A DYNAMIC VOID GROWTH MODEL

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ABSTRACT
In this paper, the problem of the dynamic growth of a single spheroidal void in a power law visco-plastic matrix material has been studied and a new void growth model which is capable of describing the deformation and fracture of ductile materials under dynamic loading conditions is presented. Particular attention is paid to inertial effect, rate-sensitivity effect and void shape.

1. INTRODUCTION

Microvoid nucleation, growth and coalescence are the dominating mechanisms of ductile fracture. For static loading, void growth problems have been studied by McClintock [12] and Rice and Tracey [2]. Gurson [6] extended the void growth model by Rice and Tracey into a pressure dependent constitutive equation where not only the void growth but also the effect of void growth on the plastic flow of a porous material has been taken into consideration. Modifications to bring the Gurson model to realistic predictions have been made by Needleman and Tvergaard [11]. Zhang et al. [8] have studied the microvoid coalescence problem and found that the plastic limit load model by Thomason [10] works very well as a coalescence criterion for the Gurson model. A so-called complete Gurson model where no empirical critical void volume fraction at void coalescence is needed has been introduced by Zhang et al. [9].

With the successful application of ductile fracture models, the need for dynamic ductile fracture models capable of describing deformation and fracture of structures under dynamic loading conditions has been steadily increasing, especially in the last ten years when fracture mechanics was introduced into the auto industry. Although a large body of work exists on quasi-static microvoid growth, void growth under dynamic loading has drawn relatively modest attention since the pioneering work of Carroll and Holt [7]. The effect of dynamic loading on the void growth naturally involves three aspects: thermal effect, strain-rate sensitivity and inertia effect. Various studies have shown (Tong and Ravichandran, Cortes) that in most cases the thermal effect is less significant compared with the effect of inertia and strain-rate sensitivity.

In this paper we focus the problem of dynamic growth of a single spheroidal void in a power law visco-plastic cell element, which has a confocal spheroidal shape. Particular attention is paid to inertial effect, rate-sensitivity effect and void shape.

2. REPRESENTATIVE VOLUME ELEMENT AND MATERIAL PROPERTIES

In this study, a spheroidal (axisymmetric) cavity with semi-axes *a* (along x_3) and *b* (along x_1 and x_2), embedded in a confocal spheroidal representative volume element with semi-axes *A* (along x_3) and *B* (along x_1 and x_2) has been considered, Figure 1.

In the representative volume element, $c \equiv \left|a^2 - b^2\right|^{\frac{1}{2}} \equiv \left|A^2 - B^2\right|^{\frac{1}{2}}$, denotes the focal distance, e_1 and e_2 the eccentricities of the inner and outer spheroids, $e_1 = c/a$, $e_2 = c/A$. The following two geometrical parameters have been used, the void volume fraction $f = ab^2 / AB^2$ and void aspect ratio

$w = a / b$. The inner and outer eccentricities can be calculated in terms of these parameters. Here we only consider the case with prolate void ($A \geq B$).

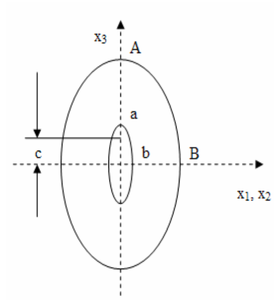

Figure 1: The representative volume element model

In orthogonal spheroidal coordinates, the iso - λ surfaces are confocal spheroids with semi-axes and eccentricity denoted *a*, *b* and *e* respectively:

$$\begin{cases} a = c \operatorname{Cosh} \lambda \\ b = c \operatorname{Sinh} \lambda \\ e = c/a = 1/\operatorname{Cosh} \lambda \end{cases} \tag{1}$$

In particular, the surface of the void and the external boundary are iso-λ surfaces corresponding to some value λ_1 and λ_2 respectively. The nonzero metric coefficients for this system of coordinates are given by (Moon and Spencer [13]):

$$\begin{cases} g_{\lambda\lambda} = g_{\theta\theta} = c^2 \left(\operatorname{Cosh}^2 \lambda - \operatorname{Cosh}^2 \theta \right) = c^2 \left(\operatorname{Sinh}^2 \lambda + \operatorname{Sinh}^2 \theta \right) \\ g_{\varphi\varphi} = c^2 \operatorname{Sinh}^2 \lambda \operatorname{Sinh}^2 \theta \end{cases} \tag{2}$$

