NUMERICAL MODELLING OF FORCED VIBRATIONS OF AN ELASTYC CRACKED BODY UNDER UNILATERAL CONTACT WITH A PUNCH

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ABSTRACT
A problem of forced vibrations of a finite elastic cracked body under unilateral contact with a punch is considered. A non-stationary two-layer numerical algorithm for computational solution of the problem based on the Fourier transformation and boundary element method is proposed. Numerical results have been obtained.

1 INTRODUCTION

Problems of forced vibrations of elastic systems with unilateral contact interaction of crack edges and a body with a punch are nonlinear problems. A distinctive feature of the problem is that its statement includes boundary conditions expressed in the form of inequalities. Consequently, capabilities of analytical methods for the considered class of problems are limited to discrete systems with few degrees of freedom and the principal role in solving such problems belongs to numerical methods.

There are two basic approaches to the computational algorithm development for the problems of forced vibrations of elastic systems under unilateral constraints. The first approach is based on reduction of a hyperbolic system of differential equations to a system of ordinary differential equations with respect to the time coordinate by sampling the source problem with respect to the space coordinates using either the finite difference method or the finite element method [1, 2]. The second approach is based on usage of a variety of algorithms to reduce the source contact problem to a sequence of problems with given contact forces, which are solved using the Fourier transform with respect to the time coordinate. The obtained boundary problem in Fourier transform space can be solved by a variety of numerical methods and, in particular, by the boundary element method (BEM). The same approach was used for investigation of diffraction of harmonic waves on a crack taking into account contact interaction of the crack edges [3].

A procedure for a joint usage of the Fourier transformation and BEM for analysis of forced vibrations of finite cracked elastic bodies under unilateral contact with a punch is considered in the present paper.

2 FORMULATION OF A PROBLEM

Let a homogeneous isotropic body occupies a finite area Ω with the border Γ and has a crack in the form of the open two-way Lyapunov curve Γ_t. Under $u_i(x, t)$, $\varepsilon_{ij}(x, t)$, $\sigma_{ij}(x, t)$ we understand the components of the displacement vector, the deformations and stresses tensors respectively in the $x \in \Omega$ point at the time moment t. Let us consider that the displacements and deformations are small, the stresses in initial undeformed condition and volume forces are absent. The spreading velocities of longitudal and transverse waves in the area Ω are marked as C_1 and C_2 respectively.

The body surface Γ consists of three uncrossed parts $\Gamma = \Gamma_u \cup \Gamma_q \cup \Gamma_p$. The displacements $\vec{g}(x, t)$ are assigned on part Γ_u; the efforts $\vec{q}(x, t)$ are assigned on part Γ_q. The collection of largest possible contact zones of the body Ω with a punch is marked as Γ_p. The punch shape and position are described by the function $\Phi(x, t)$, the function value in $x \in \Gamma_p$ point is a distance measured along external normal direction $\vec{\nu}(x)$ of the body Ω at the time moment t from this point to the punch surface. The distance $\Phi(x, t)$ is calculated relatively to undeformed condition of the body Ω.

Contact interaction of the elastic body Ω with the punch is described by linearized conditions of ideal unilateral contact:

$$\sigma_\nu(x, t) \le 0, \quad \sigma_\tau(x, t) = 0, \quad u_\nu(x, t) \le \Phi(x, t),$$

$$\sigma_\nu(x, t)[u_\nu(x, t) - \Phi(x, t)] = 0, \quad x \in \Gamma_p, \tag{1}$$

and contact interaction of the crack edges Γ_t is described by conditions:

$$\sigma_\nu^+(x,t) = \sigma_\nu^-(x,t) \le 0, \qquad u_\nu^+(x,t) + u_\nu^-(x,t) \le \delta_\nu(x),$$

$$\sigma_\tau^+(x,t) = \sigma_\tau^-(x,t) = 0, \qquad \sigma_\nu^+[u_\nu^+(x,t) + u_\nu^-(x,t) - \delta_\nu(x)] = 0, \qquad \forall x \in \Gamma_t, \tag{2}$$

where σ_ν, σ_τ – are normal and tangent components of the surface stresses vector; u_ν – is a normal component of the displacements vector; δ_ν – is an initial crack opening; signs “+”and “–“ are pertained to displacements and stresses of opposite crack edges.

