PHYSICAL MEANING OF THE FICTITIOUS CRACK OPENING DISPLACEMENT IN DUGDALE MODEL

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ABSTRACT

Dugdale model is a very useful modelling method in order to investigate crack opening displacements even though the cracked body does not satisfy the small scale yielding condition. The fictitious crack for Dugdale model, which corresponds to plastic region ahead of a crack, has a certain opening displacement although this part is continuum medium in an actual condition.

Physical meaning of the crack opening displacement of a fictitious crack in Dugdale model is investigated by comparing with elastic-plastic finite element analyses. Center cracked tensile specimen with various crack lengths is turned into an object of the analysis. Crack opening profiles including a fictitious crack and deformations ahead of a crack tip are calculated by applying the Dugdale model. The corresponding finite element analyses are also performed.

It is confirmed that the fictitious crack opening displacement in Dugdale model has close association with the integration value of plastic strain for the vertical direction to a crack line under elastic stress with tensile yield stress level. That is, the fictitious crack corresponds to the plastic deformation condensed over a crack line and is interpreted as the source of plastic deformation ahead of a crack tip.

1 INTRODUCTION

Cohesive force models are proposed in order to describe the deformation of cracked bodies in case that the linear fracture mechanics cannot describe the deformation of them quantitatively. The plastic zone ahead of a crack tip is condensed over a crack line and this condensed plastic zone is treated as the fictitious crack in these models. The cohesive force works between fictitious crack surfaces. Cohesive force models are established by applying the principle of superposition to the cracked bodies with a fictitious crack subjected to the cohesive force. Dugdale model [1] and Barenblatt model [2] for mode I crack, BCSS model [3] for mode III crack are proposed as cohesive force models. Dugdale model is widely used as a cohesive force model for its simplicity, because the cohesive force is constant (the yield stress of materials) over fictitious crack surfaces.

Cohesive force models give the ambiguous value, which is the crack opening displacement of a fictitious crack. Because the fictitious crack region correpsonds to the plastic zone, no crack opening occurs in an actual state. The appearance of fictitious crack opening is derived from the modelling of plastic zone as the "crack". It is, however, expected that the crack opening displacement in a fictitious crack has a certain physical meaning related with the behaviour of the plastic zone in the actual cracked bodies, because the platic zone length and the crack opening displacement for actual crack part are estimated with good precision by applying the Dugdale model.

Physical meaning of the crack opening displacement of a fictitious crack in the Dugdale model is investigated in this study. Crack opening profiles including the fictitious crack of center cracked tensile specimens with various crack lengths and deformation ahead of a crack tip are calculated by applying the Dugdale model. Elastic-plastic finite element analyses are also performed. By comparing both numerical calculation results, the physical meaning of crack opening displacement of a fictitious crack is investigated.

2 PHYSICAL MEANING OF THE FICTITIOUS CRACK OPENING DISPLACEMENT

Dugdale model contains the discrepancy, in other words, crack opening displacements exist in a fictitious crack region although this region is the continuum medium in an actual condition. It is interpreted that the continuum medium in plastic zone is removed from cracked bodies in case of applying the Dugdale model although cohesive force works between their surfaces. However, the crack opening profile by applying the Dugdale model is good agreement with that by finite element analyses except in the vicinity of a crack tip. Besides, the length of plastic zones by both numerical calculations are the almost same value. The results implies that the fictitious crack opening displacement in Dugdale model has a certain physical meaning.

Now, let us consider the situation of continuum medium removed from the cracked body described by Dugdale model. The stress state of medium in plastic zone is plastic flow because the constitutive relation applied to the Dugdale model is the perfect elastic-plastic. The free body made from this medium enables to extend and to shrink freely if the surrounding region under elastic condition does not exist. However, the plastic zone ahead of a crack tip in the actual cracked body cannot deform freely because the elastic body surrouds this region.

Considering above mentioned situations occuring near a crack tip, it is expected that the crack opening displacement of a fictitious crack satisfies the following equation.