The expression of the elementary volume in spheroidal coordinate is:

$$\begin{aligned} d\,\mathrm{v} &= c \operatorname{Sinh} \lambda \; g_{\lambda\lambda} \operatorname{Sin} \theta \; d\lambda \; d\theta \; d\varphi \\ &= c^3 \left(\operatorname{Sinh}^2 \lambda + \operatorname{Sinh}^2 \theta \right) \operatorname{Sinh} \lambda \operatorname{Sinh} \theta \; d\lambda \; d\theta \; d\varphi \end{aligned} \tag{3}$$

Throughout the analysis, we assume that the matrix surrounding the void responds to monotonic stressing as a visco-plastic solid with flow rule:

$$d_{ij} = \dot{\varepsilon}_e \frac{3 s_{ij}}{2\sigma_e} = \frac{\partial \phi(s)}{\partial s_{ij}}, \; \dot{\varepsilon}_e = \dot{\varepsilon}_0 \left(\frac{\sigma_e}{\sigma_0} \right)^N, \quad \phi(s) = \frac{\sigma_0 \dot{\varepsilon}_0}{N+1} \left(\frac{\sigma_e}{\sigma_0} \right)^{N+1} \tag{4}$$

where, σ_{ij} is the stress tensor, s_{ij} the deviatoric stresses tensor, d_{ij} is the rate of deformation tensor, σ_e and ε_e the effective Mises stress and strain, respectively, σ_0 is a flow stress, and $\dot{\varepsilon}_0$ are material constants, N is the rate sensitivity parameter, $N \geq 1$.

The macroscopic rate of deformation of the representative volume element is defined in terms of the velocity field, **v,** on the surface of the cell element,

$$D_{ij} = \frac{1}{V} \frac{1}{2} \int_S \left(v_i n_j + v_j n_i \right) dS \tag{5}$$

where V is the volume of the cell element, S is its outer surface and $\boldsymbol{n}$ is the unit outward normal on *S.* The average rate of work $\langle \boldsymbol{\sigma} : \mathbf{d} \rangle$ of the cell element is defined as:

$$\langle \boldsymbol{\sigma} : \mathbf{d} \rangle = \frac{1}{V} \int_V \sigma_{ij} d_{ij} dV \tag{6}$$

Using the principle of virtual work and neglecting the body forces, the above equation becomes

$$\langle \boldsymbol{\sigma} : \mathbf{d} \rangle = \frac{1}{V} \int_S \sigma_{ij} v_i n_j dS + \frac{1}{V} \int_V \rho v_i \frac{dv_i}{dt} dV \tag{7}$$

Following Molinari and Mercier [5] and using the definition of the dynamic macroscopic stress $\Sigma_{ij} = \langle \sigma_{ij} \rangle + \left\langle \rho \frac{dv_i}{dt} x_j \right\rangle$, where ρ is mass density, the relation between the microscopic and macroscopic stresses of the volume element model reads,

$$\boldsymbol{\Sigma} : \mathbf{D} = \langle \boldsymbol{\sigma} : \mathbf{d} \rangle + \left\langle \frac{1}{2} \rho \frac{d |\mathbf{v}|^2}{dt} \right\rangle \tag{8}$$

The macroscopic plastic dissipation $\Phi(\mathbf{D})$ is defined by

$$\boldsymbol{\Sigma} : \mathbf{D} \le \Phi(\mathbf{D}) = Inf \left(\langle \phi(\mathbf{d}) \rangle + \left\langle \frac{1}{2} \rho \frac{d |\mathbf{v}|^2}{dt} \right\rangle \right) \tag{9}$$

For any velocity field, **v**, satisfying conditions of homogenous boundary strain rate **D**, one can computer the overall dissipation $\Phi(\mathbf{D})$ corresponding to the velocity field considered through numerical integration over the volume *V.* When $\Phi(\mathbf{D})$ is obtained, the macroscopic yield locus can be obtained by the equation $\boldsymbol{\Sigma} = \frac{\partial \Phi}{\partial \mathbf{D}}(\mathbf{D})$.