Functions $\vec{g}(x, t)$, $\vec{q}(x, t)$ и $\Phi(x, t)$, which describes the external influence on the body Ω, are T-periodic functions:

$$\vec{g}(x, t) = \vec{g}(x, t+T); \quad \vec{q}(x, t) = \vec{q}(x, t+T); \quad \Phi(x, t) = \Phi(x, t+T).$$

The problem is to determine displacement $u_i(x, t)$, deformations $\varepsilon_{ij}(x, t)$ and stresses $\sigma_{ij}(x, t)$ that satisfy the motion equation

$$\left(C_1^2 - C_2^2\right) u_{i,ij} + C_2^2 u_{j,ii} = \ddot{u}_j + F_c, \tag{3}$$

as well as the correlation of Cauchy and Hooke’s law and periodicity conditions

$$\vec{u}(x, t) = \vec{u}(x, t+T), \quad \frac{\partial \vec{u}(x, t)}{\partial t} = \frac{\partial \vec{u}(x, t+T)}{\partial t}, \quad \forall x \in \Omega \tag{4}$$

and boundary conditions on the surface Γ and crack edges Γ_t.

There are a number of models that describe the fading vibrations process of mechanic systems. However, in most cases internal resistance, especially internal friction in materials, has the dominant role. There are two approaches used to describe phenomenological standpoint of internal friction forces in materials under cyclic deforming. The first of them is based on a viscous damper concept and consists of a usage of models of viscoelastic bodies of Kelvin, Maxwell etc. The second approach is based on a usage of nonlinear dependencies between components of stresses and displacements tensors that reflect the loop hysteresis presence under cyclic deforming. We dispensed with rendering loop hysteresis form upon investigation unresonance modes viscosity and used to describe diffusing energy so-called conditional-viscous scheme in accordance with value internal resistance forces F_c was proportional to velocity $\dot{u}$

$$F_c = c\dot{u},$$

where c – was viscous damping coefficient.

3 SOLUTION METHOD

Direct application of the Fourier transformation to the problem (1)-(4) is not possible because of the nonlinear boundary conditions of unilateral contact (1)-(2). A special iterational algorithm is used for this problem solution which allows reduction of the problem with unilateral contact to a sequence of problems with contact stresses $p_p(x,t)$ and $p_t(x,t)$ given on a possible contact surface and crack edges correspondingly. The two-layer iterational algorithm with a variable parameter was suggested for definition of unknown contact pressure $p_p(x,t)$ and $p_t(x,t)$ value:

$$p_p^{n+1}(x,t) = (p_p^n(x,t) - \rho^n(u_\nu^n(x,t) - \Phi(x,t)))\mathrm{H}(p_p^n(x,t) - \rho^n(u_\nu^n(x,t) - \Phi(x,t))), \quad (5)$$

$$p_t^{n+1}(x,t) = (p_t^n(x,t) - \rho^n(u_\nu^{n+}(x,t) + u_\nu^{n-}(x,t) - \delta_\nu(x)))\mathrm{H}(p_t^n(x,t) - \\ - \rho^n\left(u_\nu^{n+}(x,\ t) + u_\nu^{n-}(x,\ t) - \delta_\nu(x)\right), \quad (6)$$

where $p^n(x,t)$, $p^{n+1}(x,t)$ – are contact pressure on n and (n+1) iterations, $u_\nu^n(x,t)$ – are contact displacements, $\Phi(x,t)$ – is a function that describes the punch shape and position, $\mathrm{H}(\cdot)$ – is the Heaviside function, ρ^n – is a variable parameter of the iterational process. Let approach $p^n(x,t)$ and value ρ^n are defined. Two following approach are calculated by scheme (5)-(6) in supposing to $\rho^{n+2} = \rho^{n+1} = \rho^n$. Hereinafter define