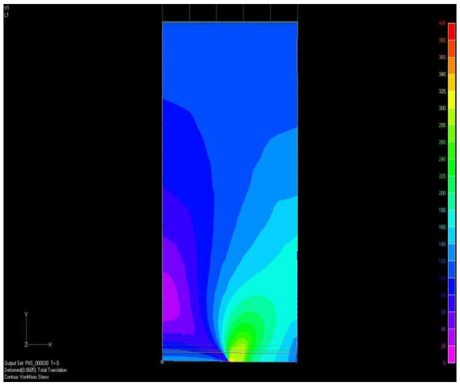
$$V(x) = (1 + \sigma_Y/E) \int_{pz} \varepsilon_y^p(x) dy, \tag{1}$$

where,

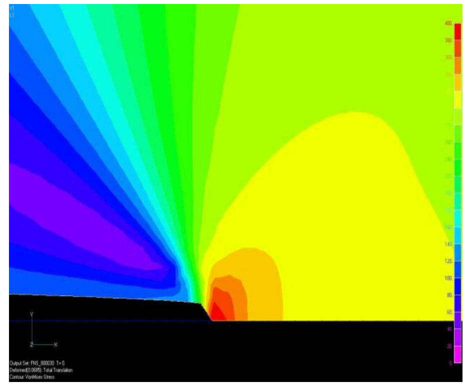
$V(x)$	:	crack opening displacement in a fictitious crack at x,
σ_Y	:	yield stress of materials,
E	:	Young's modulus,
pz	:	plastic strain existing range for y direction at x,
$\varepsilon_y^p(x)$	:	plastic strain (y component) at x,
x	:	axis defined over a crack line,
y	:	axis perpendicular to x axis,

origin of the coordinate system for eqn (1) is set to the center of specimen. In case of plane strain condition, E in eqn (1) is replaced by $E' = E/(1-\nu^2)$, where ν is Poisson's ratio.

Eqn (1) implies that the crack opening displacement of a fictitious crack region corresponds to the source of inherent deformation and is equivalent to the "source" in fluid mechanics.

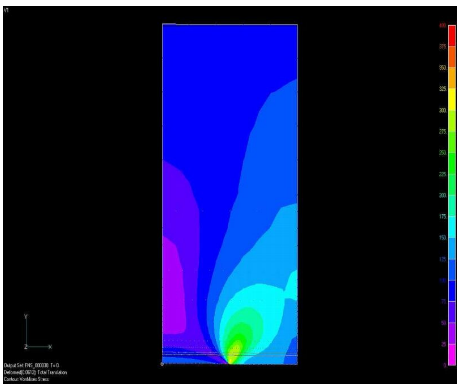


(a) Whole of FE model
(1/4 of the specimen)



(b) Near a crack tip

Figure 1: Stress distribution around a crack tip (von Mises equivalent stress, $a/W = 0.5$, $\sigma_{net}/\sigma_Y = 0.5$, plane stress condition)



(a) Whole of FE model
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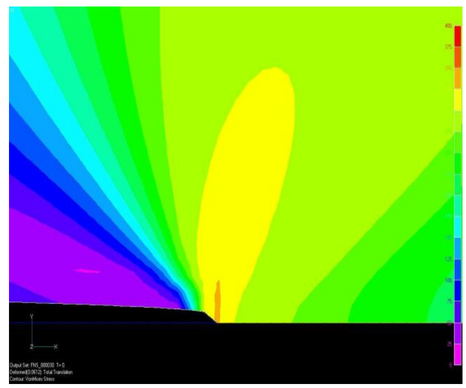

(b) Near a crack tip

Figure 2: Stress distribution around a crack tip (von Mises equivalent stress, $a/W = 0.5$, $\sigma_{net}/\sigma_Y = 0.5$, plane strain condition)

3 OUTLINE OF FINITE ELEMENT ANALYSES

Elastic-plastic finite element analyses for a center cracked specimen with three crack lengths are performed in order to investigate the deformation ahead of a crack tip. The ratio of crack length (a) to plate width (W) are 0.3, 0.5 and 0.7. The constitutive relation of material is identifed as bi-linear stress-strain curve with a second modulus of $E/100$. The value of E is equal to 206 GPa, σ_Y is equal to 294 MPa. Supposed material constants (E and σ_Y) and the constitutive relation are regard as the value of mild steel.