3. VELOCITY FIELD

Following Gåråjeu [3], the homogenous strain rate tensor **D** on the outside surface of the cell element has the following form

$$\mathbf{D} = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}, \quad D_{11} = D_{22} \tag{10}$$

A two-trail velocity field has been tried in current study.

$$\mathbf{v} = \mathbf{v}^{(1)} + \mathbf{v}^{(2)} \tag{11}$$

Where both $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$ are incompressible fields, $\mathbf{v}^{(1)}$ satisfies a condition of homogeneous boundary strain rate on the outer surface of the cell element with a strain rate tensor T as follow,

$$\mathbf{v}^{(1)} = \mathbf{T}\cdot\mathbf{x} \qquad \mathbf{T} = \begin{bmatrix} \xi & 0 & 0 \\ 0 & \xi & 0 \\ 0 & 0 & \frac{B^2}{A^2}\xi \end{bmatrix} \tag{12}$$

and $\mathbf{v}^{(2)}$ corresponds to a homogeneous strain on the entire domain, as follow

$$\mathbf{v}^{(2)} = (\mathbf{D} - \mathbf{T})\cdot\mathbf{x} \tag{13}$$

where

$$\xi = \frac{D_{11} + D_{22} + D_{33}}{2 + \frac{B^2}{A^2}} \tag{14}$$

Different to Gologau's [4] expansion velocity field, this velocity field **v** only satisfies the homogenous strain rate boundary condition on the outside surface of the cell element.

This velocity field will reduce to the classical incompressible expansion field used by Gurson in the spherical and cylindrical cases when the ellipsoid shape becomes spherical or cylindrical respectively.

4. A DYNAMIC VOID GROWTH MODEL

Based on eqn (8), the plastic dissipation function can be written as a summation of a quasi-static and a dynamic part:

$$\begin{aligned} \Phi(\mathbf{D}) &= \Phi^S(\mathbf{D}) + \Phi^D(\mathbf{D}) \\ &= \int_V (\boldsymbol{\sigma} : \mathbf{d})\,dV + \int_V \frac{1}{2}\rho\,\frac{d\,|\mathbf{v}|^2}{dt}\,dV \end{aligned} \tag{15}$$

Using the field eqn (11) in eqn (15) and transforming to spheroidal coordinates, we obtain

$$\begin{aligned} \Phi^S(\mathbf{D}) &= \int_V (\boldsymbol{\sigma} : \mathbf{d})\,dV \\ &= \frac{\sigma_0}{(m+1)\varepsilon_0^m}\frac{3}{4\pi AB^2}\int_{\lambda_1}^{\lambda_2}\int_0^{\pi}\int_0^{2\pi} \dot{\varepsilon}_e^{m+1} c^3\left(\sinh^2\lambda + \sinh^2\theta\right)\sinh\lambda\sinh\theta\, d\lambda\, d\theta\, d\varphi \end{aligned} \tag{16}$$

$$\Phi^D(\mathbf{D}) = \frac{3}{4\pi AB^2}\int_{\lambda_1}^{\lambda_2}\int_0^{\pi}\int_0^{2\pi}\frac{1}{2}\rho\frac{d|\mathbf{v}|^2}{dt}c^3\left(\sinh^2\lambda + \sinh^2\theta\right)\sinh\lambda\sinh\theta\, d\lambda\, d\theta\, d\varphi \tag{17}$$

where $\dot{\varepsilon}_e$ is a function of velocity field **v**. Following Gåråjeu's work, the estimate of the volume average in eqn (16) reads (Gåråjeu [3]):

$$\Phi^S(\mathbf{D}) \simeq \frac{\sigma_0}{(m+1)\dot{\varepsilon}_0^m}\int_f^1\left(4\bar{\xi}^2 + \frac{4\xi^2}{\eta^2}F_1\right)^{\frac{m+1}{2}} d\eta \tag{18}$$

where ξ and $\bar{\xi}$ are two constants, as follows:

$$\xi = \frac{1}{3 - e_2^2}\left(D_p + D_n\right), \quad \overline{\xi} = \frac{\left(1 - e_2^2\right) D_p - 2 D_n}{2\left(3 - e_2^2\right)}$$
$$D_p = D_{11} + D_{22}, \quad D_n = D_{33} \quad and \quad m = 1/N \tag{19}$$