$$\delta_{k+1} = p^{n+1}(x^*,t^*) - p^n(x^*,t^*),$$

where (x^*,t^*) – is a point, where normal displacement components $u^n(x,t)$ reach the most absolute value meaning. If following condition is satisfied

$$\delta_{k+1}\delta_{k+2} \geq -\alpha\delta_{k+1}^2, \quad (7)$$

where $0 < \alpha < 1$ – is an iteration process compression coefficient, then next two steps are successful and iterational process continues. If the left part of inequality (7) is positive then the parameter ρ^n can be increased assuming $\rho^{n+4} = \rho^{n+3} = \beta\rho^n$, where $\beta > 1$ - is an increase step coefficient of the iteration process. If the condition (7) is not satisfied then it is necessary to repeat the calculation of (n+1) and (n+2) approaches assuming $\rho^{n+2} = \rho^{n+1} = \gamma\rho^n$, where $\gamma < 1$ – is a reduction step coefficient of the iteration process.
The condition

$$\frac{1}{\rho^n} \max_{x \in \Gamma_p} \left| p^{n+2}(x,t) - p^{n+1}(x,t) \right| \le \varepsilon \max_{x \in \Gamma_p} \left| p^{n+2}(x,t) \right|,$$

is used as a completion of iterational process, where ε – is a parameter, that characterize inaccuracy of the approximate solution. The left part of the inequality is the largest value of two consequent approach differences and does not depend on parameter value ρ^n.

Hereinafter the Fourier transform with respect to the time coordinate and together with the BEM was used for solution of the problem sequence with given contact efforts. Fundamental singularity solutions of motion equations in Fourier transform space were used to construct a system of boundary integral equations (BIE) [4]. Two methods of BIE system construction, one corresponding to direct and the other one to indirect BEM formulation were considered. The crack is prototyped by double layer potential in both cases.
The BEM sampling equation by a collocation method was used for numerical solution of the BIE systems. One-knot elements with constant approximation of unknown functions were used as boundary elements. The linear algebraic equations system obtained as a result of sampling was solved by the Gauss method.
Boundary potentials kernels in Fourier transform space are presented by complex correlations containing cylindrical functions. Thus, the regular integrals were calculated numerically with the aid of squaring Gauss formulae during sampling problem while the integrals having singularity were calculated analytically. The integrals with a weak logarithmic type singularity were calculated as improper integrals, the integrals with a strong singularity were defined in the sense of the main value by Cauchy and hypersingularity integrals were understood at present work as the finite part by Hadamard.

4 NUMERICAL REALIZATION

The designed calculating algorithm was realized as a software package. Numerical solutions of a number of problems were obtained and convergence of the calculating algorithm with vibration frequencies close to resonance was investigated. It was determined that contact interaction of crack edges and body under a punch significantly influenced the stress intensity factors (SIF). SIF were calculated utilizing direct methods. Maximum SIF value greatly changes in the case of contact interaction. The influence of damping on the solution behavior was investigated. It was determined that damping substantially changes the problem solution in areas close to resonance domains.

5 REFERENCES

1. Bazenov V. A., Gotsylyak E. A., Kondakov G. S., Ogloblya A. I. Stability and vibrations of deformed systems with unilateral constraints. Kiev 1989. 400 p (in Russian)
2. Bobylov A. A., Suturin E. L. Application of stabilization method forresearch of forced vibrations of elastic body under unilateral contact with punch // Bulletin of Donetsk University. Series A. Natural sciences. 2002. N 2. P.122–126 (in Russian)
3. Guz A. N., Zozulya V. V. Fracture dynamics with allowance for a crack edges contact interaction // Int. Jour. of Nonlin. Science and Numerical Simulation. 2001. 2, N3. P.173-233.
4. Cruse T.A., Rizzo F.J. A direct formulation and numerical solution of the general transient elastodynamic problem // J. Math. Anal. Appl. 1968. 22, N1. P. 244-259.