Four node isoparametric element type is applied to finite element modellings. The minimum mesh size of finite element model is 0.1 mm $\times$ 0.1 mm and this size meshes are placed along a crack line. Finite element analyses are performed under both plane stress and plane

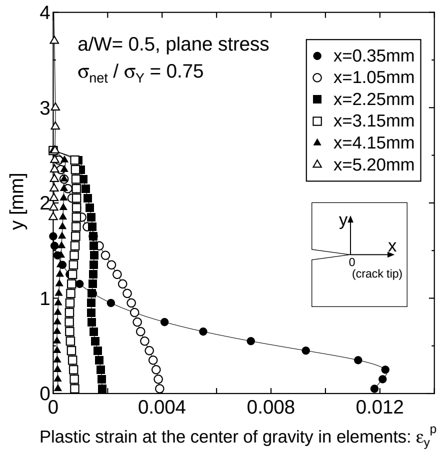

Figure 3: Plastic strain ($\varepsilon_y^p(x)$) distributions ($a/W = 0.5$, $\sigma_{net}/\sigma_Y = 0.75$, plane stress condition)

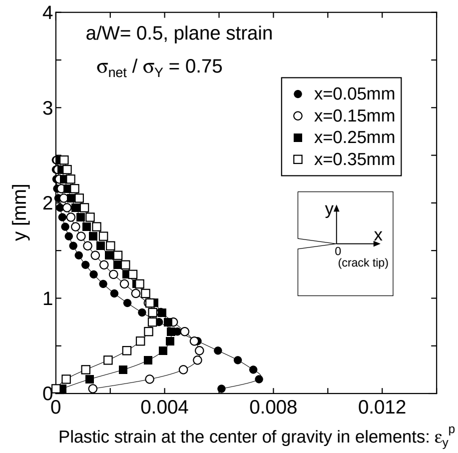

Figure 4: Plastic strain ($\varepsilon_y^p(x)$) distributions ($a/W = 0.5$, $\sigma_{net}/\sigma_Y = 0.75$, plane strain condition)

strain conditions.

Figures 1 and 2 are examples of the distribution of von Mises equivalent stress near a crack tip in both plane stress and plane strain conditions.

Figures 3 and 4 are examples of plastic strain distribution ($\varepsilon_y^p(x)$) at representative locations x. The maximum plastic strain points in these figures appear not only over a crack line but also at some distance from a crack line.

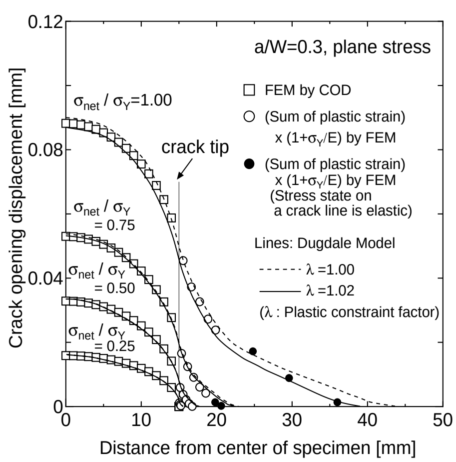

Figure 5: COD profiles ($a/W = 0.3$, plane stress condition)

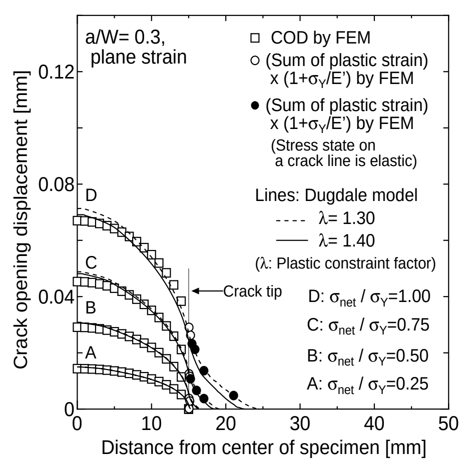

Figure 6: COD profiles ($a/W = 0.3$, plane strain condition)

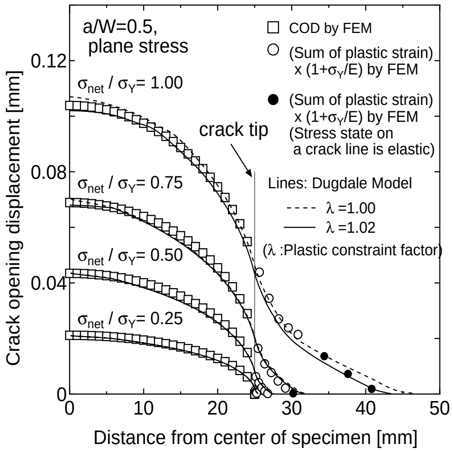

Figure 7: COD profiles ($a/W = 0.5$, plane stress condition)