After a lengthy derivation with the help of Mathematica program the dynamic plastic potential function can be written:

$$\Phi^D(\mathbf{D}) = \rho c^2 \left(FD_1 D_n^3 + FD_2 D_n^2 D_p + FD_3 D_p^3 + FD_4 D_p + FD_5 D_n D_p^2 + FD_6 D_n\right) \tag{20}$$

$$FD_4 = FD_{41}\dot{D}_n + FD_{42}\dot{D}_p \tag{21}$$

$$FD_6 = FD_{61}\dot{D}_n + FD_{62}\dot{D}_p \tag{22}$$

Where FD_i (i=1, 2, 3, 5, 41, 42, 61, 62) are analytical functions of eccentricities of the inner and outer spheroids, e_1 and e_2 (Liu, [1]).

Assuming the void remains spheroidal during the deformation, when the velocity field **v** is specialized to eqn (11), we obtain the following differential equation:

$$\dot{w} = -\frac{2D_{11} + D_{33} + \left[2D_{11} + D_{33} + 6\left(D_{33} + D_{11}\left(-1 + e_2^2\right)\right) f\right]}{\left(-3 + e_2^2\right) f} - \frac{2\left(2D_{11} + D_{33}\right)\left(-1 + e_2^2\right) w^2}{\left(-3 + e_2^2\right) f} \tag{23}$$

With the same procedure, we obtain the evolution equation of parameter a as follow:

$$\dot{a} = \frac{\left(2D_{11} + D_{33}\right) e_1^3 \left(1 - e_2^2\right)^2 + 2\left(1 - e_1^2\right) e_2^3 \left(D_{33} - D_{11}\left(1 - e_2^2\right)\right)}{\left(1 - e_1^2\right) e_2^3 \left(3 - e_2^2\right)} a \tag{24}$$

Based on the incompressibility of the matrix material, the evolution equation of void volume fraction can be derived as follow:

$$\dot{f} = \left(1 - f\right) Tr\left(\mathbf{D}\right) \tag{25}$$

5. DISCUSSIONS AND CONCLUSIONS

The void growth model is compared with the FEM results (Liu, [1]). An example is shown in figure 2. The agreement of the present model with FEM results is quite satisfactory on a wide range of strain rate except that the evolution of void shape can not be predicted precisely.

Compared to other models in the literature, this model not only included the strain rate sensitivity and void shape effect as in some previous work, but also included quantitative terms for inertial effect which related with the void shape and size.

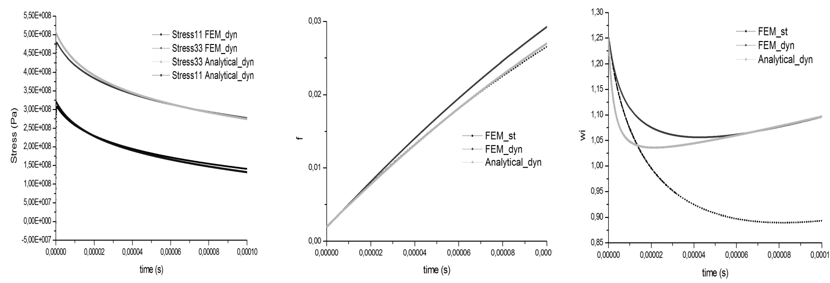

Figure 2. Comparison with FEM results. f_0=0.002, e_1=0.6, D_{11}=-300 (1/*s*), D_{33}=900 (1/s). st: static calculation; dyn: dynamic calculation.

6. REFERENCES

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