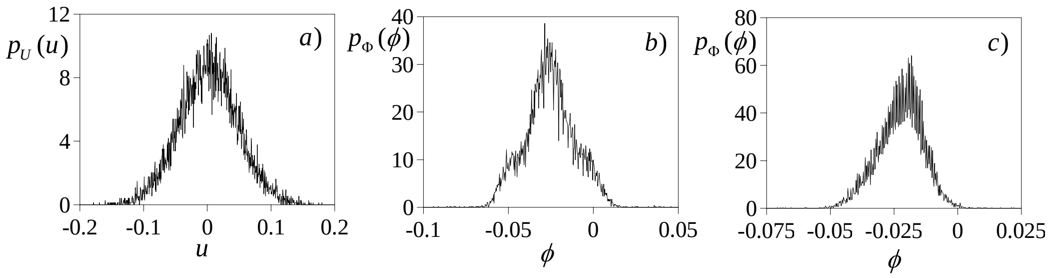

Figure 4: probability density functions of the response process: a) displacement at restrain (input signal); b) rotation below the crack; c) rotation above the crack

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