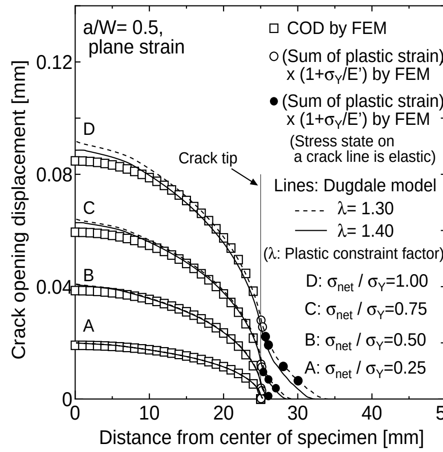

Figure 8: COD profiles ($a/W = 0.5$, plane strain condition)

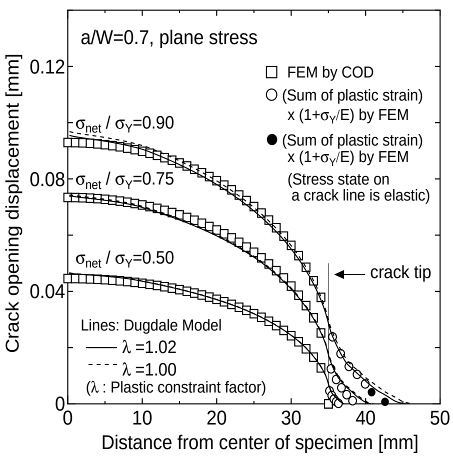

Figure 9: COD profiles ($a/W = 0.7$, plane stress condition)

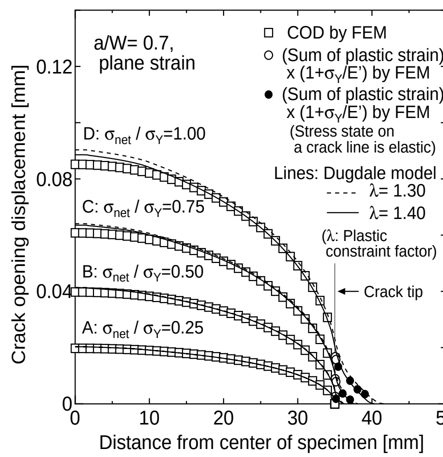

Figure 10: COD profiles ($a/W = 0.7$, plane strain condition)

4 RESULTS AND DISCUSSIONS

To confirm the validity of eqn (1), numerical calculation results by finite element analyses and by the Dugdale model are compared. Figures 5 $\sim$ 10 are the crack opening displacements by both calculations. $\sigma_{\rm net}$ in these figures means the applied net stress.

It is well known that numerical calculation results by the Dugdale model give rather large crack opening displacements. For adjusting crack opening displacements by the Dugdale model, the plastic constraint factor (λ in Figures 5 $\sim$ 10) is introduced into the Dugdale model, i.e. σ_Y is replaced by $\lambda\sigma_Y$. Figures 5 $\sim$ 10 indicate that the suitable value of plastic constraint factor in plane stress condition is 1.02 and in plane strain condition is 1.30

respectively.

Open and solid cricles in Figures 5 $\sim$ 10 is the “fictitious ” crack opening displacements defined by eqn (1). $\varepsilon_y^p(x)$ in eqn (1) is obtaind by finite element analyses. The difference between open and solid marks means the stress state on a crack line ($y = 0$). Open marks region corresponds to the plastic on a crack line. Solid marks is introduced in order to highlight that the stress state on a crack line is elastic even though the condition at some distance from a crack line shows plastic.

Fictitious crack opening displacements by finite element analyses are good agreement with the calculation results by the Dugdale model with suitable plastic constraint factors both plane stress and plane strain conditions. Considering that these plastic constraint factors are identified by the comparison of crack opening displacements in an actual crack region, it is concluded that the hypothesis of eqn (1) is reasonable.

5 CONCLUDING REMARKS

The fictitious crack for Dugdale model, which means plastic region ahead of a crack, has a certain opening displacement although this part is the continuum medium. The physical meaning of crack opening displacement of a fictitious crack in the Dugdale model is investigated by comparing with the results by elastic-plastic finite element analyses.

It is confirmed that the fictitious crack opening displacement in Dugdale model has close association with the integration value of plastic strain along the vertical direction to a crack line under elastic stress with tensile yield stress level. That is, the fictitious crack is interpreted as the source of plastic deformation ahead of a crack tip, which is represented by eqn (1).

ACKNOWLEDGEMENTS

This paper owes much to the helpful support of Mr. Yoshiaki Tabuchi, graduated student of Naval Architecture and Marine Systems Engineering course, Kyushu university for his graduation thesis.

This research fund was Grant-in-Aid for Scientific Research (B)(2)(No.16360440) by Japan Society for the Promotion of Science.

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